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A Multi-Cone X-Ray Imaging Bragg Crystal Spectrometer

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A multi-cone x-ray imaging Bragg crystal spectrometer[a)]

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This article describes a new x-ray imaging Bragg crystal spectrometer, which – in combination with a streak camera or a gated strip detector – can be used for time-resolved measurements of x-ray line spectra at the National Ignition Facility (NIF) and other high power laser facilities. The main advantage of this instrument is that it produces perfect images of a point source for each wavelength in a selectable spectral range and that the detector plane can be perpendicular to the crystal surface or inclined by an arbitrary angle with respect to the crystal surface. These unique imaging properties are obtained by bending the x-ray diffracting crystal into a certain shape, which is generated by arranging multiple cones with different aperture angles on a common nodal line. All files MUST be submitted through the online system at: http://rsi-htpd.peerx-press.org/

I. INTRODUCTION

At the National Ignition Facility (NIF) and other high power laser facilities, time-resolved measurements of the x-ray line emission from the, there produced, small (point-like) high energy density plasmas are commonly performed with Hall's conical x-ray crystal spectrograph,[1,2] in combination with a fast streak camera or a gated strip detector. Hall's spectrometer design, which is depicted in Figs. 2 and 3 of ref. 2, was adapted to the experimental constraints at those high power laser facilities, which provide only limited diagnostic access to the target due to the arrangement of the laser beams; and, therefore, it has two unusual features: The orientation of the detector plane is perpendicular to a nodal line on the conical crystal surface, and the x-ray source is not located on the cone's axis, so that the ray pattern lacks rotational symmetry, which is a basic requirement for imaging. Since in ref. [1], Hall described his spectrometer as a modification of the von Hamos spectrometer,[3] we point out that contrary to his design, the cylindrical von Hamos geometry satisfies this basic requirement for imaging. A consequence of the absence of rotational symmetry in Hall's design is that rays, which are incident, with the same Bragg angle, at different points on the crystal surface are NOT reflected to the same point in the detector plane but to different points, whose distribution depends on the Bragg angle and the crystal size. The spectral resolution of Hall's spectrometer is therefore ill defined, even though the distribution of intersection points of the reflected rays with the detector plane can be peaked for a small range of Bragg angles and a small crystal size.

The here proposed multi-cone Bragg crystal spectrometer overcomes these shortcomings of the Hall geometry and satisfies the condition for imaging by arranging the crystal points on many (ideally, an infinite number of) cone surfaces, whose axes pass through the x-ray point source. A rotational symmetry of the ray pattern is thus guaranteed for each Bragg angle, so that perfect images of the point source are produced for each wavelength in an arbitrarily large spectral range. These image points fall on a straight line - or a narrow slit of a streak camera - in a vertical detector plane.

The paper is organized as follows: Section II presents the equations for a single-cone geometry with a vertical detector plane. Section III describes the geometry of a multi-cone structure. Section IV gives instructions for the fabrication of a multi-cone crystal substrate. Plans for experimental tests of multi-cone structures with visible light and x-rays are discussed in Section V; and conclusions are presented in section VI.

II. EQUATIONS FOR A SINGLE-CONE CRYSTAL

Figure 1 illustrates the x-ray imaging properties of a 'single-cone' crystal with a half-aperture angle, α, for the case, where the cone's axis passes through the x-ray point source, **S**. The x-axis is along a nodal line on the cone's surface; and the point source, **S**, is at a distance **z = D** from this nodal line. The point **S*** is the mirror image of **S** with respect to the x-axis. The detector plane is parallel to the z-axis and defined by the equation **x = L**.

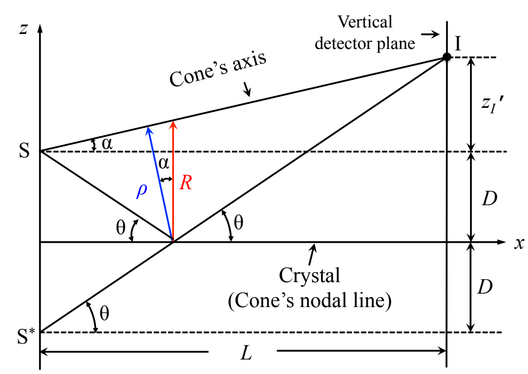

FIG 1: X-ray imaging properties of a single-cone crystal

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Since the point source **S** is on the cone's axis and since this axis is an axis of rotational symmetry, the images **I** of the source **S** that are obtained for any Bragg angle must also be on this axis. And, evidently, there can be only one Bragg angle **Θ**, for which the image, **I**, is simultaneously on the cone's axis and the detector plane – see Fig. 1. In the remainder of this section, we derive the equations, which apply to this special situation.

Considering the incident and reflected rays in Fig. 1, we find the following two relations for the Bragg angle Θ:

(1) $$\tan(\theta) = \frac{D}{x}$$

(2) $$\tan(\theta) = \frac{2D + z_I^{'}}{L};$$

Equation (1) expresses a unique correspondence between **Θ** and **x**, where **x** is the x-coordinate of the point of incidence on the crystal's nodal line, that is established by the incident ray, if the source is at (x=0, y=0, z=D). Equation (2), on the other hand, is a relation between **Θ** and the z-coordinate, $z_I = D + z_I^{'}$, of the image point, **I**, that must be satisfied by the reflected ray, if the image is to be on the vertical detector plane, **x** = **L**.

By equating the RHS of equations (1) and (2), we find the relations, which express the cone's half-aperture angle **α** and z-coordinate of the image point, **I**, as functions of **x**:

(3) $$\frac{D}{x} = \frac{2D + z_I^{'}}{L}$$ or

(3') $$\frac{z_I^{'}}{L} = \frac{D}{L}\left(\frac{1}{\frac{x}{L}} - 2\right) = \tan(\alpha)$$

Using equation (3'), we can also express **R** and ρ as functions of **x**:

(4) $$R = D + x\tan(\alpha) = D + x\frac{D}{L}\left(\frac{1}{\frac{x}{L}} - 2\right)$$ or

(4') $$\frac{R}{L} = \frac{2D}{L}\left(1 - \frac{x}{L}\right)$$

(5) $$\frac{\rho}{L} = \frac{R}{L}\cos(\alpha)$$

where, **R** is the length of a line segment on the bisectrix of the angle between the incident and reflected rays that extends from a position **x** on the cone's nodal line to the cone's axis; and ρ is the cone's radius at the position **x**.

III. A MULTI-CONE CRYSTAL STRUCTURE

The equations (1) through (5) for a single-cone crystal determine the values of **Θ**, **x**, **R**, and ρ for the special case, where the image point **I** is on a vertical detector plane, if the values of **α**, **D,** and **L** are given as input parameters. Conversely, we can also choose **x**, **D,** and **L** as input parameters and solve these equations for **Θ**, **α**, **R**, and ρ, so that **α** becomes a dependent variable of **x**.

Proceeding in this way, we obtain a multi-cone structure, where a cone with a certain half-aperture angle **α** is assigned to each x-value, where each cone axis passes through the source point **S**, where all the cones have a common nodal line along the x-axis, and where the image points **I**, which are associated with each **x**-value or **Θ**-value, fall on a straight line in the vertical detector plane. The geometry of such a multi-cone structure is shown in Fig. 2.

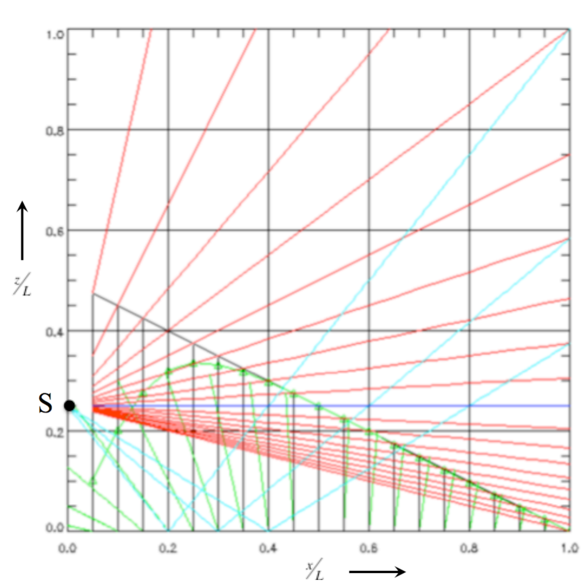

FIG 2: Geometry of a multi-cone structure for D/L = 0.25

Figure 2 was created by calculating the values of **Θ**, **α**, **R**, and ρ for 20 x-values. The red lines, whose extensions pass through the point source **S**, are the cones' axes. The vertical black lines**,** which extend from the x-axis to the cone axes and the tilted black line, given by equation (4'), represent **R**. The green curve and green symbols represent the value (not the direction) of ρ, which is given by equation (5), whereas the tilted green lines, which connect each **x**-value with its associated cone axis represent both, the value and direction of ρ. Also shown are the incident and reflected rays (blue lines) for a few **x**-values.

IV. FABRICATION OF MULTI-CONE SUBSTRATES

A multi-cone crystal will be produced by attaching a thin (~100 μ thick) crystal to a multi-cone substrate. - If the surfaces of crystal and substrate are sufficiently smooth and polished, this can be done by using the molecular contact forces between crystal and substrate; otherwise one may have to use glue. - A multi-cone substrate can be produced by a 3D printer or a programmable milling machine from a CAD-file, which can be generated by following a 'two-steps'-procedure:

Step 1: We draw semicircles with the radii $\rho_i = \rho(x_i)$, in the vertical planes, $\mathbf{x} = \mathbf{x_i}$, which are parallel to the y,z-plane, about the points, which are marked by the green symbols in FIG. 2. These circles are given by the equation:

$$(z-\rho_i)^2 + y^2 = \rho_i^2 \quad (\text{where } -\rho_i \le y \le \rho_i) \tag{6}$$

Solving equation (6) for **z**, yields:

$$z = \rho_i - \sqrt{\rho_i^2 - y^2} \tag{7}$$

The points on those vertical semicircles are, therefore given by the coordinate vectors

$$\vec{r}_{vc} = [x_i, y, z] = \left[x_i, y, \rho_i - \sqrt{\rho_i^2 - y^2}\right] \tag{7}$$

Step 2: The vertical semicircles, obtained in *step 1*, are tilted by the angle $\alpha_i = \alpha(x_i)$. - The points of these tilted semicircles are described by the coordinate vectors

$$\vec{r}_{tc} = \left[x_i - z\sin(\alpha_i), y, z\cos(\alpha_i)\right] \tag{8}$$

where z is given by equation (7).

The vertical and tilted semicircles – which look like the ribs of a boat – are shown in FIG. 3 and FIG. 4, respectively.

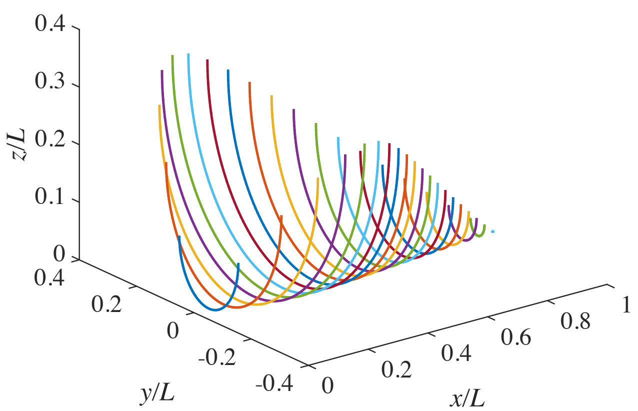

FIG. 3: Vertical semicircles from *Step_1*

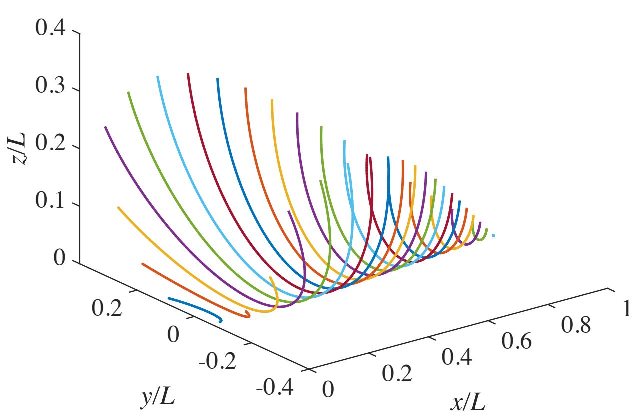

FIG. 4: Tilted semicircles from *Step_2*

V. TESTS OF MULTI-CONE SUBSTRATES

Tests with visible light and x-ray of the imaging properties of single- and multi-cone substrates and crystals will be carried out as a 2016 summer-student project at the Princeton Plasma Physics Laboratory. For the optical tests, we plan to use metal substrates with a reflecting Al-coating. These substrates will be produced by a programmable milling machine or high-precision 3D printers. The substrates' parameters will be close to those of single-cone crystals for the two Hall spectrometers with (**L=116 cm** and **D = 12 cm**), which are presently being designed for the National Ignition Facility (NIF). However, our test substrates will have an area of 12 cm x 12 cm and will be much larger than the crystals, with an area of 3 cm x 3.5 cm, of these two Hall spectrometers, in order to study the imaging properties for a large range of Bragg angles and investigate the effects of the substrate's size. We hope to demonstrate that the imaging properties of the multi-cone substrate will not be affected by the substrate's size and it will also produce perfect images for a large Bragg angle and that, on the other hand, this will not be the case for a single-cone substrate in a Hall configuration.

Comments for the referee: *We already have results from a first, still very crude, test of a multi-cone 220 crystal with visible light. These results look quite promising and will be presented in our poster at this conference. However, since we expect to have more accurate results soon (presumably by August 2016), we do not want to publish the present, still crude, results in this article.*

VI. CONCLUSIONS

As Hall's single-cone crystal spectrometer, the here proposed multi-cone spectrometer is well adapted to the constraints at high-power laser facilities, since it makes use of the same experimental arrangement. However, in contrast to Hall's spectrometer, the multi-cone spectrometer maintains rotational symmetry of the ray pattern for each Bragg angle, so that - for each wavelength in an arbitrarily large spectral range - perfect images of a point source can be produced along the slit of a streak camera that is perpendicular to nodal line of the crystal. And since the imaging properties are independent of the crystal size, a multi-cone spectrometer is capable of providing spectra with a high spectral resolution and a high photon throughput

ACKNOWLEDGMENTS

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