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An Information-theoretical Approach

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External vs Internal Triggering of Substorms: An Information-Theoretical Approach

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The role of external triggering of substorms through northward turning of the interplanetary magnetic field has been examined in a number of recent studies [Hsu and McPherron, 2002; Morley and Freeman, 2007]. While Hsu and McPherron [2002, 2004] argue that the strong association between external triggers defined by Lyons et al. [1997] and substorm onsets could be responsible for most substorms, Morley and Freeman [2007] argue that the association between northward turnings and substorm onsets are coincidental rather than causal, because the same external triggers are also closely associated with an artificial list of substorm onsets generated with the Minimal Substorm Model [Freeman and Morley, 2004], which has no requirement of northward turning. We examine an expanded list of substorms [Frey et al., 2004; Frey and Mende, 2006] using conditional redundancy, an entropy-based measure of conditional dependency, to examine whether northward IMF turning as an external trigger provides any additional information about substorm onset beyond knowing that there has been a period of sustained loading of energy flux (southward IMF). Our analysis reveals that only a few percent additional information is provided by the northward turning criterion, which is consistent with the statistics of surrogate datasets of external triggers constructed to coincide with 2% of substorms. We therefore conclude that northward turning of the IMF is, in general, coincidentally, rather than causally, associated with substorm onsets.
1. Introduction

The solar wind transfers energy into the magnetosphere, and correlation studies have established that energy flux is particularly strong when the interplanetary magnetic field (IMF) is southward. The magnetosphere responds to this energy flux in a complex manner, leading to large-scale activity such as strong convection or substorms. Understanding the underlying cause of substorm onset remains a central problem in space physics. One significant question is whether substorms are triggered externally (for example by changes in the IMF) or whether they are primarily an internal response that results when stored energy in the magnetotail exceeds a critical threshold.

Lyons [1995] proposed that the expansion phase of substorms results from a reduction in the large-scale convection electric field imparted to the magnetosphere from the solar wind following a period of strong magnetospheric convection (growth phase). This proposal was supported by observations suggesting that northward IMF turnings following a period of IMF $B_z$ that has been negative for more than 30 minutes triggers substorm onset [Caan et al., 1975; Rostoker, 1983; McPherron et al., 1986]. A subsequent observational study that employed coordinated observations to identify substorm onsets and compare with external triggers suggested a high likelihood to see an external trigger in conjunction with a substorm onset, where a trigger satisfies the following requirements [Lyons et al., 1997]:

1. Growth Phase Requirement: $B_z < 0$ for 22 of the previous 30 minutes
2. Turning Initiation: $B_z(t_0 + \Delta t) - B_z(t_0) \geq 0.375nT\Delta t/\text{min}$
3. Sustained:
   - $\nabla B_z(t : t + 10) \geq 1.75nT/\text{min}$
• $B_z(0 \leq t - t_0 \leq 3 \text{min}) \geq B_z(t) + 9.15 \times (t - t_0) \text{nT/} \text{min}$

• $B(z(3 \text{min} \leq t - t_0 \leq 10 \text{min}) \geq B_z(t) + 0.45 \text{nT}$

4. No other point in the previous 10 minutes is a trigger

A more quantitative investigation was subsequently performed by Hsu and McPherron [2002, 2003, 2004] using a substorm onset database determined from a decrease in the AL index in association with Pi2 pulsations. In that work they examined the association number between substorm onsets and triggers (which is like a cross-correlation) and found that there is a strong association between substorm triggers as defined above and substorm onsets. In their subsequent work, Hsu and McPherron [2004] examined the relative number of events and found that while most of the substorms could be associated with a trigger that nearly 40% were not associated with a trigger, which they considered to be evidence that substorms are caused by an internal instability. However, they suggested that the internal instability is susceptible to external perturbations as evidenced by the strong association of the substorms with external triggers.

Internal triggering of substorms is believed to result when sustained energy flux is stored up in the magnetotail leading to stretching of the tail and intensification of cross-tail current. When the energy stored in the tail attains a critical threshold energy is released as the result of a physical processes such as an internal instability [Lui, 1996; Cheng and Lui, 1998] or reconnection [Birn and Hones, 1981; McPherron, 1991]. The underlying dynamics of storage and release have been captured by simpler models such as a dripping faucet model Baker et al. [1990] or integrate and fire model such as the Minimal Substorm Model (MSM) [Freeman and Morley, 2004] or through circuit analogues [Horton...
and Doxas, 1996]. The basic underlying feature of storage and release models is that there is an energy input \( dE/dt = P(\text{solarwind}) \) that results when IMF is southward and the energy level is reset to a “ground state” dependent on the external boundary conditions (bc) \( (E \rightarrow E_0(\text{bc})) \) when the energy level exceeds a critical level \( E > E_c \).

Morley and Freeman [2007] reevaluated the data set considered by [Hsu and McPherson, 2002] considering the importance of internal vs external triggering. While their analysis confirmed that there is a high association number between external triggers and substorm onsets, they also considered whether the association was causal or coincidental.

To gain some insight, they defined an internal trigger as a period when the IMF has been southward at least 22 of the previous 30 minutes (equivalent to criterion 1 of the external trigger). Using only these “internal” triggers they found that there is also a strong association between “internal” triggers and substorm onsets. As a test of the relevance of northward turning for substorm onset, they considered a stream of solar wind data and constructed an alternative data set of substorm onsets using the integrate and fire MSM model. This alternative set of substorm onsets only depends on the energy input from the solar wind and has no dependence on northward turning of the IMF. However, when they performed a comparison of the external triggers with the alternative substorm data set, they found a strong association number in spite of the fact that these substorm onsets had no requirement of northward turning. The conclusion that they drew was that although northward turnings are correlated with substorms, they are not causally correlated with substorms. Following up on this study, [Freeman and Morley, 2009] used a superposed epoch analysis of IMF \( B_z \) with respect to substorm onset time to show that the tendency...
of the IMF to turn northward close to substorm onset could be explained simply by a bias of substorms to occur during southward IMF irrespective of a coincident rapid northward turning of the IMF, and similar results were found for MSM substorms, which have no northward turning requirement. [Newell and Liou, 2011] similarly noted that although the mean $B_z$ has a northward turning (reversion to the mean) starting 20 minutes before onset, a similar reversion to the mean was found for random elevations of solar wind driving based on several coupling functions, further supporting the concerns of Morley and Freeman [2007]. These analyses cast doubt on the hypothesis that northward turning of the IMF is causally related to substorm onset by providing simple alternative explanations for the association of the external trigger and the onset. However, these analysis do not directly address the question of whether northward IMF is causally related to substorm onset.

To address this issue we utilize information theory to analyze the following question: do external triggers (that satisfy 1-4 above) provide any additional information about substorm onsets beyond what is known from the energy flux into the system (criterion 1), and if so, how much more information is known. To answer this question we will utilize redundancy, which is an entropy-based measure of dependency.

2. Redundancy as a measure of dependency

Fraser [1989] and Prichard and Theiler [1995] pioneered the use of redundancy as a generalization of mutual information to examine multi-dimensional systems. To examine dependency between a set of variables that are measured, it is useful to consider whether

$$P(x_1, x_2, ..., x_n) \neq P(x_1)P(x_2)...P(x_n).$$

(1)
with $P(x_1, x_2, ..., x_n)$ the joint probability of measuring a combination of variables and $P(x_1), ..., P(x_n)$ the probability of measuring each of the variables separately. This relationship is preferable to examining cross-correlations because it allows more generally for nonlinear dependencies, which, in the case of substorms, should be considered given that substorm response is highly nonlinear. The question posed in Eq. 1 can be quantified using the following definition of redundancy as a discriminating statistic [Prichard and Theiler, 1995]

$$R(x_1; ...; x_m) = \sum_i H_1(x_i) - H(x_1; ...; x_m)$$

which measures how much additional information is known about the relationship of set of variables $(x_1; ...; x_m)$ when they are measured simultaneously rather than independently.

In the expression for redundancy, $H_1(x_i)$ is the entropy of measuring variable $x_i$ defined as

$$H_1(x_i) = -\sum_{\mathcal{N}} p(\hat{x}_i) \log p(\hat{x}_i)$$

where $p(\hat{x}_i)$ is the probability that the variable, $x_i$, lies in the partition, $\hat{x}_i$, of a set of discrete partitions of the domain, $\mathcal{N}$. Similarly, the joint entropy is obtained using

$$H(x) = -\sum_{\mathcal{N}} p(\hat{x}) \log p(\hat{x})$$

with $x = (x_1, ..., x_n)$ and $\hat{x} \in \mathcal{N}$.

In the case that none of the variables are related to each other, there is no redundancy and $R = 0$. Here, we are more interested in looking at conditional dependencies that are better described by marginal redundancy, which provides a measure of how much a
variable, $x_m$ depends on a set of other variables, $(x_1, ..., x_{m-1})$

$$R_M(x_1, ..., x_{m-1}; x_m) = R(x_1; ..., x_m) - R(x_1; ..., x_{m-1}).$$ (5)

In this study, we are particularly interested to know how an output, $x_m$, depends on another variable, $x_1$, given a vector of other inputs, $(x_2, ..., x_{m-1})$. The conditional redundancy

$$R_C(x_1|x_2, ..., x_{m-1}; x_m) = R_M(x_1, ..., x_{m-1}; x_m) - R_M(x_2, ..., x_{m-1}; x_m)$$ (6)

provides such a measure and allows us to determine if a given variable provides additional information beyond what we know from another set of inputs or whether that variable contains redundant information.

We can now state the question raised by the analysis of Morley and Freeman [2007] as a conditional redundancy. Is there any additional information about substorm onsets provided by external triggers (defined by conditions 1-4) given that the condition for “internal” triggering (condition 1) is known. This information is quantified by $R_C(\text{ext}|\text{int}; \text{onsets})$ and can be compared with $R_M(\text{ext}, \text{int}; \text{onsets})$ to obtain the fraction, $\mathcal{F}$, of additional information provided by knowledge of external triggers

$$\mathcal{F} \equiv \frac{R_C(\text{ext}|\text{int}; \text{onsets})}{R_M(\text{ext}, \text{int}; \text{onsets})}$$ (7)

In the case that onsets are mostly determined by external triggering independent of the loading rate $\mathcal{F} \to 1$, while in the case that there is no dependence on external triggers, $\mathcal{F} \to 0$. 
3. Database for substorm analysis

For our analysis, we consider the substorm onset list obtained by Frey et al. [2004] and Frey and Mende [2006]. These onsets were obtained using the FUV instrument on the IMAGE spacecraft. Substorms were identified if they fulfilled the following criteria: (1) a clear local brightening of the aurora has to occur, (2) the aurora has to expand to the poleward boundary of the auroral oval and spread azimuthally in local time for at least 20 min, (3) a substorm onset was only accepted as a separate event if at least 30 min had passed after the previous onset. The dataset contains over 2400 substorms during 2000-2005.

We also examined triggers in solar wind data defined by criteria (1-4) above using satellite measurements. Our primary source of data was the ACE satellite. We augmented the ACE data with WIND observations when ACE data was not available. Data gaps less than 5 minutes in duration were filled in using linear interpolation as in the study of Morley and Freeman [2007]. The data was propagated using the minimum variance method to \( \text{GSM}(x,y,z) = (17,0,0) \ R_E \) [Weimer et al., 2003].

For our analysis, we construct three variables (int, ext, onset) which take on the value of 0, 1, or NaN. Variable int=1 if criterion (1) is satisfied at the time of observation, int=0 if criterion (1) is not satisfied, and int=NaN if inadequate data is available to address criterion (1). Similarly, ext=0,1,NaN when criteria (1-4) are not all satisfied, all satisfied, or not enough data is available to assess criteria (1-4). In evaluating criteria (1-4) \( \Delta t \) is taken to be 1 minute. The variable onset=1 when there is a substorm onset and onset=0 otherwise. The data streams are obtained for every minute of data during the period
between the first and last substorm of the Frey et al. [2004]; Frey and Mende [2006] data list.

It is useful to first examine the statistics of substorm onsets and external triggers. In Figure 3 we show (a) the intersubstorm intervals obtained by considering the difference in onset times of the ordered list of substorms. Consistent with the data selection criterion of Frey et al. [2004] there are no intersubstorm intervals less than 30 minutes. A peak does appear at around 1 hour and a second peak at around 3 hours. The three hour peak has been previously reported [Borovsky et al., 1993; Prichard et al., 1996] and was interpreted as a periodic component that occurs in spite of random solar wind driving. They suggested that the three-hour timescale is an intrinsic property of the magnetosphere related to internal dynamics. It is also interesting to note a broad peak on the intersubstorm interval between 10-15 hours and at 25 to 30 hours. These peaks are likely an orbital bias that result from the 14.2 hour orbital period of the IMAGE spacecraft. Such a periodicity would arise because the imager was turned off during passage through the radiation belt and targeted the auroral oval, which would repeat on a timescale of 14 hours. For our analysis, this orbital bias is not relevant, because we consider timescales that are less than five hours as shown in panel (b).

Northward turning intervals were also identified in the data, and we have examined the statistics of northward turning intervals, which we have shown in panel (c) with the same time resolution as panel (b). It is to be noted that the smallest interval is 10 minutes consistent with criterion (4) for an external trigger. The distribution of northward turning intervals appears to fall off exponentially like those of a Poisson process suggestive that
northward turning is a somewhat random process. A comparison of panels (b) and (c) shows that the statistics of intersubstorm onset intervals is quite different than northward turning intervals.

4. Analysis of substorm redundancy

In this section, we compute the conditional redundancy using the int, ext, and onset datastreams. It is necessary to compute

\[ R_C(\text{ext}|\text{int}; \text{onset}) = R_M(\text{int}, \text{ext}; \text{onset}) - R_M(\text{int}; \text{onset}) \]
\[ = R(\text{int}, \text{ext}, \text{onset}) - R(\text{int}, \text{ext}) - R(\text{int}, \text{onset}) + R(\text{int}) \]
\[ = H(\text{ext}, \text{int}) + H(\text{int}, \text{onset}) - H(\text{ext}, \text{int}, \text{onset}) - H(\text{int}) \]

The entropies are computed from the joint probabilities functions: \( p(\text{ext}, \text{int}, \text{onset}) \), \( p(\text{int}, \text{onset}) \), \( p(\text{ext}, \text{int}) \), \( p(\text{int}) \) via Eq. 4. It is useful to consider a few limiting cases. If onsets could be entirely determined from criterion (1), then \( p(\text{int}, \text{onset}) = p(\text{onset}) = p(\text{int}) \) in which case \( R_C(\text{ext}|\text{int}; \text{onset}) \rightarrow 0 \). In the case where onsets have no dependence on criterion (1), then \( R_C(\text{ext}|\text{int}; \text{onset}) \rightarrow H(\text{ext}) + H(\text{onset}) - H(\text{ext}, \text{onset}) \equiv M(\text{ext}, \text{onset}) \), where \( M \) is the mutual information [Prichard and Theiler, 1995].

Because data only appears in binary, the distributions for \( p(x_1, x_2, x_3) \) are simply computed by converting data to binary numbers \( (x_1 = 1, x_2 = 0, x_3 = 1 \rightarrow 101 = 5) \). A number is obtained for each measurement, then the numbers are sorted and instances are counted. Division of the total instances of each number by the total number of observations provides the probability for that number. Entropy involves summing \( p \log p \) for each number.
Because the entropy is based on probabilities, the statistics will depend on the time resolution. To explore this effect, we constructed resampled variables, \( \bar{a} \), that consider whether a trigger or onset occurs within a window of \( \pm h \) minutes around time, \( t \). More specifically, 
\[
\bar{a}(t) = \begin{cases} 
1 & \text{if any of } \{a(t-h), \ldots, a(t+h)\} = 1, \\
NaN & \text{if all of } \{a(t-h), \ldots, a(t+h)\} = NaN, \\
0 & \text{otherwise.}
\end{cases}
\]
A similar windowing was used in prior studies of association number to improve statistics [Hsu and McPherron, 2002; Morley and Freeman, 2007]. In the present study, it should be noted that both the triggers and onsets are both windowed, so the overlap between a trigger and onset occurs when they are separated by \( 2h \). In the following analysis, we present results with \( h = 5 \), which provides good statistics without overly smoothing the results.

The results of our analysis are shown in Figure 2. Panel (a) shows in blue the conditional redundancy of the dependence of substorm onsets (at a given time lag, \( \tau \)) on external triggers given the internal trigger [i.e. \( R_C(\text{ext}(t)|\text{int}(t); \text{onset}(t+\tau)) \)] and for comparison the conditional redundancy when the list of external triggers is randomized in red. It is apparent that \( R_C \) is elevated with respect to random triggers around substorm onset and there are secondary elevations at 50 minutes and a 3 hours after the external trigger.

The secondary elevations correspond with peaks seen in the statistics of intersubstorm intervals shown in Figure 3. To place these elevations into context, in panel (b) we show \( R_C(\text{int}(t)|\text{ext}(t); \text{onset}(t+\tau)) \), which indicates how much additional information about substorm onset is provided from the growth phase requirement beyond that which is known from the northward turning requirement. The peak is a factor of 25 larger than that seen in panel (a) suggestive that external triggers only provide an additional 4% more
information. The internal trigger peak occurs at 15 minutes following onset with a broad
distribution with a full width at half maximum around 1 hour. Secondary elevations seen
in panel (b) occur at 2 hours and 3 hours and do not coincide with the peaks seen in panel
(a).

To further interpret the value of the conditional redundancy obtained in our analysis,
we also perform a comparison with a surrogate dataset of external triggers constructed
using the onset list. Holding the number of external triggers fixed, we select a percentage
of the onsets to coincide with the onsets, and the remainder of the external triggers are
randomized. Panel (c) shows the conditional redundancy when 20 percent of the external
triggers are selected to coincide with onsets and the remainder of the external triggers are
randomized. As expected, the peak coincides with substorm onset and is dramatically
elevated (factor of 50) compared with that due to northward turning.

Panel (d) shows the significance of external triggering and the fractional information
$F$ is indicated by the color. The significance is obtained from $S = |R_C - \mu|/\sigma$ where $\mu$
and $\sigma$ are the mean and spread of the surrogate dataset of randomized triggers shown in
panel (a). There is clearly a peak of significance suggestive that some substorms may be
triggered; however, because the fractional information for the peak is small, $F \approx 0.04$, it
is likely that only a few substorms are triggered. The increase of $F$ away from the onset
peak results from a reduction of information from substorm growth phase

To further quantify how many substorms are triggered, we construct surrogate datasets
as in panel (c) where the fraction of substorms that are triggered varies from 0 to 100%.
The results presented in Figure 3 show that the value of conditional redundancy obtained in panel (a) is consistent a dataset constructed with 2% triggered substorms.

5. Conclusions

Prior work has suggested that northward turning of the IMF is closely associated with substorm onset. Questions have arisen about whether this association is coincidental or causal. In this paper, we have provided a quantitative analysis, based on conditional redundancy, that demonstrates that northward turning of the IMF is, in general, coincidental with substorm onsets rather than causal. This finding is consistent with the analysis of Freeman and Morley [2009] and Newell and Liou [2011], which suggested that the association between northward turning of the IMF is most likely an indicator of a reversion to the mean rather than a trigger. Increased driving of the magnetosphere through other coupling functions also showed a similar northward turning of the IMF consistent with a reversion to the mean. These results are consistent with the study of Morley and Freeman [2007], which suggested that artificial substorms, essentially driven only by a coupling function, are well associated with northward IMF turnings. Newell and Liou [2011] also points out that southward IMF turnings would be equally likely to be associated with substorm onset.

This example also demonstrates the feasibility of using information-theoretical tools, such as conditional redundancy, to determine whether correlations are coincidental or causal in nature. Because these tools are founded on statistics, they should be applied to datasets of sufficient size to ensure convergence of the multivariate probabilities, and the results should be compared with surrogate data sets to establish its significance.
These techniques could be used to address causal roles of solar interplanetary structures on the wave environment of the inner magnetosphere and radiation belt responses or to understand identify the causal roles of waves, field aligned currents, electron precipitation, and ion outflows in the coupled magnetosphere-ionosphere system [e.g. Strangeway et al., 2005].

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References


Figure 1. Statistics of substorms and external triggers. Panels (a) and (b) show the statistics of intersubstorm intervals (on hour and minute timescales). Panel (c) shows the statistics of external triggers defined as sustained northward IMF turning following a growth phase.
Figure 2. Conditional redundancy describing how much additional information about onsets, $R_C(\text{ext}(t)|\text{int}(t); \text{onset}(t + \tau))$, is added by knowing external trigger events (ext) given the growth phase requirement (int), as a function of $\tau$ for: (a) external triggers satisfying Lyons [1995] (blue) and random external triggers (red), (b) $R_C(\text{int}(t)|\text{ext}(t); \text{onset}(t + \tau))$, (c) 20 percent of external triggers coincide with onset list and the remainder are random, and (d) the significance of the external trigger compared with random triggers with the fractional information, $\mathcal{F}$, shown in color.
Figure 3. The conditional redundancy of datasets constructed with a percentage of onsets externally triggered and the remainder of the external triggers randomized is compared with the peak of conditional redundancy from Figure 2, which is consistent with a dataset of external triggers constructed with 2% accuracy.