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Ion Cyclotron Emission Studies: Retrospects and Prospects*

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Abstract

Ion Cyclotron Emission (or ICE) studies emerged in part from the papers by A.B. Mikhailovskii in $'70s$. Among the discussed subjects were electromagnetic compressional Alfvénic cyclotron instabilities with the linear growth rate $\sim \sqrt{n_\alpha/n_e}$ driven by fusion products, α-particles which draw a lot of attention to energetic paticle physics. The theory of ICE excited by energetic particles was significantly advanced at the end of 20^{th} century motivated by first DT experiments on TFTR and subsequent JET experimental studies which we highlight. More recently ICE theory was advanced by detailed theoretical and experimental studies on ST (or spherical torus) fusion devices where the instability signals previously indistinguishable in high aspect ratio tokamaks due to high toroidal magnetic field became the subjects of experiments. We discuss further prospects of ICE theory applications for future burning plasma (BP) experiments such as those to be conducted in ITER device in France where neutron and gamma rays escaping the plasma create extremely challenging conditions fuison alpha particle diagnostics.

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I. INTRODUCTION

The energetic particle physics research can be traced back several decades to publications on cyclotron excitation of Alfvénic instabilities by fusion alpha particles [1] and on thermonuclear 'drift' instabilities, i.e., those which are caused by the spatial inhomogeneity of alphas [2]. In those and later studies the fusion charged products driven instabilities in general were dubbed as thermonuclear instabilities.

The cyclotron thermonuclear instabilities represent unique excitations due to the velocity space gradient of the ion distribution function with direct access to the energy of fast ions. Typically the excitation of such instabilities is treated theoretically in a perturbative manner. This is justified by the relatively small density of energetic particles (EP) of fusion plasmas in general [3],

$$n_\alpha/n_e \ll 1, \tag{1}$$

and expected linear dependence of the cyclotron instability growth rate on EP density. However, original studies indicated that the instability's dependence on growth rates $\sim \sqrt{n_\alpha/n_e}$ was predicted for the cyclotron excitation in homogeneous plasmas [4, 5] (see Sec.II for introduction of the strong instability theory). This stimulated a lot of interest due to potentially strong growth and the dangerous effects on the plasma, since even small density EP population may have deleterious consequences for the plasma discharge, as we discuss below.

A serious interest was expressed theoretically and experimentally to the problem of cyclotron instabilities, which was motivated by observations of the Ion Cyclotron Emission (or ICE) in tokamaks. ICE occurs when the magnetic field pick up coils measure the signal at integer harmonics of thermal ion cyclotron frequency at the low field side of the tokamak. ICE was commonly acknowledged as driven by the super-thermal fast ions such as beam ions, ICRH minority ions, or fusion charged products. Perhaps the most convincing argument in support of ICE excitation by fast ions was presented by Cottrell et al. [6] where the author demonstrated that the intensity of ICE spectrum growth was linearly proportional to the fusion product density for Ohmic and beam-heated discharges over six orders of magnitudes of ICE (and fast ion) power. Similar correlation was reported by JT-60U, TFTR and LHD groups. In more recent LHD experiments, NBI was applied perpendicularly to the direction of the equilibrium magnetic field. In LHD ICE was timed with TAE excitations [7]. It is

Cottrell's publication [6] that brought the attention of the community to the problem of ICE excitation wheras ICE theory was already available to great extend by that time (see review [8]).

Nevertheless in TFTR DT plasmas the correlation between ICE and neutron signal did not have such linear proportionality. It had more complicated dependences which is not clearly understood as was pointed out in Ref.[9]. In Sec.II we review the growth rate nonlinear dependence on the fast ion density coming from the strong instability theory $\gamma \propto \sqrt{n_\alpha/n_e}$ and point out that ICE growth rate could have other power dependences.

At the beginning of the development of ICE theory, the analysis of the experimentally observed instabilities was done in the homogeneous limit ignoring the mode structure, i.e. assuming the oscillations have CA polarization such as dominantly parallel magnetic field perturbation. It was also assumed that the perturbations were due to the plasma compressibility and thus were coupled to the shear Alfvén branch. The EP/wave interactions were considered with realistic drift ion motion in the presence of CA waves [10] (see also recent publications on those interactions in Refs.[11, 12]). Nevertheless, this simplified homogeneous plasma limit allowed one to make the case for ICE diagnostic applications in ITER burning plasmas [6, 13] (see also recent review with the summary and current understanding of ICE [3]).

Important further developments occurred in the early 2000s in connection with ST (or spherical torus) fusion experiments, where both high and ion subcyclotron frequency instabilities of certain class cavity modes were observed. In particular in NSTX they were identified as high frequency compressional Alfvénic (CA or fast magnetosonic) modes driven by fast ions injected during the NBI heating [14–18].

This review is written in the introductory style. We are giving preference to understanding the physics rather than to rigorous derivations of the underlying equations and refer the reader to the original publication reference. The paper is organized as follows. We start with the strong instabilities of the homogeneous plasma in Sec. II. Then in Sec.III we move to observations of high frequency modes in STs which are very important for CA eigenmode (CAE) validations. The current status of ICE instability theory is summarized in section IV, which is followed by a short account of the nonlinear ICE studies in Sec V. We summarize and discuss ICE theory development in section VI.

II. HOMOGENEOUS PLASMA AND STRONG CYCLOTRON INSTABILITIES

Although the instabilities covered in this section were not reported for tokamaks their theoretical discoveries motivated significant interest in EP physics in the late $60's$ due to predicted strong growth, $\gamma/\omega \sim \sqrt{n_\alpha/n_{pl}}$, where ω is the frequency of the underlying instability, n_α and n_{pl} are the densities of energetic particles and the background plasma. However, it turns out that in realistic plasmas the proposed thermonuclear instability is stabilized because of the toroidal drift of fast ions. Let us highlight key theoretical aspects of this problem by deriving the expression for their growth rate.

To introduce this instability, we adopt a homogeneous plasma approximation which can be justified in realistic tokamak plasmas for the processes on a time scale faster than the EP drift motion, i.e. they are justified if the growth rate is large enough for the oscillation to grow faster than the particle drift across its localization region. Let us introduce the plasma permittivity tensor, $\hat{\epsilon}$, in the familiar and convenient for our analysis form:

$$(\hat{\epsilon} - 1)\,\boldsymbol{E} = \frac{4\pi i}{\omega}\boldsymbol{j}. \tag{2}$$

(A concise derivation of the permittivity tensor expression can be found in B. B. Kadomtsev textbook [19].) Here we made use of the standard notations for the electric field vector, $\boldsymbol{E}$, and the perturbed plasma current, $\boldsymbol{j}$. From the derivation of Ref.[19] it follows that in the low frequency limit, $\omega \ll \Omega_i$, the diagonal terms $\hat{\epsilon}_{11} = \hat{\epsilon}_{22} \simeq c^2/v_A^2$ are described by thermal ions contributions, whereas the non-diagonal terms are primarily due to thermal electrons. Here Ω_i is the thermal plasma ion gyrofrequency. At low frequencies the non-diagonal terms vanish, $\hat{\epsilon}_{12} = \hat{\epsilon}_{21} \simeq 0$, whereas they contribute to the mode dispersion at higher frequencies comparable to Ω_i.

The contribution by the fast ions can be readily computed expressing their perturbed current via the EP distribution function $\boldsymbol{j}_\alpha = z_\alpha \int \boldsymbol{v} f_\alpha d\boldsymbol{v}$. Thus, the fast ion tensor is proportional to the fast ion density so that the dispersion relation after some algebra takes the form

$$1 - \frac{k_\perp^2 v_A^2}{\omega^2} - \frac{n_\alpha}{n_e}\frac{v_A^2}{v_\alpha^2}\frac{\omega J_{2n}\left(2k_\perp v_h/\omega_{c\alpha}\right)}{\omega - n\omega_{c\alpha}} = 0, \tag{3}$$

where EP density is small according to Eq.(1). It follows from this expression that the fast ion contribution cannot be ignorable if the frequency is sufficiently close to thermal ion cyclotron frequency.

This dispersion in the limit of zero fast ion density results in the compressional or fast Alfvén wave. At finite EP density Eq.(3) can be solved using the perturbation technique and considering $\omega = \omega_0 + \omega_1$, where the plasma oscillations satisfy the compressional Alfvén dispersion:

$$\omega_0 = n\omega_{c\alpha} = \omega_{cA} \equiv k_\perp v_A. \tag{4}$$

For the perturbation correction, ω_1, we find from Eq.(3)

$$\left(\frac{\omega_1}{\omega_0}\right)^2 = \left(\frac{n_\alpha}{2n_e}\right)\left(\frac{v_A}{v_\alpha}\right)^2 J_{2n}\left(\frac{2nv_\alpha}{v_A}\right). \tag{5}$$

And since the Bessel function here changes sign, ω_1 can be imaginary, which corresponds to the cyclotron instability. One can see that the above expression gives the sufficiently strong growth rate even when the EP density is small since $\omega_1 \sim n_\alpha^{1/2}$.

The dispersion relation, Eq.(4), is relevant for the so-called compressional Alfvén eigenmodes or CAEs in STs in particular [15, 18]. As we pointed out in the introduction, CAE instabilities are believed to be responsible for the Ion Cyclotron Emission (ICE) excited in fusion tokamak plasmas. There are two known theoretical limits of ICE theory with respect to the instability growth rate. The first one is the strong CAE instability theory when the mode grows faster than the EP drift motion or the growth rate satisfies $\gamma > \Delta\tau^{-1}$, where $\Delta\tau = min\left(\tau_{res}, \Theta qR/v_\parallel\right)$ is the characteristic time of wave-particle interaction, τ_{res} is the time of particle interaction with the perturbation near the local resonance, $\Theta qR/v_\parallel$ is the time which the particle spends in the localization domain of the unstable mode. Here we followed the line of arguments of Ref. [15] where the example of NSTX plasma was used for which $\Delta\tau = 0.7 \times 10^{-6}s$ is found at $\Theta = 1$, $R = 100cm$, $q = 1$, and $\chi = 0.5$. The validity condition for the strong instability theory in NSTX example is then:

$$\frac{\gamma_{CAE}}{\omega} > 0.1\frac{\omega_{cD}}{\omega}.$$

The second limit is known as the slow instability limit. In it the EP drift motion should be included in the wave-particle interaction. The cyclotron instability example considered above is of the first type, i.e. it is the strong instability. The instabilities of this type were proposed to explain ICE in TFTR and in JET DT tokamak experiments in Refs. [20, 21]. In fact, in both papers a smooth transition from the linear or perturbative in EP density dependence of the growth rate to square root dependence is found numerically at

$n_\alpha/n_e \sim 10^{-6}$. Such values and even higher ones for EP density were reported for TFTR experiments but no clear picture of ICE signal transition to square root dependence was demonstrated using the experimental data [9]. ICE signal delay shown in Fig.6 of Ref.[6] clearly exhibits a linear dependence of ICE signal vs. n_α/n_e in JET DT plasmas.

Two seminal studies were begun with A. B. Mikhailovskii as the contributing author. In the first one Kaladze and Mikhailovskii have started investigations of the cyclotron instabilities by introducing trapped and passing particle dynamics [4]. The authors suggested that the cyclotron instability of interest was due to the positive velocity space gradient or bump on tail (BOT) of the EP velocity distribution. The difference in passing and trapped particle drift motion dynamics on the cyclotron instabilities was included in their theory. In particular, it was concluded that trapped ions can excite the compressional Alfvén waves in tokamaks at $k_\|$. It was also found that due to bounce period averaging the resulting growth rate of the instability is significantly smaller then in the case of strong instability (see the discussion in the previous paragraph).

The second noteworthy earlier paper by Mikhailovskii is Ref.[22] where the universal drive or spatial gradient of EP distribution function excites the shear Alfvénic plasma oscillations. For the sake of simplicity, Ref.[22] assumed the distribution function, which is Maxwellian in velocity and isotropic in pitch angles. This low frequency excitation was considered earlier for the first time in Ref.[2].

III. OBSERVATIONS OF HIGH FREQUENCY MODES IN ST SUPPORTED ICE INTERPRETATIONS

Initial studies of cyclotron instabilities related to Ion Cyclotron Emission (ICE) in STs were based on the analysis of the high frequency spectra of the Mirnov signal. It was shown experimentally that the frequencies of the instability correlate with the Alfvén velocity. This property was helpful to identify the magnetic activities as the instabilities of the Compressional branch of Alfvénic Eigenmodes (CAEs, also called fast Alfvén or magnetosonic eigenmodes) [20, 23–26]. We should note that another kind of high frequency instabilities are often seen in that frequency range with different dispersion properties. These instabilities correspond to the excitation of the global shear Alfvén eigenmode or GAE [27]. It is now believed that the instabilities of GAE modes have subcyclotron excitations due to their dispersion, $\omega_A = \omega_{cA} k_\parallel / k_\perp$, whereas CAEs at higher eigenfrequencies (above the thermal ion cyclotron frequency) are used to explain the ICE phenomena in tokamaks [20, 25, 26]. Here it makes sense to consider the excitation of CAE modes by beam ions and how their instabilities are proposed to diagnose future burning plasma experiments.

Observations of CAEs in ST devices allowed measurement of the dispersion and the polarization properties of each peak of ICE relevant instabilities, including their internal structures. The main reason for this is the intrinsically low equilibrium magnetic field in STs, so that the frequency spacing between the neighboring CAEs $\delta f = v_A/r$ is small but measurable [14]. Let us construct the CAE localization heuristically following earlier publications [24, 25]. In particular, in Ref. [24] the eigenmode equation was postulated (later derived more rigorously in Ref.[25]) taking the relation (4) squared and making use of the inequality $k_\parallel / k_\perp \ll 1$. For CAEs the dominant perturbed quantity is the parallel component of the equilibrium magnetic field, $\delta B_\parallel$, and we can readily write in the cylinder

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} \delta B_\parallel + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \delta B_\parallel = -\frac{\omega^2}{v_A^2(0)} \frac{n((r)}{n_0} (1 + \epsilon \cos\theta)^2 \delta B_\parallel, \tag{6}$$

where the plasma density profile takes the form $n(r) - n_0 (1 - r^2/a^2)^{\sigma_i}$. The right-hand side of this equation has a minimum that is a localized region of the mode structure as we illustrate in Fig.(1). A radially localized normal mode solution of Eq.(6) can be found by considering that the potential well is formed by its narrow radial width and its shallow but long poloidal extension, i.e., short poloidal wavelength in comparison with the radial wavelength. This justifies the choice of the eikonal for the CAE mode structure (cf. [24]):

$$\delta B_\parallel (r, \theta) = b(r, \theta) \exp\left[-i\omega t + im(\theta + \epsilon_0 \sin\theta) - in\varphi\right], \tag{7}$$

where φ is the toroidal angle, m is the poloidal mode number. The poloidal mode number is assumed to be large so that the equation for CAE mode amplitude, $b(r, \theta)$, implies slow variation in both directions but sill requiring $\partial b/\partial \ln r \gg \partial b/\partial\theta$. The envelop of CAE mode structure can be found iteratively with the zeroth iteration allowing only the poloidal variation in the eikonal (7). The first and second iterations produce the solution envelops [28]

$$b(r, \theta) = b_0 \phi_k \left(\frac{\sqrt{2}\theta^2}{\Theta}\right) \phi_s \left(\frac{\sqrt{2}(r - r_0)}{\Delta}\right), \tag{8}$$

where ϕ_s is the Chebyshev-Hermite function and the characteristic widths of those functions are $\Delta^2/a^2 = \kappa\sqrt{2\sigma_i/(1+\sigma_i)(\epsilon_0 - \alpha_0)}/(2m+1)$ and $\Theta^2 = 1/(\epsilon_0 - \alpha_0)(m + 1/2)$, where

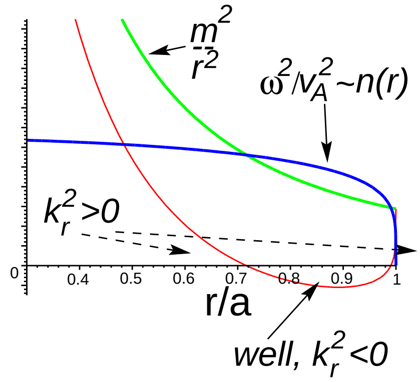

Figure 1. *A schematic of the radial dependence of the CAE potential (RHS of Eq.*(6) *plus* m^2/r^2 *term) and how it is formed and is related to the density profile* $(\omega^2/v_A^2\,(r))$.

$\alpha_0 = B_\theta^2/2B_\varphi^2$ taken at r_0 on LHS. The assumptions of Ref.[28] included $m \gg nq\,(r_0)$ and $\varepsilon > 1-\kappa^2$.

A variational method of solving the eigenmode equation (6) was adopted in Ref. [29] and assumes low values of the poloidal mode numbers m. While for Δ/a Ref. [29] has found a similar expression to Eq.(8) the poloidal, localization was somewhat different, namely:

$$\Theta^{-4} \simeq n^2 q^2 \left(\kappa^2 - \frac{R_0}{R}\right),$$

where only the leading order terms in n^2 are kept. Although the poloidal mode number dependence, $m \sim nq$, is the same in both treatments the numerical factors are somewhat different due to the close to expected eikonal which was enforced by Eq.(7). Predictions for the mode frequency as a function of the CAE mode numbers differed, with the dominant reason the fact that the poloidal mode number m in Ref.[28] is consistent with used approximations.

Because of the complexity of the above eikonal the realistic CAE dispersion that we will see is difficult to analyze and compare with dedicated experiments. For example, in the aforementioned papers [28, 29] used theoretical methods are rather approximate. Instead a heuristic dispersion relation of CAEs adopted in Ref. [30] is worth mentioning here. In analogy with the dispersion (4) the paper [30] finds the characteristic length and the corresponding "quantum" numbers in each of three relevant directions: toroidal mode number n with the major radius R as the characteristic length; radial mode number S with the radial width of the effective potential in radial direction and the poloidal mode number M with the plasma minor radius, i.e., one can write

$$\omega_{MSn}^2 \simeq v_A^2 \left(\frac{M^2}{r^2} + \frac{S^2}{L_r^2} + \frac{n^2}{R^2} \right). \tag{9}$$

The numerical solutions obtained in Rev.[30] using the ideal MHD code NOVA agreed with the dispersion relation (9) which is consistent with the eikonal (7). The above dispersion was checked numerically only for $n = 0, 1$ due to strong coupling of the dominant compressional Alfvénic polarization of CAEs and the shear Alfvénic harmonics at higher n numbers.

Another paper, Ref.[31], debated the validity of the presented approach that is based on the eikonal Eq.(7), although the treatment of Ref.[31] is not suitable for large ellipticity tight aspect ratio plasmas. More recent CAE studies were dealing with the ST plasmas and managed to decouple the shear Alfvén and the compressional Alfvén plasma oscillations [32]. The decoupled equations for CAEs in Ref. [32] completely ignore interactions with the kinetic Alfvén waves (KAW) and thus ignore an important dissipation mechanism which was recently considered as a way to channel the energy from the beam ions to thermal electrons [33]. In that paper it was shown that for CAEs resonantly interacting with the fast ions it is natural to have special spatial resonance with the shear Alfvén waves at some location in the radius. KAWs, due their relatively short radial wavelength, $k_\perp \rho_i \sim 1$, are prone to have a strong parallel electric field and thus a linear damping rate on electrons. We should note that this mechanism was overlooked in the conventional ICE theory. The CAE to KAW power channeling is a very important dissipation mechanism which has to be brought into consideration in the codes targeting CAEs.

Nevertheless, the modeling of CAEs with the mentioned codes was very important for the interpretation of the experimental results such as from NSTX and from MAST. CAE dispersions of Eq.(9) was brought into consideration in Refs. [14, 18, 30]. The comparison illustrated clear frequency separation for the modes at the subsequent "quantum" numbers of CAEs according to Eq.(9). These CAE dispersion properties were validated after ST experimental measurements of high frequency subcyclotron instabilities were interpreted [14, 28]. Moreover, the CAE dispersion seems to be instrumental in helping experimentalists identify the kind of the instabilities by following the mode frequency evolution, i.e. their dispersions, without knowing the polarizations [34].

We show examples of CAE observed signals from NSTX and MAST devices in left and right Figs. (2) respectively. In the left figure from NSTX, two CAE unstable peaks cluster clearly around $2MHz$ and $1MHz$, which corresponds to the largest frequency separation, $\sim 1MHz$, associated with the radial mode number S (see Eq.(9)). Smaller frequency separation is due to the poloidal mode number, M, $\Delta f \simeq 120kHz$, whereas the smallest separation in frequency could also be distinguished around each peak in the left figure (2). It is due to the toroidal mode number n at $\Delta f \simeq 20kHz$. Similar frequency separations were reported in Ref.[29] which is mentioned above.

An experimental study of CAE modes from MAST was done in a similar manner and is illustrated in the spectrogram of the right figure (2). Although the spectrogram shows a low part of the frequency spectrum clear peaks indicated by their toroidal mode numbers correspond to the ICE unstable CAEs at much higher frequencies. The values of those CAE frequencies were recovered by assuming that measured signals are actually aliases of real peaks. The inferred values of the frequency separation were similar to the values we see in NSTX. That is, the poloidal mode number separation is on the order of $150kHz$ and, the toroidal mode numbers are separated by $\Delta f = 10 - 15kHz$, whereas the whole observed band is estimated to be at the value of its frequency $f = 1 - 2MHz$.

At first glance the spectra shown in Figs.2 resemble the harmonics of ICE: descrete peaks in the Mirnov coil spectrum, almost equally spaced in frequencies. However the edge cyclotron frequency of the background thermal deuterium ion was $\sim 2.2MHz$ in NSTX and $\sim 2.5MHz$ in MAST in those shots which are much larger than the peak frequency separation shown in the figures. Nevertheless the frequency separations corresponding to various mode numbers as we highlighted above served as an important confirmation of CAE's dispersion and their theory. It allows potentially the development of the diagnostic tools to study fusion plasmas and burning plasmas in particular as we will be discussing.

Due to this understanding the theory of CAEs helped to find its way for validations in spherical tokamaks. Furthermore, recent theoretical investigations resulted in the prescription for how to decouple CAE baseline eigenmode equations from the shear Alfvén oscillations, allowing for a focus on CAE properties only [32]. A similar approach, but with a slightly reduced model for the eigenmode equations, was further developed and applied to the MAST plasma [35], where the modes co- and counter- propagating in the direction of the plasma current were reported. As an illustration of those studies, we show the computed CAE radial mode structures in Fig. (3). The CAE shown has a mode structure which is similar to the one we used in this review (see Eq.(8)), although both theoretically [36] and

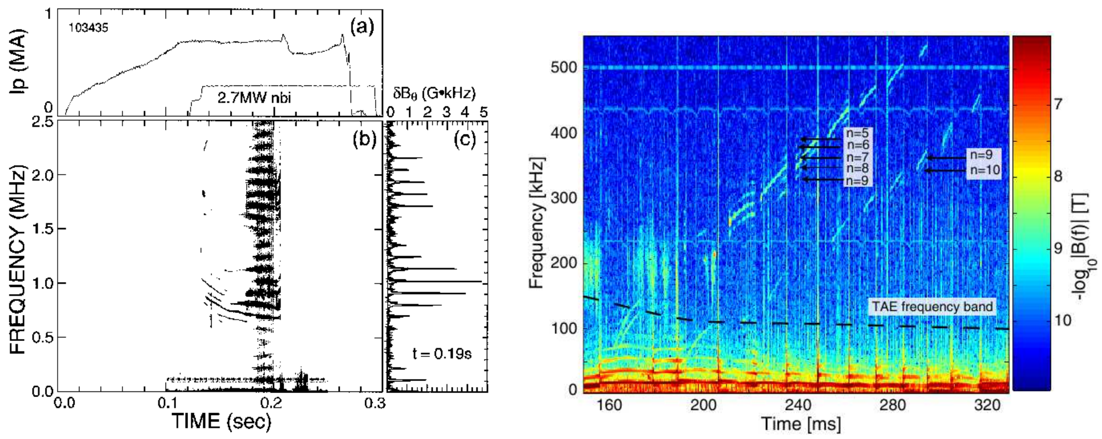

Figure 2. *CAE frequency spectra from two devices, NSTX (left), and MAST (right). Both spectra were measured by the magnetic pick-up coils installed outside of the plasma near the equatorial plane. On the right (MAST) figure the toroidal mode numbers and thus the smallest frequency split corresponding to the toroidal mode numbers are indicated.*

numerically [32] CAEs that were localized at the HFS were predicted as well.

A special study was undertaken in Ref.[32] which showed that the aspect ratio plays a critical role in the poloidal localization of CAEs. In "tokamak" approximation, i.e., with high aspect ratio, the poloidal localization hardly can be seen. On the other hand, in ST-like devices with low aspect ratio such as $R/a \sim 1.3$ found CAEs are well localized at the low field side, which is similar to the results of the WHALES code shown in Fig.3.

Finally we summarize this section by stating that it is due to CAE observations in ST devices and to theoretical advances that it became possible to understand the nature of modes responsible for ICE. Let us examine the theory of ICE excitation in its current understanding.

IV. ICE THEORY

The observations of ICE were done in tokamaks with the frequency spectrum signal having peaks at the harmonics of the edge background ion cyclotron frequency [6, 37, 38] (see also recent review paper by the author [3], Sec.4.2.3). In the previous section the CAE theory was viewed using the ideal MHD eigenmode equations which are sufficient to interpret and understand the experimental data from tokamaks and STs. It was shown more recently

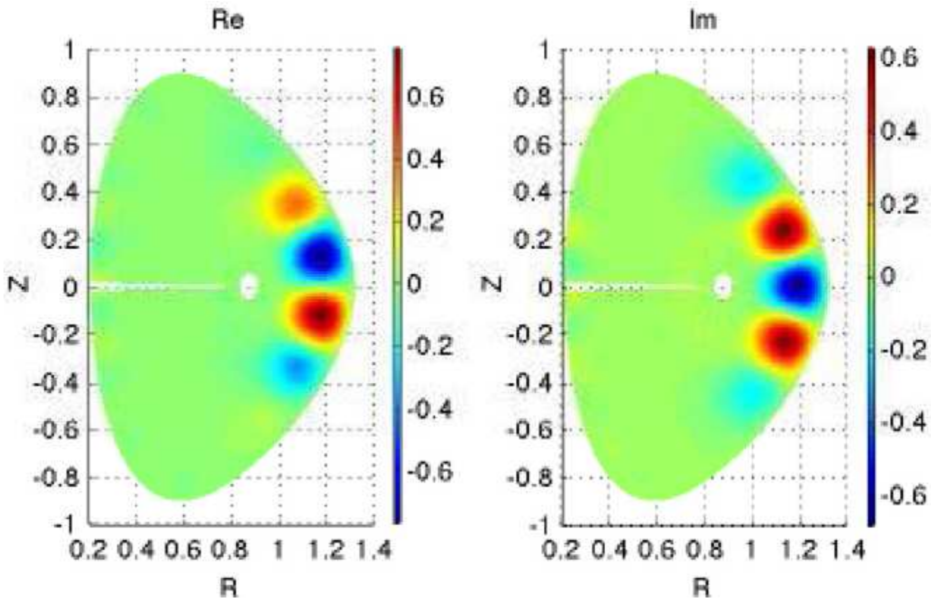

Figure 3. *Contour map of CAE poloidal structure obtained by the WHALES code. Shown are the real and imaginary values (as indicated) of η_ψ variable representing the normal to the magnetic surface component of the mode displacement. The depicted mode corresponds to $n = 10$ with the mode frequency $f = 2.23MHz$.* [35].

that for more accurate treatment of the modes the Hall term has to be included in the CAE framework to compute the dependence of the solutions on the poloidal phase velocity [26, 39, 40]. One particular consequence of this is the shift of the eigenmode frequencies away from each other, depending on the sign of m. For the plasma cross section with the ellipticity κ the eigenfrequencies of CAEs were found to have the asymmetry in the poloidal mode number sign as follows (using the notations of Ref.[26])

$$\omega_{CAE} = k(\kappa)\, v_{A*} \left[\frac{\sigma_m v_A (\ln n)'}{2\omega_B(\kappa)} + \sqrt{1 + \left(\frac{v_A n'}{2\omega_B(\kappa)\, n} \right)^2} \right]_{r_*}, \tag{10}$$

where the location of the eigenmode is given by

$$2 + r(\ln n)' - \sigma_m \frac{v_A}{\omega_B(\kappa)} \left(r (\ln n)' \right)' \sqrt{1 - \frac{2 + r(\ln n)'}{\left(r(\ln n)' \right)'}}. \tag{11}$$

Other notations here are: $k(\kappa) = |m| \sqrt{\frac{1+k^{-2}}{2}}/r$ is the poloidal wave vector, $\omega_B(\kappa) = \frac{eB}{Mc}\sqrt{\frac{1+k^2}{2}}$ is the cyclotron frequency, and $\sigma_m = m/|m|$ is the sign of the poloidal wave number. Further analytic and numerical treatment of the CAE eigenproblem can be found in

Ref.[36]. As we point out in the discussion section VI the dispersion 10 should be important in applications of ICE for diagnostics of burning plasmas since this will affect the stability of the CAE modes.

The stability theory of CAEs in applications to ICE is developed for various cases. We highlight its main elements following the author's earlier work [25, 41] which was strongly influenced by A.B. Mikhailovskii reviews on cyclotron instabilities by fast ions and in particular by Ref. [8]. We should say that a similar approach to CAE instabilities was developed in Ref. [42], where the slow instability limit was implied although no applications to ICE in experiments was made. Other papers dealing with ICE instabilities were limited to the strong instability as discussed above [20, 21] (see also recent publications on those interactions in strong instability limit Refs.[11, 12] and in earlier papers [10, 13]). More recent formulations of CAE growth rates in applications to STs analyze the growth rate expressions accounting for EP drift frequency contributions which carefully address not only the cyclotron but EP drift frequencies [43, 44].

Let us outline the formulation of CAE growth rates derivation. The growth rate is computed via the anti-Hermitian part of the permeability tensor of a specie j, $\hat{\boldsymbol{\epsilon}}_j^A$:

$$\gamma \simeq -\frac{\omega}{2} \frac{\sum_j \Im \int E_1^* \hat{\epsilon}_{j11}^A E_1 d^3r}{\int E_1^2 \left|\hat{\epsilon}_{11}\right| d^3r}, \tag{12}$$

where E_1 is the component of the perturbed electric field in the direction of the mode propagation which is expected to be in the poloidal direction if $k_\theta \gg k_r$, and the integration is taken over the plasma volume occupied by the mode. The permeability tensor appears in the above expression for the anti-Hermitian part of the perturbed current, $\tilde{\boldsymbol{j}}^A$, of the plasma specie j which in its turn can be expressed via the perturbed distribution function:

$$\int E_1^* \hat{\epsilon}_{j11}^A E_1 d^3r = \frac{4i\pi}{\omega} \int \boldsymbol{E}^* \cdot \tilde{\boldsymbol{j}}^A d^3r = \frac{4i\pi}{\omega} e \int \boldsymbol{E}_1^* \cdot \boldsymbol{v}_\perp \tilde{f}_j d^3v d^3r, \tag{13}$$

where $\boldsymbol{v}_\perp$ is the vector of the perpendicular particle velocity. Then after some algebra one arrives at (for more complete derivation we refer to Ref.[41])

$$\int E_1^* \hat{\epsilon}_{j11}^A E_1 d^3r = -\frac{8\pi^2 e^2 B}{\omega_c \omega^2 T} \int dP_\varphi d\mu d\mathcal{E} \tau_b \sum_{l,p} \frac{F_{lp}^{\prime *} \left(\omega - \omega_*^T\right) F_{lp}}{\omega - l\bar{\omega}_c - \bar{\omega}_D - p\omega_b} f_j, \tag{14}$$

where we made use of the set of COM (constants of motion) variables, $\mathcal{E}$ - particle energy, P_φ - toroidal canonical momentum, μ - adiabatic moment, τ_b is the ion drift bounce frequency,

f_j is the equilibrium distribution function of the fast ions, and the functions F'_{lp} account for the wave particle interactions[41], the set of frequencies in the denominator includes CAE eigenfrequency, cyclotron, drift and drift bounce frequencies. A similar expression was obtained earlier by Mikhailovskii[8] in the limit of zero banana width for well-trapped and strongly passing ions. As in Ref.[8], this equation contains the sum over the bounce resonances to account for slow growth of the unstable CAE mode. The bounce resonances were summed in the following novel way [41].

The resonant denominator in Eq.(14) is expandable into the integrable form involving the delta function of the resonance condition:

$$\frac{1}{\omega - l\bar{\omega}_c - \bar{\omega}_D - p\omega_b} = \frac{\mathcal{P}}{\omega - l\bar{\omega}_c - \bar{\omega}_D - p\omega_b} - i\pi\delta\left(\mathcal{R}\right). \tag{15}$$

The delta function is allowed to transform and integrate the sum over the bounce harmonic index p. Finally the expression for the growth rate of the CAE becomes

$$\frac{\gamma}{\omega_c} = \frac{\omega^3}{\omega_p^2\omega_c^2}\frac{\sqrt{2}ecB}{\sqrt{\pi}\Delta r_0 R_0}\sum_{l,\sigma}\int dP_\varphi d\mathcal{E} d\mu I^2 \frac{E_1^2}{E_0^2}\frac{\mu l J_l^2}{z^2}\left[\frac{\partial}{\partial\mathcal{E}} + \frac{l\omega_c}{\omega B}\frac{\partial}{\partial\mu}\right]\tilde{f}_j,$$

where ω_p is the plasma frequency, $E_1 = E_0 f\left(r,\theta\right)$ is the CAE structure in the poloidal cross section required for proper averaging of the local growth rate expression, Δ is its radial width, $I^2 = 8\pi/\left|\frac{d}{dt}\left(l\omega_c + \omega_D\right)\right|$ is to be taken at the point where the ion has the cyclotron resonance with the mode, J_l is the Bessel function of order l with the argument $k_1\rho_L$, and ρ_L is the Larmor radius of the fast ions. The resonance condition is then $\omega - l\omega_c\left(r,\theta\right) - \omega_D\left(r,\theta\right) = 0$ which is to be evaluated along the EP drift trajectory.

An important element of the ICE theory is the quest for the damping mechanisms which can shape the spectrum of unstable CAEs in linear regimes. Several damping mechanisms were considered in recent publications on the alpha channeling [45], including thermal electron and ion Landau cyclotron resonant dampings. However, as we said above in Sec.(III), an important and often dominant damping mechanism in simulations which were not previously studied is the coupling of the KAW structures to CAEs [33]. KAW damps on thermal electrons due strong parallel electric field.

In connections with CAE dampings we would like to note that the instabilities of CAEs do not necessarily lead to ICE. They can occur at arbitrary frequencies between the integer harmonics of plasma ion cyclotron frequency, $\omega \neq l\omega_{ci}$. A substantial discussion on allowed CAE eigenfrequencies from the point of view of the mode damping is presented in the other author paper Ref.[45]. In particular it was shown that in STs CAEs could be excited at low frequencies $\omega < \omega_{ci}$ such as shown in figures 2. This is because the thermal electron Landau damping is small near $l\omega_{ci}$ in conventional tokamaks whereas it is large away from $\omega = l\omega_{ci}$.

V. CAE NONLINEAR EVOLUTION

There are known publications where the nonlinear aspects of CAE evolution were considered as we briefly list in this section. A pioneering study was done in Ref.[46] (its main results are summarized in Ref.[8]). The study was focused on the trapped fast ion excitation of some compressional mode characterized by the magnetosonic polarization. The evolution of the CAE mode was considered for the initial stage of the instability in which the EP velocity distribution function is given by the source distribution, which represents a shifted Maxwellian distribution in velocities. Because of this assumption, the considered case can be applicable to the initial stage of the discharge or to when the plasma is heated quickly and alphas do not slow down. The authors concluded, based on numerical results, that the thermal broadening of the α-particle source grows logarithmically with time. The considered scenario is not directly applicable to the burning plasmas which are designed for slow evolutions, so that the α-particle distribution function remains nearly slowing down at all times[47].

In another publication we would like to highlight, a set of quasilinear (QL) diffusion equations considered[48] for the case in which the resonance between the oscillations and the fast ions is cyclotronic with and without contributions from the EP bounce resonances. The resonance's overlap due to Coulomb collisions was also considered. The performed theoretical analysis is applicable to the ICRH problem according to that paper and could be used in developing the ICE nonlinear simulations.

Relatively recent studies of ICE in both linear and nonlinear regimes have been done numerically [49, 50]. In those publications particle-in-cell simulations address ICE evolution using the hybrid model computations where ion drift motion is advanced numerically while thermal electrons are treated as a fluid. Simulation allowed the authors to more accurately model both stages of ICE in experiments. In particular, the frequency dependence of expected CAE instabilities had a shape similar to those observed for ICE in JET DT experiments.

VI. DISCUSSION AND SUMMARY

We overviewed the ICE theory in its present status. We noted that the ICE linear theory seems to be well developed and can be developed further focusing on its potential application to the present and future experiments. It had been theoretically predicted, and now observed, that many CAE instabilities with narrow spectrum peaks will overlap and form a broader peak near each ion cyclotron frequency harmonic. The main difference between the more recent observations from STs and those of earlier studies is that the frequency spectrum of the observed CAE instabilities in high toroidicity plasma is discrete, so that properties of each mode can be and was measured separately [14, 34, 51].

From previous studies it is known that, due to high magnetic fields in tokamaks, the ICE frequency spectrum is populated more densely by the unstable modes. This makes the ST plasma very attractive for studying the properties of individual CAE modes and for verifying the theoretical predictions. In particular, NSTX presents a unique opportunity to study mode numbers, structure, polarization, amplitude etc. We note that, based on the CAE observations on NSTX, one would expect that ICE spectrum contains more complicated, fine structures than previously reported [6, 52] perhaps in plasmas with intermediate values of the equilibrium magnetic fields. In addition one should look for the ICE-like high harmonics of the ion cyclotron frequency features in the magnetic activity of STs.

Three important problems associated with ICE could be of interest for the fusion community.

First, developing an understanding of ICE for diagnostic purposes in burning plasma conditions makes a perfect case for both the linear and nonlinear development of ICE theory. This was considered in several recent publications, see Refs.[11, 12], and in a recent review [3] in which the summary and current understanding of ICE theory and experiments were included. A pioneering observational analysis from JET by Cottrell pointed out at the proportionality of the ICE signal to the neutron flux over six orders of magnitude of its amplitude[6]. Another example was discussed recently on the ICE activity in DIII-D experiments in which its signals were observed in clear correlation with the off-axis fishbones [53]. Such correlations serve as a compelling case for using ICE for fusion products diagnostics (see Fig.15 f with the Mirnov pick-up coil data in that paper).

Second is the idea of the anomalous thermal ion heating, energy channeling, pointed out in

Ref.[54]. In that paper, CAEs were proposed as mediators for energy channeling from the fast ions to thermal ions. In this mechanism, fast ions excite the CAE modes during NBI heating as demonstrated in NSTX. The modes in turn stochastically transfer their wave energy to thermal plasma ions due to the cyclotron damping. Recently, some dedicated studies use the special CAE to KAW decoupling technique [55] to verify the theoretical predictions. Using model parameters, the paper demonstrates the feasibility of this technique and states that more detailed experimental observations of CAE internal amplitudes are required for validations of this idea.

One similar idea of thermal ion anomalous energy diffusion and associated plasma heating is behind the so-called "alpha channeling" [56] when certain plasma oscillations are excited by the external antennae. Recently, CAEs were suggested to explain strong fast ion energy diffusion in TFTR [57] which was required for alpha channeling to work [45, 58]. This argument pointed to further experiments, even on existing devices, that might verify and further extend this very unusual, but possibly extremely useful, alpha-channeling effect. Moreover, the demonstration of the role played by these modes, together with their theoretical description, carries significant implications for how this effect might be extrapolated with confidence to achieve significant improvements in the tokamak reactor concept. Since this mechanism could mean a big reduction in the cost of electricity produced by fusion, the newly inspired confidence in extrapolating these results may lead to important follow up experiments.

Third, the already mentioned coupling of CAEs to KAW [33] could explain the anomalous electron thermal transport observed in NSTX [59]. The deficit of heating the thermail electrons could account for up to $30-40\%$ of the total NSTX electron transport. In the core of Ref.[33] explanation is the heating power channeling by means of CAE/KAW coupling of thermal electrons. The short wavelength KAW is prone to have a strong parallel electric field and thus very effectively heat thermal electrons. The KAW was found to be localized poloidally on the high field side (HFS) and is in local resonance with CAEs. The radial width of the KAW is comparable to the beam ion Larmor radius. The eigenfrequency of CAE modes satisfies the following dispersion relation, which includes the contribution from beam ions: $\omega^2 = k_{\|}^2 v_A^2 \left[1 + (n_\alpha/n_e)\left(3k_\perp^2\rho_\alpha^2/4 - \omega^2/\omega_c^2\right)\right]$. Although the coupling of CAEs and KAWs is demonstrated there is no theoretical explanation of this effect which could be used in the plasma simulation codes like TRANSP.

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Response to the referee comments

The author thanks the referee for thoughtful comments. Addressing them will hopefully improve the paper and after the resubmission.

Following is my point by point response to the referee report. I also put my amendments in blue color to find them easily.

(i) *The paper deals mainly with the theory of CAE instabilities. Although presumably the CAE modes are responsible for the ICE, the CAEs do not necessarily lead to ICE. It is necessary to explain when the destabilization of compressional Alfven waves leads to the radiation with the peaks at* $\omega = l\omega_c$ *from the plasma.*
The manuscript is corrected and a statement is added at the end of Sec.IV on p.14 about the role of thermal electron damping in shaping the ICE spectra.

(ii) *It is not clear whether the experimental results shown in Fig. 2 are relevant to ICE. What were the gyrofrequencies in NSTX (left panel in Fig. 2) and MAST (right panel)? Do the peaks of* δB_θ *correspond to harmonics of the beam ion gyrofrequency or plasma ion gyrofrequency?*
This point is good to discuss as indeed it clears some potential confusion coming from Fig.2 on p.11 which I did not notice working on the draft. This point is discussed on p. 10.

(iii) *The intensity of the ICE spectrum is linearly proportional to the fusion product density. A question arises, why so much attention in the paper is paid to instabilities with the growth rate* $\gamma \propto \sqrt{n_\alpha}$ *?*
Section II takes only 2.5 pages of the draft which is about 22 pages in total. Besides I believe that because the whole cyclotron instability study and thus ICE theory started with the strong instability theory it deserves such attention. We discuss this on p.3, second paragraph.

(iv) *It is necessary to change the Abstract. The current Abstract does not reflect the fact that the theory of CAE instabilities is reviewed in the paper.*

The abstract is rewritten reflecting the content more accurately, in particular pointing out at the CAE theory review.

In addition, it is not clear why Mikhailovskij's research is highlighted in the Abstract and which his pioneering papers on ICE the author means. This should be explained in the paper.

This part of the abstract is also rewritten more accurately. The abstract makes more clear connection with Mikhailovskii works.

My opinion is that it were experimental observations (first of all, by Cottrell et al [6]) rather than any theory, that attracted attention to the ICE phenomenon.

I agree that ICE attracted most of the attention after Cottrell's publication. However even before that publication papers by Mikhailovskii, such as his review of ′86, contained the theoretical formalism sufficient for ICE theory to develop. I used it myself a lot when i was publishing the ICE formalism in Ref.[25]. I make this point clear on p.3 first paragraph.

Concerning theory, I have to say that Refs. [4], [5] were not pioneer works on fast-ion induced instabilities. In addition, Refs. [4], [5] and other cited works by Mikhailovskij were carried out at the end of 70s, not in 60-70s, as the author claims.

Parts of the paper citing [4,5] are made more clear and accurate. Also years of publications are corrected.

(v) *I cite from p.3: "The EP/waves interactions were then considered with realistic drift ion motion in the presence of CA waves". In which works? I suggest to cite the original works.*

I meant to cite Dendy's earlier work immediately following Cottrell paper. I added it to the draft.

(vi) *The last two paragraphs in p.16 are not clearly written. In particular, it is difficult to understand what effects are "very unusual" and what the author means* *saying about "deficit".*

Those two paragraphs are rewritten and hopefully address concerns by the referee.

(vii) *It is necessary to remove misprints. In particular, to replace "satisfy" with "satisfies" in p. 17 and the year (1995) with (1968) in Ref. [2].*

The resubmitted draft includes corrections noted and suggested by the referee.

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