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Defocusing of an ion beam propagating in background plasma due to two-stream instability

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The current and charge neutralization of charged particle beams by background plasma enables ballistic beam propagation and has a wide range of applications in inertial fusion and high energy density physics. However, the beam-plasma interaction can result in the development of collective instabilities that may have deleterious effects on ballistic propagation of an ion beam. In the case of fast, light-ion beams non-linear fields created by instabilities can lead to significant defocusing of the beam. We study an ion beam pulse propagating in a background plasma, which is subject to two-stream instability between the beam ions and plasma electrons, using PIC code LSP. The defocusing effects of the instability on the beam can be much more pronounced in small radius beams. We show through simulations that a beamlet produced from an ion beam passed through an aperture can be used as a diagnostic tool to identify the presence of the two-stream instability and quantify its defocusing effects. The effect can be observed on the National Drift Compression Experiment-II (NDCX-II) facility by measuring the spot size of the extracted beamlet propagating through several meters of plasma.

Beam-plasma systems have a wide range of applications for astrophysics[1–3], atomic physics[4], high-energy accelerators and colliders[5–7], inertial confinement fusion, high energy density plasma physics in particular, fast ignition[9– 11] and heavy ion fusion[12–15], and basic physics phenomena[8, 16, 17]. For heavy ion fusion, the background plasma presents means of current and charge neutralization, enabling the ballistic propagation of the beam over long distances [18-22]. However, the beam can be subject of numerous collective instabilities during its propagation in plasma [23, 24]. Relatively short propagation in plasma and large variation of beam and plasma parameters during the beam compression was shown experimentally [21, 22] and theoretically [25] to be safe from deleterious effects of instability for current and past National Drift Compression NDCX-I and NDCX-II experiments. In this paper we consider effects of the longitudinal two-stream instability between plasma electrons and beam ions on ion beam propagation (two-stream instability between beam and plasma ions do not have time to develop for NDCX set-up unlike that of Refs.[8,29]). It has been long known that the two-stream instability can strongly affect the return current formation in plasmas [26-34]. Under some conditions the two-stream instability causes plasma electrons to follow the ion beam with the mean velocity approaching the beam velocity. Without the instability electrons in return current of an ion beam propagating in plasma would flow in the direction of the beam velocity with the velocity proportional to the beam velocity times ratio of the beam density to background plasma density, $v_b n_b/n_p$; this flow originates from the inductive electric

field induced by the beam pulse in plasma and is formed in response to time-dependent beam self-magnetic field [35]. The two-stream instability generates an electrostatic field that in turn generates the plasma current due to nonlinear effects of the order $-en_p v_b \gg -en_b v_b$, which is large compared with the beam current, if the plasma density is higher than the beam density $n_p \gg n_b$ and can result in complete current and consequently self-magnetic field reversal. The reversed azimuthal magnetic field no longer has a focusing effect on the beam and enhances radial defocusing of the beam, instead. The return current enhancement was experimentally observed in Refs. [30, 31] for electron beams and was demonstrated using particle-in-cell simulations for electron [28] and ion beams [26, 34]. The oscillations of the potential due to the two-stream instability were measured in Refs.[8, 32] for an ion beam and agree with theoretical predictions. The oscillatory electro-static electric fields generated by the two-stream instability can significantly modulate the beam density longitudinally. Furthermore the time-averaged oscillating longitudinal electric field can produce radial ambipolar fields via ponderomotive force which can enhance radial defocusing of the beam during propagation. The modulation of a low energy ion beam by an oscillating wakefield was demonstrated [36] using two-dimensional particle-in-cell simulation which can be employed to produce ultra-short beam pulses. The enhancement of self-electric and self-magnetic fields by magnetization and rotational effects were also shown by 2-D PIC simulation for a high energy ion beam propagating in magnetized plasma [37].

For NDCX-II parameters (Li or He ion beam in the energy range of 0.3-4MeV, the beam current is 0.6A and beam radius is 1-3cm [38]) the longitudinal two-stream instability does not affect the beam transport during compression and propagation over few meter long plasma [26, 34]; the transverse displacements and the distortions created by the two-stream instability on the beam density profile are small and hence are difficult to detect. In this paper we propose a diagnostic method, whereby a thin beamlet is extracted from the original beam and propagated through a background plasma. Measurement of the spot size of the extracted beamlet can reveal whether or not the instability is present. We demonstrate the effect of the two stream instability on the beam transport making use of electromagnetic particle-in-cell simulations for NDCX-II parameters.

We have simulated ion beam transport in background plasma with parameters similar to those of the proposed NDCX-II experiment: a Li^+ ion beam with axial directed velocity $v_b = c/30$ corresponding to a directed axial kinetic energy 3.66 MeV, beam density $n_b = 2 \times 10^9 / \text{cm}^3$, and beam radius $r_b = 2.5 \text{ cm}$ propagates in a background carbon plasma with density $n_p = 0.55 \times 10^{11} / \text{cm}^3$ for a propagation distance $L_z = 300 \text{ cm}$. The axial beam velocity was a constant $v_b = c/30$ with no velocity tilt. The ion beam profile was a radial flat top profile and an axial profile was a Gaussian pulse with pulse width duration $\Delta t = 20 \text{ ns}$, see Fig.1. The simulations were performed making use of LSP PIC code [39] and a moving-frame algorithm. The characteristic development time of the two-stream instability is determined by the linear growth rate $\gamma \sim \omega_p (\omega_b / \omega_p)^{2/3}$,

where $\omega_p = \sqrt{n_p e^2 / \epsilon_0 m_e}$ is the electron plasma frequency, e denotes unit charge, ϵ_0 is the permittivity of free space, and $\omega_b = \sqrt{n_b e^2 / \epsilon_0 m_i}$ is the beam ion plasma frequency with m_i denotes the beam ion mass. The growth rate of the two-stream instability for the simulation parameters is $\gamma \sim 2.1 \times 10^7 \text{ Hz}$ and instability e-folding time is about 4.8 ns. It takes 100ns for the beam to propagate for 1 meter of plasma, therefore it is expected that the instability develops and saturates after a meter of propagation.

Figure 1 shows evolution of the beam density profile during propagation in the 3 meters of background plasma. The two-stream instability develops and saturates around $t = 240 \text{ ns}$ as evident from Fig.2 that shows the phase space of the beam. The two-stream instability saturates at amplitudes of the plasma waves that accelerate

plasma electrons up to the velocity twice of that of the ion beam; and the mean electron velocity at the saturation state of instability approximately equals to the ion beam velocity [30, 32-34]. The longitudinal modulation of the beam density can be significant about 100% [34] as evident from Fig.1. However radial variations of the beam profile are not significant, as shown in Fig.1(a): at $t = 100$ ns, $r_b \cong 2.65$ cm and at $t = 300$ ns $r_b \cong 2.97$ cm, yielding $\Delta r_b / r_{b0} \sim 0.12$, where Δr_b and r_{b0} are the change in beam radius and the initial beam radius, respectively. Therefore, the two-stream instability will be difficult to detect for the full beam propagation in plasma over 3 meters of available propagation space.

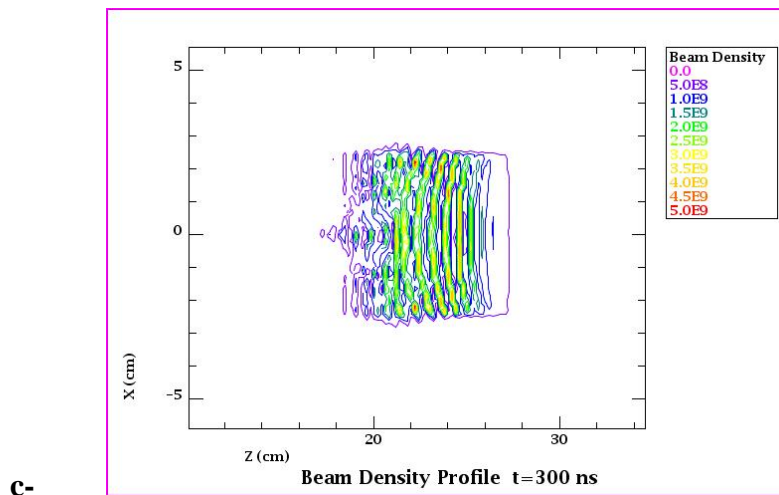
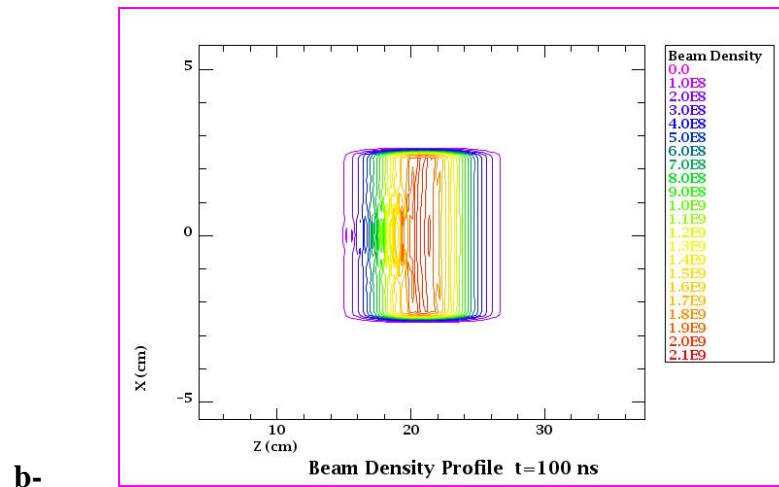
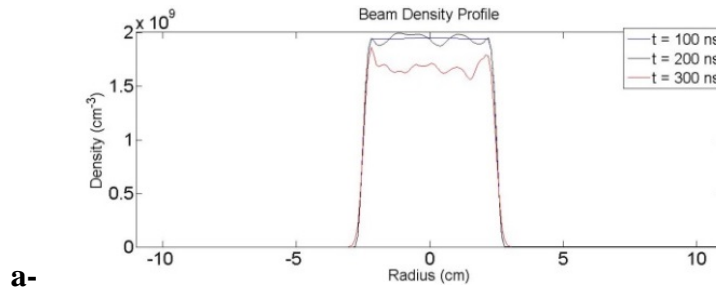


Fig.1. The evolution of the beam density profile during propagation in the background plasma for NDCX-II experiment: a Li^+ ion beam with axial directed velocity $v_b = c/30$, $n_b = 2 \times 10^9 / \text{cm}^3$, $r_b = 2.5 \text{ cm}$ propagates in a background carbon plasma with density $n_p = 0.55 \times 10^{11} / \text{cm}^3$. Fig.1(a): Radial beam density profiles at the beam pulse central cross section at $t = 100, 200$ and 300 ns . Fig.1 (b): beam density contour plots at $t = 100 \text{ ns}$ (1 m of propagation in plasma), Fig.1(c): beam density contour plots at $t = 300 \text{ ns}$ (3 m of propagation in plasma).

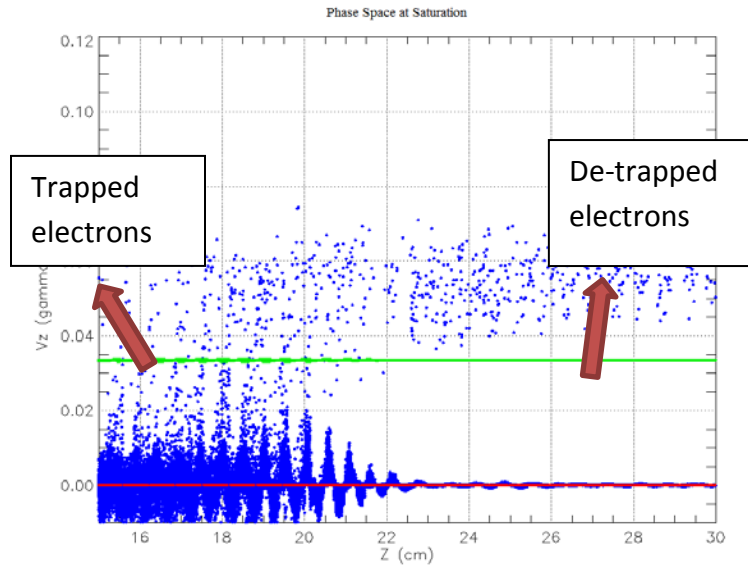


Fig.2. Phase-space plots of the ion beam and plasma electrons and ions at $t = 240 \text{ ns}$ corresponding to saturation of the two-stream instability. The beam parameters are the same as in Fig.1. Blue dots show plasma electrons; red- plasma ions; green – beam ions.

Note that our simulations do not show any development of neither transverse two-stream nor hose instability [23, 24, 30]. The hose instability may not have time to develop possibly due to the fact that the beam is not relativistic. The transverse two-stream instability was shown not to develop when the self-consistent profile of the return current and self-magnetic field of the beam are both taken into account as proven in Ref.[40]. (If the self-magnetic field of the beam is not taken into account and the beam density profiles have sharp edges, the beam is subject to the two-stream transverse instability [40]). Similar conclusions were drawn in experimental study of Ref.[30].

Because the radial expansion due to two-stream instability is of order of few millimeters, one way to detect such an expansion is to aperture the beam to a small radius of few millimeters at the injection location into plasma. Propagation in a background plasma of a beamlet of radius $r_b = 0.1 \text{ cm}$ was simulated using PIC code LSP with the same other parameters as in Fig.1. Figure 3 shows evolution of the beam density profile in the beam frame as it propagates through background plasma.

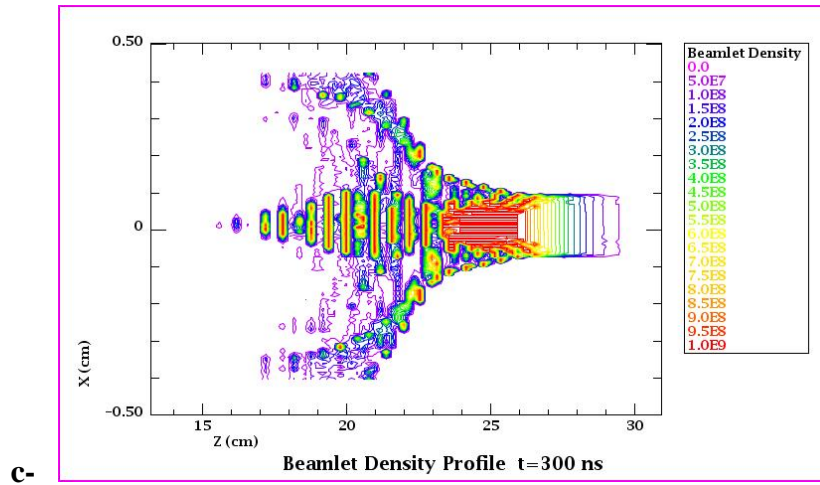
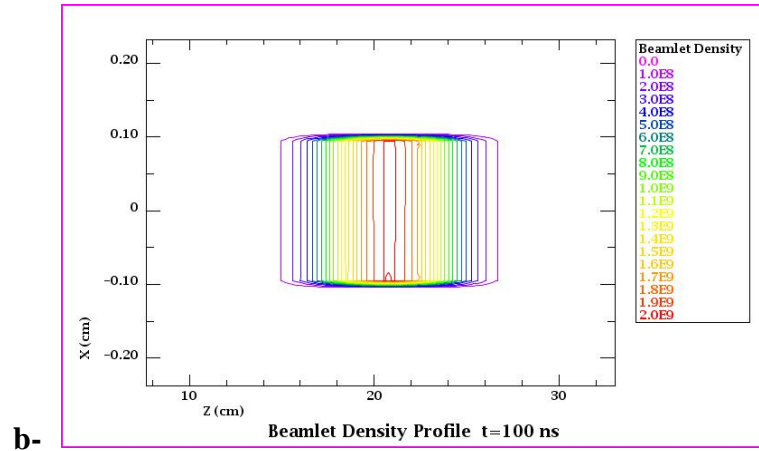
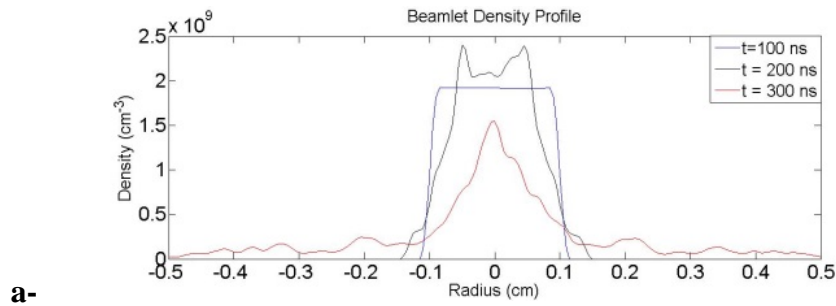
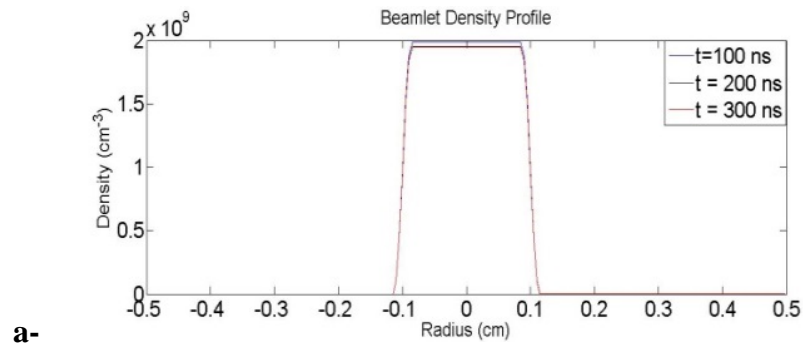


Fig.3. The evolution of the beam density profile during propagation in the background plasma similar to Fig.1 but beam radius is $r_b = 0.1$ cm instead of $r_b = 2.5$ cm. Fig.3(a): Radial beam density profiles at the beam pulse central cross section at $t = 100, 200$ and 300 ns. Fig.3 (b): beam density contour plots at $t = 100$ ns (1 m of propagation in plasma), Fig.3(c): beam density contour plots at $t = 300$ ns (3 m of propagation in plasma).

The results of simulation of the beamlet ($r_b = 0.1$ cm) propagation in the background plasma indicate that unlike the original beam, the extracted beamlet becomes significantly distorted and defocused due to effects of the two-stream instability. In fact the ratio of the relative beam radius change, $\Delta r_b / r_{b0} > 4$ for $t = 300$ ns as

evident from Fig.3. Therefore, measuring the spot size of an extracted beamlet can be a valuable tool to detect the presence of two-stream instability as a novel diagnostic method not employed previously to the best of our knowledge.

We have also performed simulations to demonstrate that the radial spreading of the beamlet is due to the defocusing effects of the two-stream instability and not because of numerical effects and is bigger than the spread due to initial transverse emittance of the beam. For a perpendicular ion temperature $T_{\perp i} = 1\text{eV}$ the radius increase due to thermal velocity is approximately 1.1 mm for a propagation time of 300 ns corresponding to 3 meters of propagation in plasma. This is significantly smaller than approximately 4 mm radial spread due to the two-stream defocusing effect. Moreover, the thermal spread can be easily identified because it is linear with distance and can be calculated from the measured beam emittance, whereas the defocusing effect due to the two-stream instability is highly nonlinear with distance because the beam particles are accelerating radially under time-varying transverse fields. Note that all simulations performed for this work assumed a cold beam and plasma species $T_{\perp i} = T_{\parallel i} = 0\text{eV}$ for simplicity. To illustrate absence of artificial numerical effects, we performed a simulation, where the ion beamlet is charge and current neutralized by an electron beam of same velocity and density profile. In this case the instability does not develop because there is no relative streaming between beam ions and electrons. Figure 4 shows the evolution of the ion beam density for this case. As is evident from Fig. 4 there is no modulation of the ion beam density, which demonstrates that, as expected, the two-stream instability is entirely absent. Moreover, both the density contour plots and the radial profile show almost no space change during propagation, which proves that the defocusing of the ion beamlet in a stationary background plasma is entirely due to strong forces generated by the (longitudinal) two-stream instability that resonantly moves with the ion beam and thus can affect radial profile of the beam.



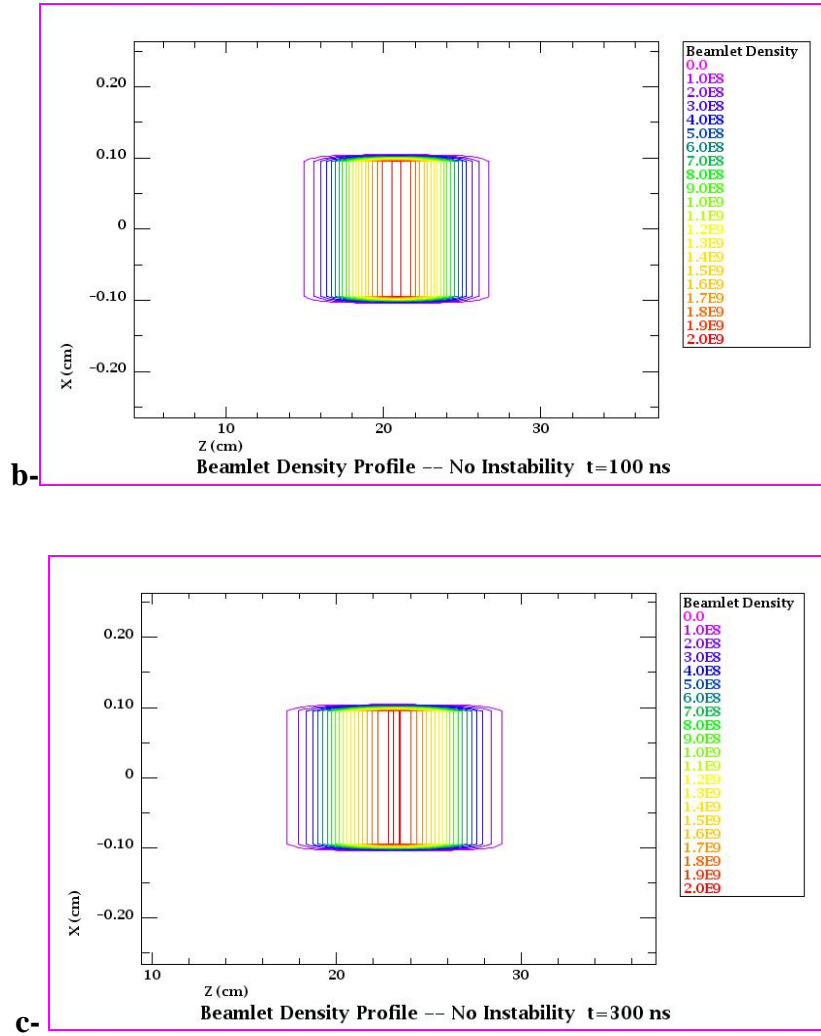


Fig.4. The evolution of the beam density profile during propagation in the background plasma similar to Fig.3 but beam is neutralized by an electron beam with the same velocity and density profile, no background plasma is present. Fig.4(a): Radial beam density profiles at the beam pulse central cross section at t= 100, 200 and 300 ns. Fig.3 (b): beam density contour plots at t = 100 ns (1 m of propagation in plasma), Fig.3(c): beam density contour plots at t = 300 ns (3 m of propagation in plasma).

In initial NDCX-II experiments the beam energy may be lower and have some residual velocity tilt after inductive acceleration. The defocusing force scales as $F_{defocusing} \sim m_e v_b^2 / 2r_b$ [26]. With such a force for the beam radius to double under action of the defocusing forces, the beam should propagate distance [26]

$$L_{defocus} \sim r_b \left(\frac{m_b}{m_e} \right)^{\frac{1}{2}}.$$

This distance is independent of the beam velocity and plasma density. Therefore the defocusing effect should be also observed for the beam with smaller beam energy. The velocity tilt is known to reduce the two-stream instability growth rate as much as a factor of two [25]. However, 3 meter of propagation in plasma should be sufficient for two-stream instability to develop even with some velocity tilt [26].

Note also that the neutralization of the beam charge occurs on the plasma period scale, which is less than one nanosecond for these beam parameters. In general, the fast variations of the beam density on the scales faster than the plasma period- a ns duration for our parameters may drive large amplitude plasma waves, see e.g. Ref. [41]. However the beam pulse temporal variations occur on much longer time scales than the plasma period and wake-like excitation of the plasma waves is not expected.

In absence of the external magnetic fields, the beam can be neutralized during propagation in an undersense plasma, $n_b > n_p$ but with large plasma volume, so that total electron charge exceeds the beam charge [42]. In this case the two stream instability does not develop, because neutralizing electrons follow the beam with the same velocity [42]. As shown in Fig.4 in this case the beam-defocusing effect disappears and beam should propagate ballistically. This can be tested in future experiments as well.

In summary, we have shown that the beam-driven longitudinal two-stream instability between beam ions and plasma electrons can generate strong electric fields which can significantly distort the ion beam profile under certain conditions. We proposed that for NDCX-II parameters the defocusing effect of the instability can be observed if a thin beamlet is extracted from the beam source of radius of 1 millimeter and propagated in 3 meters of plasma. Such an experiment will provide an experimental tool to detect the presence of the two-stream instability in the beam-plasma system.

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