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Andrei Khodak

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Princeton Plasma Physics Laboratory

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Adaptation of General Purpose CFD Code for Fusion MHD Applications*

Andrei Khodak

Princeton Plasma Physics Laboratory

P.O. Box 451

Princeton, NJ, 08540 USA

akhodak@pppl.gov

Abstract— Analysis of many fusion applications such as liquid metal blankets requires application of Computational Fluid Dynamics (CFD) methods for electrically conductive liquids in the geometrically complex regions and in the presence of a strong magnetic field. Current state of the art general purpose CFD code allows modeling of the flow in complex geometric regions, with simultaneous conjugated heat transfer analysis in liquid and surrounding solid parts. Together with Magneto Hydro Dynamics (MHD) capability general purpose CFD code will be a valuable tool for design and optimization of fusion devices.

This presentation describes an introduction of MHD capability into a general purpose CFD code CFX part of the ANSYS Workbench. The code was adapted for MHD problems using magnetic induction approach. CFX allows introduction of user defined variables using transport, or Poisson equations. For MHD adaptation of the code three additional transport equations were introduced for the components of the magnetic field, with additional Poisson equation for electric potential. Lorentz force is included in the momentum transport equation as a source term. Fusion applications usually involve very strong magnetic field with the values of the Hartmann number of up to tens of thousands. In this situation system of MHD equations became very rigid with very large source terms, and very strong gradients of the variables. To increase system robustness, special measures were introduced during iterative convergence process, such as under-relaxation using source coefficient for momentum and magnetic field equations.

MHD implementation in general purpose CFD code was tested against benchmarks specifically selected for liquid metal blanket applications. Results of numerical simulations using present implementation closely match analytical solution for the Hartmann number of up to 15000 for two-dimensional laminar flow in the duct of square cross-section, with conducting and non-conducting walls. Results for three dimensional test cases are also included in the presentation.

Keywords—CFD; MHD; fusion

I. INTRODUCTION

ANSYS CFX is a general purpose CFD code, allowing solving hydrodynamics and heat transfer problems. It is used at PPPL for thermal analysis of the complex systems involving fluid flow and heat transfer in liquids and solids [1]. The code was adapted for MHD problems using magnetic induction

approach. The modified code was benchmark tested against fusion relevant cases presented in [2].

II. MODIFICATION OF THE CODE

To include magnetic flux density induction three additional transport equations were added:

$$\frac{\partial \vec{B}^*}{\partial t} + (\vec{V} \cdot \vec{\nabla}) \vec{B}^* = \vec{\nabla} \cdot \left(\frac{1}{\mu \sigma} \vec{\nabla} \vec{B}^* \right) + (\vec{B}^* \cdot \vec{\nabla}) \vec{V} \quad (1)$$

where:

$$\mu = 4\pi 10^{-7} \text{ - magnetic permeability of vacuum [N/A}^2\text{]}$$

$$\sigma \text{ - electric conductivity [S/m]}$$

Electric potential is added via Poisson equation:

$$\nabla^2 \varphi = -\vec{B} \cdot (\vec{\nabla} \times \vec{V}) \quad (2)$$

Lorentz force is added to momentum transport equations as an additional source:

$$\vec{F}_{mag} = \frac{1}{\mu} ((\vec{B} \cdot \vec{\nabla}) \vec{B}) \quad (3)$$

Momentum transport equations also use modified pressure:

$$\frac{p^*}{\rho} = \frac{p}{\rho} + \frac{|\vec{B}|^2}{2\mu} \quad (4)$$

Poisson equation is added to insure solenoidal nature of magnetic field using additional scalar:

$$\nabla^2 \psi = \vec{\nabla} \cdot \vec{B}^* \quad (5)$$

This scalar is used to project magnetic field obtained by equation (1) onto solenoidal space.

$$\vec{B} = \vec{B}^* - \nabla \psi \quad (6)$$

III. VALIDATION OF THE CODE

Significant effort is currently under way to benchmark various CFD codes for on test cases relevant to fusion MHD applications [2]. Such cases involve high external magnetic fields leading to high values of Hartman number:

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$$Ha = B_0 a \sqrt{\sigma / \rho \nu} \quad (7)$$

where:

- B_0 - external magnetic field intensity [T]
- a - length scale [m]
- ν - fluid kinematic viscosity [m²/s]
- ρ - fluid density [kg/m³]

This paper presents validation of the code against some of the test cases presented in [2]:

A. Square Non-Conducting Duct flow

Fully developed flow of conducting fluid subject to uniform transverse magnetic field, walls are non-conducting, case A1. Analytical solution is available for this case [3]. Problem set-up is presented on Fig. 1.

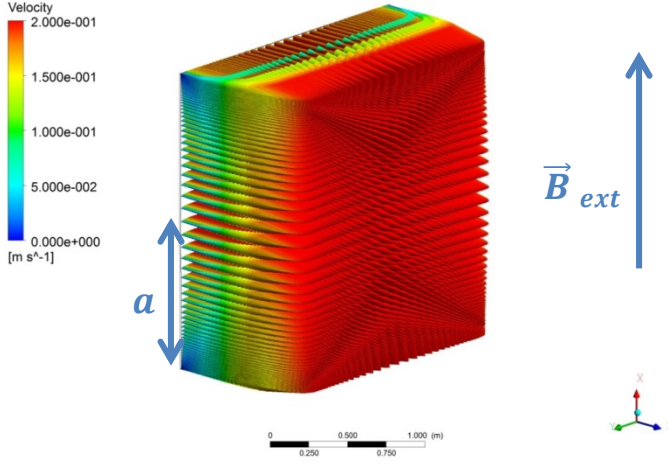


Fig. 1. Velocity vectors on the cross-section of the square non-conducting duct.

Incompressible fully developed flow is considered:

$$\rho = const \quad (8)$$

$$\frac{\partial p}{\partial z} = const \quad (9)$$

The following wall boundary conditions are imposed:

$$\vec{V}_{wall} = 0 \quad (10)$$

$$\vec{B}_{wall} = \vec{B}_{ext} \quad (11)$$

$$\left(\frac{\partial \varphi}{\partial n}\right)_{wall} = 0 \quad (12)$$

$$\left(\frac{\partial \psi}{\partial n}\right)_{wall} = 0 \quad (13)$$

The flow is characterized by non-dimensional flow rate:

$$\bar{Q} = \int_{-1}^1 d\tilde{y} \int_{-1}^1 \tilde{V} d\tilde{x} \quad (14)$$

where:

$$\tilde{x} = \frac{x}{a} \quad \tilde{y} = \frac{y}{a} \quad \tilde{V} = \frac{\rho \nu V}{a^2 \left(-\frac{\partial p}{\partial z}\right)}$$

Very thin boundary layers are created on the channel walls, requiring grid compression towards the wall for proper resolution. Table 1 shows comparison of numerical results with analytical solution. Results show that for higher Ha numbers compression ratio need to be increased to improve accuracy.

TABLE I. SQUARE NON-CONDUCTING DUCT FLOW RESULTS

Ha	\bar{Q}		Difference	Mesh Size in each direction	Cell Size Ratio
	Analytical Solution	Present Calculation	%		
500	7.6812E-03	7.6799E-03	-0.017%	100	100
5000	7.9031E-04	7.9021E-04	-0.012%	100	100
5000	7.9030E-04	7.9021E-04	-0.011%	200	100
5000	7.9025E-04	7.9021E-04	-0.004%	100	1000
10000	3.9641E-04	3.9656E-04	0.036%	100	100
10000	3.9659E-04	3.9656E-04	-0.007%	100	1000
15000	2.6483E-04	2.6480E-04	-0.013%	100	1000

Fig. 2 presents axial velocity profiles. Velocity profile is flat in the middle of the channel with the value of the velocity close to $\tilde{V} = 1/Ha$.

Fig. 3 and Fig. 4 show axial magnetic field distributions. Fig. 3 also shows electric current lines.

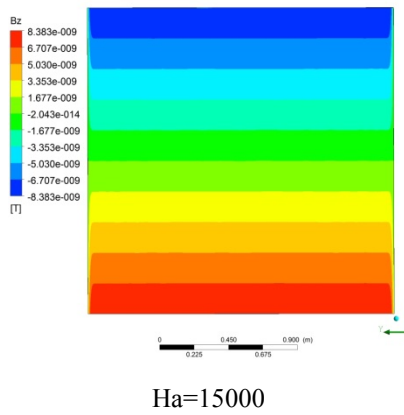
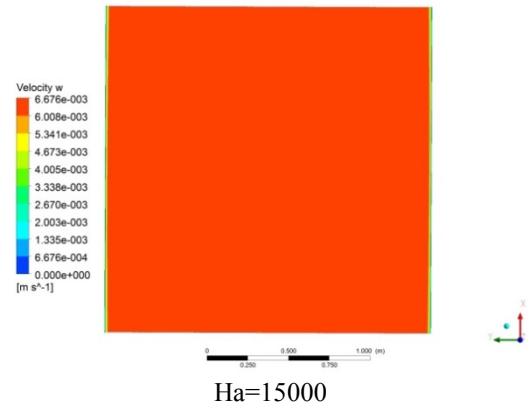
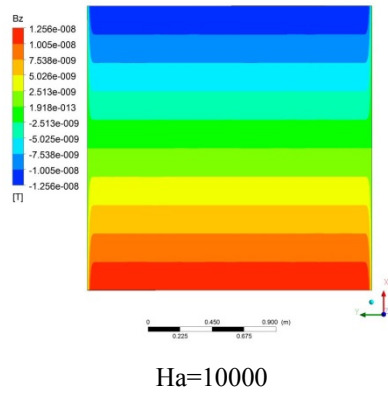
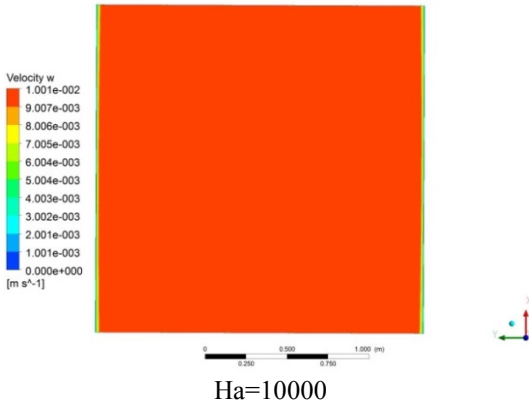
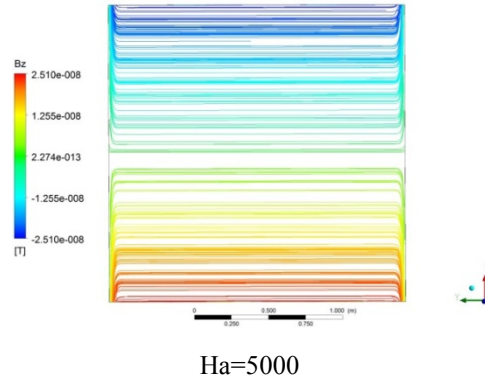
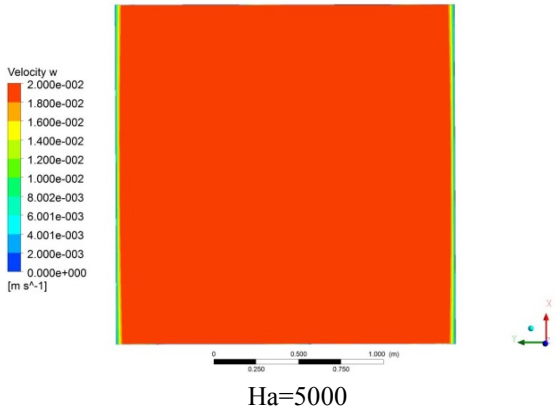
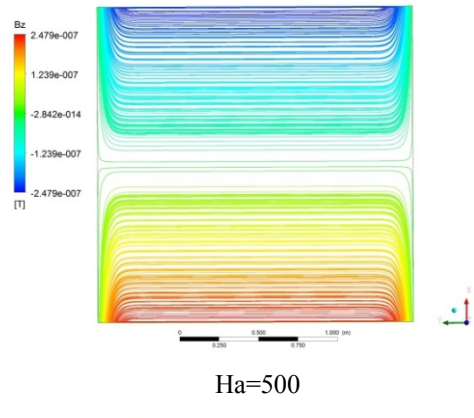
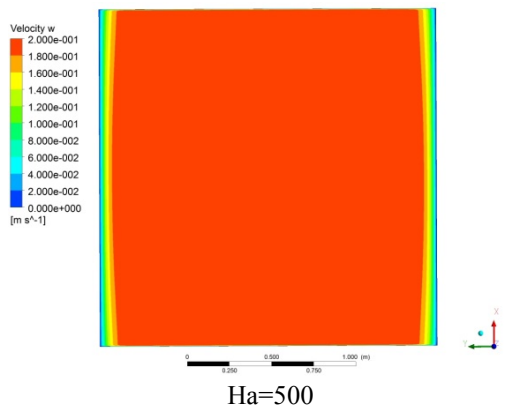


Fig. 3. Electric current lines colored with axial magnetic field.

Fig. 2. Velocity contours on the cross-section of the square non-conducting duct.

Fig. 4. Axial magnetic field contours on the cross-section of the square non-conducting duct.

B. Square Duct flow with Conducting Walls

Fully developed flow of conducting fluid subject to uniform transverse magnetic field, conducting walls are perpendicular to magnetic field, case A2 in [2]. Walls parallel to magnetic field are non-conducting. Analytical solution is available for this case [4]. Fig. 5 presents problem set-up.

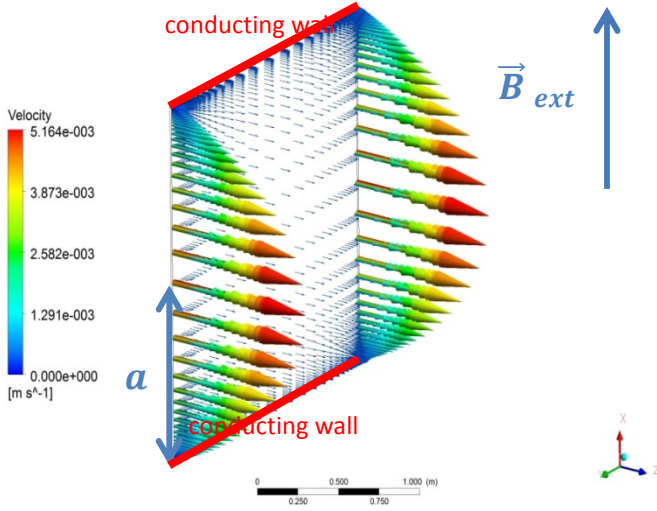


Fig. 5. Velocity vectors on the cross-section of the square non-conducting duct.

Incompressible fully developed flow is considered using conditions (8, 9). Boundary conditions are the same as in case A1 (10 - 13) except for conducting walls where conditions proposed by Shercliff in [5] are used for part of the magnetic field parallel to the wall:

$$\left(\frac{\partial B_z}{\partial n}\right)_{wall} = \frac{B_z}{c_w \mu \sigma} \quad (15)$$

where:

$$c_w = \frac{t_w \sigma_w}{a \sigma} \quad \text{- relative conductance of the wall}$$

$$t_w \quad \text{- wall thickness}$$

$$\sigma_w \quad \text{- wall electric conductivity}$$

Table 2 shows comparison of numerical results with analytical solution. Results show that for higher Ha numbers compression ratio need to be increased to improve accuracy.

Fig. 6 presents axial velocity profiles. Velocity profile is concentrated near the walls parallel to the magnetic field. Fig. 7 and Fig. 8 show axial magnetic field distributions. Fig. 8 also shows electric current lines.

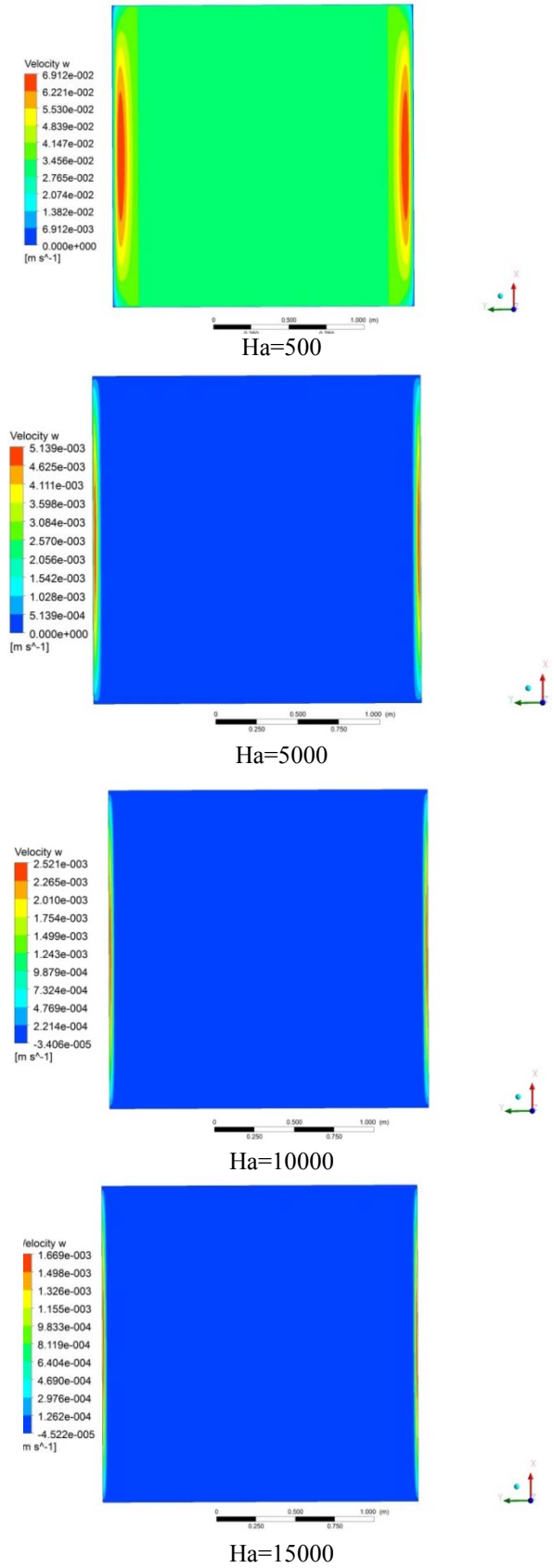


Fig. 6. Velocity contours on the cross-section of the square duct with conducting walls.

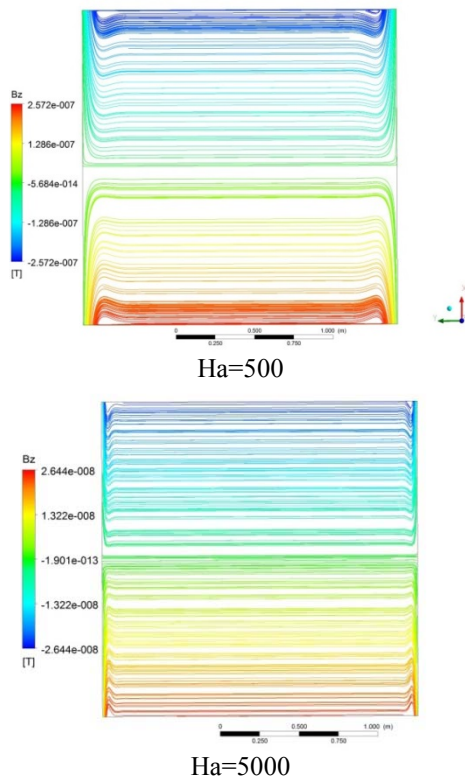


Fig. 7. Electric current lines colored with axial magnetic field.

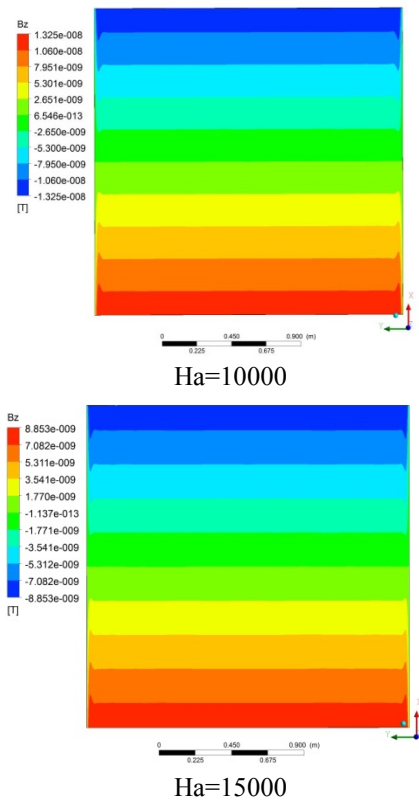


Fig. 8. Axial magnetic field contours on the cross-section of the square duct with conducting walls

TABLE II. SQUARE DUCT FLOW WITH CONDUCTING WALLS

Ha	\bar{Q}		Difference	Mesh Size in each Direction	Cell Size Ratio
	Analytical Solution	Present Calculation	%		
500	1.4054E-03	1.4084E-03	-0.213%	100	1000
5000	1.9074E-05	2.1638E-05	-13.446%	100	1000
5000	1.9074E-05	1.9143E-05	-0.361%	100	10000
10000	5.1693E-06	5.2273E-06	-1.123%	100	10000
10000	5.1693E-06	5.1906E-06	-0.412%	100	20000
15000	2.4250E-06	2.4438E-06	-0.773%	100	20000

IV. CONCLUSIONS

Customized ANSYS CFX code was able match analytical values of flow rate for fully developed square duct laminar flow.

High gradients in near wall layers require very dense mesh. Mesh with compression ratio of up to 20000 was used.

Since source terms have adverse effect on solution stability, under laxation was needed proportional to the source term magnitude and convergence was very slow especially for higher Ha numbers.

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