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Eun-Hwa Kim, Jay R. Johnson,
Ernest Valeo, and Cynthia K. Phillips

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Global Modeling of ULF waves at Mercury

2 Eun-Hwa Kim¹, Jay R. Johnson¹, Ernest Valeo² and Cynthia K. Phillips²

4 Corresponding author: E.-H. Kim, Princeton Center for Heliophysics, Princeton Plasma Physics
Laboratory, Princeton University, P.O. Box 0451, Princeton, NJ 08543-0451, USA
6 (ehkim@pppl.gov)

¹ Princeton Center for Heliophysics,
Princeton Plasma Physics Laboratory,
Princeton University, Princeton, New
Jersey, USA

² Princeton Plasma Physics Laboratory,
Princeton University, New Jersey, USA

8 **Abstract**

ULF waves in the ion cyclotron frequency range waves have been often observed at Mercury's magnetosphere. Although previous statistical studies have shown that ULF waves are primarily compressional near the equator and transverse with linear polarization at higher latitude, the underlying reason for this distribution of wave polarization has not been understood. In order to address this key question, we have developed a two-dimensional, finite element code that solves the full wave equations in global magnetospheric geometry. Using this code, we show that (1) efficient mode conversion from the fast compressional waves to the ion-ion hybrid resonance occurs at Mercury consistent with previous calculations; (2) such mode-converted waves globally oscillate similar to field-line resonance at Earth; and (3) compressional wave energy is primarily localized near the equator, while field-aligned transverse, linearly polarized waves generated by mode conversion at the ion-ion hybrid resonance radiate to higher latitude. Based on these wave solutions, we suggest that the strong transverse component of observed ULF waves at Mercury in high magnetic latitude can be explained as excitation of the field-line resonant waves at the ion-ion hybrid resonance.

24 1. Introduction

26 NASA's Mariner 10 and MESSENGER (MErcury Surface, Space ENvironment,
GEOchemistry, and Ranging) spacecraft missions provide a wealth of observations within study
Mercury's magnetosphere [e.g., *Ness et al.*, 1974, *Slavin et al.*, 2008, 2009, *Anderson et al.*,
28 2008, 2011], and both spacecraft detected ultra-low frequency (ULF) waves in the ion cyclotron
frequency range [*Russell*, 1989; *Boardsen et al.*, 2009a, b, 2012].

30 A recent statistical study of ULF waves at Mercury based on MESSENGER data [*Boardsen et*
al., 2012] reported the occurrence and polarization of the detected waves. The waves are usually
32 observed between radial distances of 1.1 to 1.8 R_M with maximum occurrence near $R_M \sim 1.4$,
where R_M is Mercury's radius (2440km), and between magnetic latitudes of $\pm 40^\circ$ with strong
34 occurrence near the magnetic equator. These waves are also detected primarily on the night side
with two maximum peaks around MLT 6 and 21. Measurements of wave polarization indicate
36 that the waves are mostly compressional near the magnetic equator, but the transverse
component is dominant at latitudes greater than $\sim 20^\circ$. The transverse fluctuations are also
38 linearly polarized.

The origin of these ULF waves has been of great interest since they were first detected by
40 Mariner 10 [*Russell*, 1989]. Because of their linearly polarization, they were thought to result
from field-line resonance in the multiple ion plasmas [e.g., *Russell*, 1989; *Othmer et al.*, 1999;
42 *Glassmeier et al.*, 2003, 2004; *Kim and Lee*, 2003; *Klimushkin et al.*, 2006]. However, it was
later argued that the waves detected by Mariner 10 are not field-line resonances because the
44 observed waves also have a significant magnetic compressional component [*Blomberg*, 1997;
Southwood, 1997; *Kim et al.*, 2008].

46 Field-line resonances at Mercury are expected to occur at the ion-ion hybrid (IIH) resonances
and should have purely transverse polarization [Kim *et al.*, 2008]. When incoming fast
48 compressional waves (FWs) match the IIH resonance condition, strong absorption of incoming
FWs can occur leading to excitation of the IIH resonance [e.g., Karney *et al.*, 1979]. Numerical
50 simulations in slab coordinates showed that this mode-converted IIH waves are guided by the
ambient magnetic field line (\mathbf{B}_0) and have linear polarization [Kim *et al.*, 2008, 2011, 2013].
52 However, the simulations did not describe the two-dimensional magnetic curvature effect, which
can separate the regions of compressional wave dominance from regions dominated by the
54 global transverse mode converted wave.

Similar to Kim *et al.* [2008], Boardsen *et al.* [2012] also concluded that the bulk of the wave
56 observations from MESSENGER cannot be explained by ion cyclotron waves or field-line
resonance because of high compressibility of the detected waves near the magnetic equator.
58 Instead, they proposed that the observed waves could be mainly composed of short wavelength
(~100 km) electromagnetic ion Bernstein mode waves (IBWs) in a high β proton plasma. Using
60 ray tracing calculations, they confirmed that electromagnetic IBWs generated by loss cone
instability can explain MESSENGER observations where the compression straddles the equator
62 [Boardsen *et al.*, 2014]. Although the IBWs in the ray tracing calculations can exhibit transverse
polarization off the equator, they reflect at magnetic latitudes around 12° , which is much less
64 than typically observed (most dominant at 20° extending up to 40°).

Therefore, a key question that remains concerning ULF waves at Mercury is why the waves
66 are primarily compressional near the equator and transverse with linear polarization at higher
latitude. If the waves are driven by the same source, does the polarization change because of
68 propagation effects or because of mode conversion processes? In order to address these questions,

we have developed a two-dimensional full wave code using the finite element method and
 70 unstructured triangle mesh, which has been used to examine general plasma waves in Tokamak
 [e.g., *Brambilla*, 1999] and field-line resonances in the magnetosphere [e.g., *Lu et al.*, 2003]. The
 72 code we developed describes three-dimensional wave structure including mode conversion when
 plasma waves are launched in two-dimensional axisymmetric background plasma with arbitrary
 74 magnetic field topology. Using the newly developed wave code, we examine ULF waves at
 Mercury that have been detected inside Mercury's magnetosphere by Mariner 10 and
 76 MESSENGER spacecraft.

2. Two-Dimensional Full Wave Code

78 We utilize a two-dimensional, finite element code that solves the full wave equations in global
 magnetospheric geometry. Assuming a time dependence $\exp(-i\omega t)$, where ω is an angular
 80 frequency, the linear and cold plasma wave equation takes the form,

$$\nabla \times \nabla \times \mathbf{E} - \frac{\omega^2}{c^2} \boldsymbol{\varepsilon} \cdot \mathbf{E} = 4\pi i \frac{\omega}{c^2} \mathbf{J}_{ext}, \quad (1)$$

82 where \mathbf{E} is the perturbed electric field, \mathbf{J}_{ext} is the (localized) external current source launching the
 waves within our model, $\boldsymbol{\varepsilon}$ is the dielectric tensor, which is expressed in coordinates aligned
 84 along and across the local \mathbf{B}_0 direction,

$$\boldsymbol{\varepsilon} = \varepsilon_S (\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}}) + i\varepsilon_D \mathbf{I} \times \hat{\mathbf{b}} + \varepsilon_P \hat{\mathbf{b}}\hat{\mathbf{b}}, \quad (2)$$

86 where $\hat{\mathbf{b}} = \mathbf{B}_0 / |\mathbf{B}_0|$ and $\varepsilon_{S,D,P}$ are dispersion functions defined by *Stix* [1992]. Because wave Eqs.
 (1)-(2) include all electron and ion effects, the code enables us to study a wide range of fluid
 88 waves from MHD waves to Langmuir oscillations in a consistent manner when the system is
 arbitrarily inhomogeneous, but in this paper we confine our study to wave in the ion cyclotron
 90 frequency range.

In this frequency range, typically $\varepsilon_p \gg \varepsilon_{S(D)}$ and $E_{\perp} \gg E_{\parallel}$, where subscript $\parallel(\perp)$ is along
 92 (perpendicular) to \mathbf{B}_0 , so accurately capture this disparity, the field is represented in terms of its
 projections along and perpendicular to \mathbf{B}_0 . We pick a local orthogonal basis $(\hat{\mathbf{b}}, \hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\eta}})$, where the
 94 unit vector $\hat{\mathbf{b}}$ is along the magnetic field line from north to south, $\hat{\boldsymbol{\phi}}$ is the azimuthal direction
 and $\hat{\boldsymbol{\eta}}$ is normal to the field line pointing outward ($\hat{\boldsymbol{\eta}} = \hat{\mathbf{b}} \times \hat{\boldsymbol{\phi}}$), with the assumption of $\mathbf{B}_0 \cdot \hat{\boldsymbol{\phi}} = 0$,
 96 appropriate for a 2D model with azimuthal symmetry.

Eq. (1) can then be expressed in cylindrical coordinates $(\hat{\mathbf{r}}, \hat{\mathbf{z}}, \hat{\boldsymbol{\phi}})$ with

$$98 \quad E = \sum_m \left(E_b^m \hat{\mathbf{b}} + E_n^m \hat{\boldsymbol{\eta}} + E_{\phi}^m \hat{\boldsymbol{\phi}} \right) \exp(im\varphi), \quad (3)$$

where m are azimuthal wavenumbers. For a dipole field, $\hat{\mathbf{b}} \equiv 3\sin\theta\cos\theta\hat{\mathbf{r}} + (3\cos^2\theta - 1)\hat{\mathbf{z}}$, where
 100 $\cos\theta = z / \sqrt{r^2 + z^2}$.

In order to adapt to multiple singular lines, we solve Eq. (1) on an unstructured triangular
 102 mesh. We represent the variation of the field within each triangle by vertex-based linear finite
 elements local basis function, $F_{j,k}$, where j labels each triangle and $k = 1, 2, 3$ labels each of its
 104 vertices. That is, $F_{j,k}$ varies linearly between 1 at vertex k^{th} and 0 at the other vertices, and is
 identically 0 outside triangle j . Since we assume axisymmetry in the azimuthal direction, thus the
 106 electric field is represented as:

$$E(r, z) \exp(im\varphi) = \sum_{j,k} E_{j,k} F_{j,k}(r, z) \exp(im\varphi). \quad (4)$$

Eq. (4) is cast into matrix form by taking its inner product in turn with each $F_{j,k}$, and integrating
 over the whole plasma volume by parts to obtain the weak variational form [e.g., *Brambilla*,
 110 1999],

$$\int d\varphi r dr dz \left\{ (\nabla \times \mathbf{F}_{j,k}) \cdot (\nabla \times \mathbf{E}) + \mathbf{F}_{j,k} \cdot \boldsymbol{\varepsilon} \cdot \mathbf{E} + i \frac{\omega}{c^2} \mathbf{F}_{j,k} \cdot \mathbf{J}_{ext} \right\} = 0. \quad (5)$$

112 Substitution of Eq. (4) into Eq. (5) yields a sparse matrix system that is amenable to solution by
standard algorithms.

114 One advantage of using the finite element method is that the local basis functions can be
readily adapted to boundary shapes [e.g., *Lu et al.*, 2003] and can be packed in such a way as to
116 provide higher resolution in regions where solutions may exhibit singular behavior. In order to
generate magnetospheric geometry, we have adapted the DISTMESH algorithm [*Persson and*
118 *Strang*, 2004]. The algorithm constructs a 2D triangular given a specified boundary and a target
density function. The density of the mesh can be specified based on the expected wavelength
120 obtained from solution of the local dispersion, except close to resonances, so that the most
efficient resolution is used.

122 3. Wave Solutions at Mercury's magnetosphere

As shown in Figure 1, we adopt Mercury's dipole field model by assuming the magnetic field
124 strength at Mercury's surface is 3.1×10^{-7} T [*Anderson et al.*, 2008]. Because sodium is one of the
major heavy ions at Mercury [e.g., *Zurbuchen et al.*, 2008, 2011; *Raines et al.*, 2013, 2014],
126 similar to previous numerical studies [*Kim et al.*, 2008, 2011] we adopt an electron-proton-
sodium plasma with constant electron density ($N_e = 1 \text{ cm}^{-3}$) and sodium concentration ratio
128 ($\eta_{Na} = N_{Na} / N_e = 15\%$).

We solve Eq. (1) in the domain as $0 < r/R_M < 2.5$ and $-2 < z/R_M < 2$ as shown in Figure 1
130 and Eq. (1) using absorbing boundary conditions (which only allows outgoing wave solutions) at
the edge of the solution domain except at Mercury's surface, which is assumed to be a perfect
132 conductor leading to wave reflection. In this calculation, for simplicity, the azimuthal wave

number m of the source is assumed to be 0. Although for an Alfvén resonance, $m = 0$ would not
 134 allow mode conversion, when heavy ions are present, mode conversion can still occur even for m
 = 0.

136 Similar to the waves detected at Mercury, we select wave frequency of $f = 0.95$ Hz (thus
 $\omega = 2.98$ Hz) near $(r_{ext}, z_{ext}) = (2R_M, 0)$ with

$$138 \quad \mathbf{J}_{ext} = [\exp(-(r - r_{ext}) / \Delta r_{ext}) + \exp(-(z - z_{ext}) / \Delta z_{ext})](\hat{\eta} - i\hat{\phi}), \quad (6)$$

where $\Delta r_{ext} = 0.125 R_M$ and $\Delta z_{ext} = 0.25 R_M$, respectively. Based on the wavelength of dispersion
 140 relation of (a) left-handed polarized waves ($n_{\parallel}^2 = \varepsilon_L$), (b) IHH waves ($n_{\parallel}^2 = \varepsilon_S$) propagating along
 \mathbf{B}_0 , and (c) fast wave modes ($n_{\parallel}^2 = \varepsilon_L \varepsilon_R / \varepsilon_S$) propagating perpendicular to \mathbf{B}_0 , we generate the
 142 triangle mesh producing 43349 nodes and 85702 triangles. The computing time is approximately
 450 seconds CPU time to obtain a solution.

144 The dispersion relation of FW modes can be written as

$$n_{\perp}^2 \cong \frac{(\varepsilon_R - n_{\parallel}^2)(\varepsilon_L - n_{\parallel}^2)}{(\varepsilon_S - n_{\parallel}^2)}. \quad (7)$$

146 Eq. (7) exhibits two cutoffs at $n_{\parallel}^2 = \varepsilon_{R(L)}$, thus when FW modes propagate perpendicular to \mathbf{B}_0
 (i.e., $n_{\parallel}^2 = 0$), the FW modes partially reflect where $\varepsilon_L(\omega = \omega_L) = 0$ or $\varepsilon_R(\omega = \omega_R) = 0$. Eq. (7) also

148 has a resonance where,

$$n_{\parallel}^2 = \varepsilon_S, \quad (8)$$

150 which is called the IHH resonance because it occurs between the two ion cyclotron frequencies.
 When Eq (8) is satisfied, mode conversion from FW modes to IHH resonance occurs and the
 152 mode-converted IHH waves propagate along \mathbf{B}_0 with dispersion relation of $n_{\parallel}^2 = \varepsilon_S$. As the mode
 converted wave propagates to higher magnetic latitude, these waves are partially reflected where

154 $\varepsilon_S(\omega = \omega_{bb}) = 0$ [Johnson et al., 1995; Johnson and Cheng, 1999; Kim et al., 2009], where ω_{bb}
 is the Buchsbaum resonance frequency [Buchsbaum, 1960].

156 Figure 2 shows the wave solutions of the fluctuating electric (**E**) and magnetic (**B**) fields. In
 this figure, both the FW and the mode-converted IHH resonant wave modes are clearly seen. The
 158 FW mode is directly launched near (r_{ext}, z_{ext}) and propagates to the inner/outer magnetosphere.
 This wave mode appears in all electric and magnetic field components. Because the FW mode is
 160 launched near the magnetic equator as marked “A” in Figure 2b, the waves propagating to inner
 magnetosphere partially reflect near $r/R_M = 1.43$, where $\omega = \omega_L$ (as marked “B” in Figure 2b)
 162 The rest of the energy penetrates deep into the magnetosphere and totally reflects at Mercury’s
 surface due to the reflecting boundary condition. The FWs reflected from Mercury’s surface
 164 propagate to the outer magnetosphere with different propagation angle (as marked “C” in Figure
 2b), and are eventually absorbed near the outer boundary.

166 The IHH resonant waves appear only in E_η , E_b , and B_ϕ in Figure 2 showing long wavelength in
 the $\hat{\mathbf{b}}$ direction and fine structure in the $\hat{\boldsymbol{\eta}}$ direction for $1.4 \leq L_M \leq 1.8$, where L_M is Mercury’s L-
 168 shell, which is consistent with Kim et al. [2015] Because the waves launched at $L_M = 2$ are pure
 FW modes, the waves showing field-aligned structures in E_η and B_ϕ are unmistakable evidence
 170 of mode conversions.

These waves also show a wide range of field-aligned harmonic numbers that increase with
 172 along L_M . For instance, the fundamental and 3rd harmonics are seen at $L_M \sim 1.5$ and 1.7,
 respectively. These results are predicted by previous 1D full wave calculations. Kim et al. [2011]
 174 showed when the plasma contains 10-17% sodium at $L_M = 1.5$, strong absorption of the FW
 mode occurs in fundamental wave modes, which is consistent with 2D results. They also show

176 that the wave frequency with higher field-aligned harmonic numbers has stronger absorption at a
single magnetic field line when the plasma contains a higher concentration of heavy ions.

178 In Figure 2, we also over-plot the locations where $\omega = \omega_{bb}$, which corresponds to the cutoff of
the field-aligned propagation mode-converted waves. Waves in higher L_M encounter the cutoff
180 location before they reach the ground, but because of the wavelength and power are enough large
the to penetrate the evanescent region between Buchsbaum resonance to the ground, we found
182 wave power near the surface. Therefore, the IIIH resonant waves globally oscillate along the
magnetic field-line similar to the field-line resonance at Earth [e.g., *Lee and Lysak, 1989*].
184 Conversely, field-line resonances occur at Mercury when the IIIH resonance conditions are
satisfied [e.g., *Kim et al., 2008*] and because such resonances globally oscillates, they reach
186 higher magnetic latitudes.

In Figure 3, the Poynting flux (S) is also calculated in order to examine the flow of wave
188 energy. Because the FW mode is launched near the magnetic equator at $L_M = 2$, S_η shows most of
the FW energy propagates toward the inner magnetosphere for $L_M < 2.0$ and the outer
190 magnetosphere for $L_M > 2.0$. Near $L_M = 1.43$, the FW cutoff is clearly seen in the drop off
Poynting flux but because wave power is strong enough to tunnel through the evanescent region,
192 the rest of FW energy reflects at the surface as we expected in Figure 2. The reflected waves also
propagate outward at higher latitude. It is noted that spatial structure of S_η are same as the FW
194 structure shown in E_η and E_b . On the other hand, S_b clearly shows the fine structure of field-
aligned IIIH wave modes. Because the FW energy launched at the magnetic equator, wave energy
196 clearly flows poleward, but because IIIH waves reflect near the surface, equatorward energy flow
is also seen near $L_M = 1.6$ and 1.9 .

198 Based on eigenmode solution shown in Figure 2, we reproduce the time-dependent wave
solutions, such as,

$$200 \quad E(r, z, t) = E(r, z) \exp(-i\omega t), \quad (9)$$

as shown in Media 1. The Media 1 clearly shows how FW and mode-converted IHH waves
202 propagates in time.

4. Discussion and Summary

204 In this letter, we utilized a two-dimensional wave code in multi-fluid plasmas that solves the
full wave equations in global magnetospheric geometry. Because this wave code adopts a finite
206 element method and unstructured triangle mesh, it provides good resolution of plasma waves in
planetary magnetospheres where mode conversion is expected to occur.

208 Using this code, we showed how field-aligned propagating ULF waves at Mercury can be
generated by externally driven FWs via mode conversion at the ion-ion hybrid resonance.
210 Consistent with the previous real time simulations in slab geometry [*Kim et al.*, 2008, 2013], the
mode-converted IHH waves are strongly guided by \mathbf{B}_0 and have linear polarization in E_{μ} and B_{ϕ} .
212 Although the IHH waves are partially reflected at the Buchsbaum resonance at higher magnetic
latitude, these waves penetrate the wave stop gap and globally oscillate. Thus the IHH waves can
214 propagate to magnetic latitudes over 40° for realistic magnetic field and densities at Mercury.
These results are consistent with the observed waves that can propagate over magnetic latitude of
216 40° with strong magnetic transverse component [*Boardsen et al.*, 2012].

Some wave events detected by MESSENGER often have a narrow frequency band near 1 Hz
218 in a wide range of L-shell. For instance, waves with 1Hz are observed in $1.3 < L_M < 1.8$ and $0 <$
MLT $< 42^\circ$ as shown in Figure 4 from *Boardsen et al.* [2012]. This observation is consistent with

220 the finite element calculation, which the IHH waves with a single wave frequency in a wide range
of L shell ($1.4 < L_M < 1.8$).

222 The simulations also show that field-aligned harmonic number increases along with L-shell
with consistency to 1D full wave calculations. *Kim et al.* [2011, 2015] estimated the efficiency of
224 mode conversion of radially propagating FWs at the IHH resonance using a simplified 1D slab
cold plasma model that captures the essential features of the IHH resonance. In particular, *Kim et*
226 *al.* [2015] showed that a relatively large absorption occurs in wide range of heavy ion density
concentration ratios. Unfortunately these results then suggest that the heavy ion density
228 concentration ratio at Mercury cannot be simply inferred from the frequency of field-aligned IHH
waves as previous suggested [*Kim et al.*, 2008; *Kazakov and Fülöp*, 2013].

230 Although we obtained general characteristics of FLR at Mercury, there are several limitations
in our work. First, we adopt a cold plasma model even though kinetic effects in hot plasmas are
232 believed to be important for the generation and propagation of ULF waves at Mercury [e.g.,
Glassmeier et al., 2003; *Slavin et al.*, 2009; *Boardsen et al.*, 2012, 2014]. In hot plasmas, the FW
234 mode can be converted into the short wavelength ion Bernstein wave (IBW) or ion cyclotron
wave (ICW) modes (or their higher harmonics) at the IHH resonance [e.g., *Wukitch et al.*, 2005].
236 The mode-converted IBW modes have short-wavelength in the radial direction and long-
wavelength in field-aligned direction, thus the mode-converted IBW mode is expected to
238 propagate at large normal angle because of $k_{\perp} \gg k_{\parallel}$. Therefore, a short wavelength IBW mode
at Mercury could be either externally generated via mode conversion at the IHH resonance or
240 internally generated from plasma instability [*Boardsen et al.*, 2014]. It should be noted that the
wave occurrence is much higher near the magnetic equator than off the equator and wave
242 characteristics at higher latitude are mainly transverse while near the equator the waves are

strongly compressible. Thus the compressionally dominant waves near the magnetic equator
244 could be mixed with internally generated and externally mode-converted IBW modes while
waves at higher magnetic latitudes could be mode-converted IHH waves (or IBWs).

246 Second, we assume electron-hydrogen-sodium plasmas and a constant sodium density for
simplicity. When lighter ions, such as He^+ and He^{++} , are included in Mercury's magnetosphere,
248 the cutoff condition of the field-aligned waves, such as the Buchsbaum resonance, shift toward
the outer magnetosphere and lower magnetic latitude. In addition, depending on the heavy ion
250 density ratios, the wave stop gap between the locations where $\omega = \omega_{bb}$ and $\omega = \omega_{ci}$ becomes
wider or narrower [Kim et al., 2015]. Thus the mode-converted waves can be tunnel through the
252 wave stop gap similar to left-handed polarized EMIC waves at Earth [Johnson et al., 1989, 1995,
Johnson and Cheng, 1999] or localized between Buchsbaum locations [Kim et al., 2009]
254 depending on wave tunneling effects.

Finally, we adopt a simple dipole magnetic field model with a perfectly conducting boundary
256 condition at Mercury's surface rather than realistic magnetic configurations [e.g., Anderson et al.,
2008, 2010, 2011] or conductivity [Anderson et al., 2014]. Because the boundary condition with
258 finite conductivity can damp the field-line resonance at Earth [Lee and Takahashi, 2006], the
realistic boundary condition might be important in Mercury's magnetosphere and these subject
260 will be left as future work.

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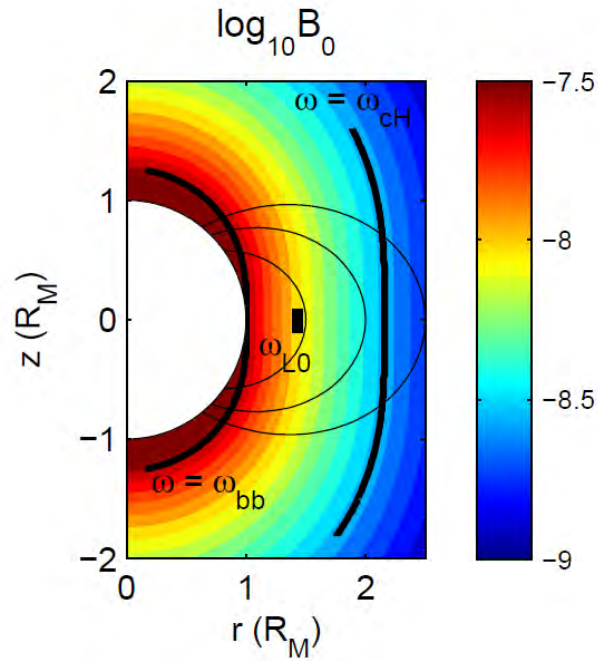
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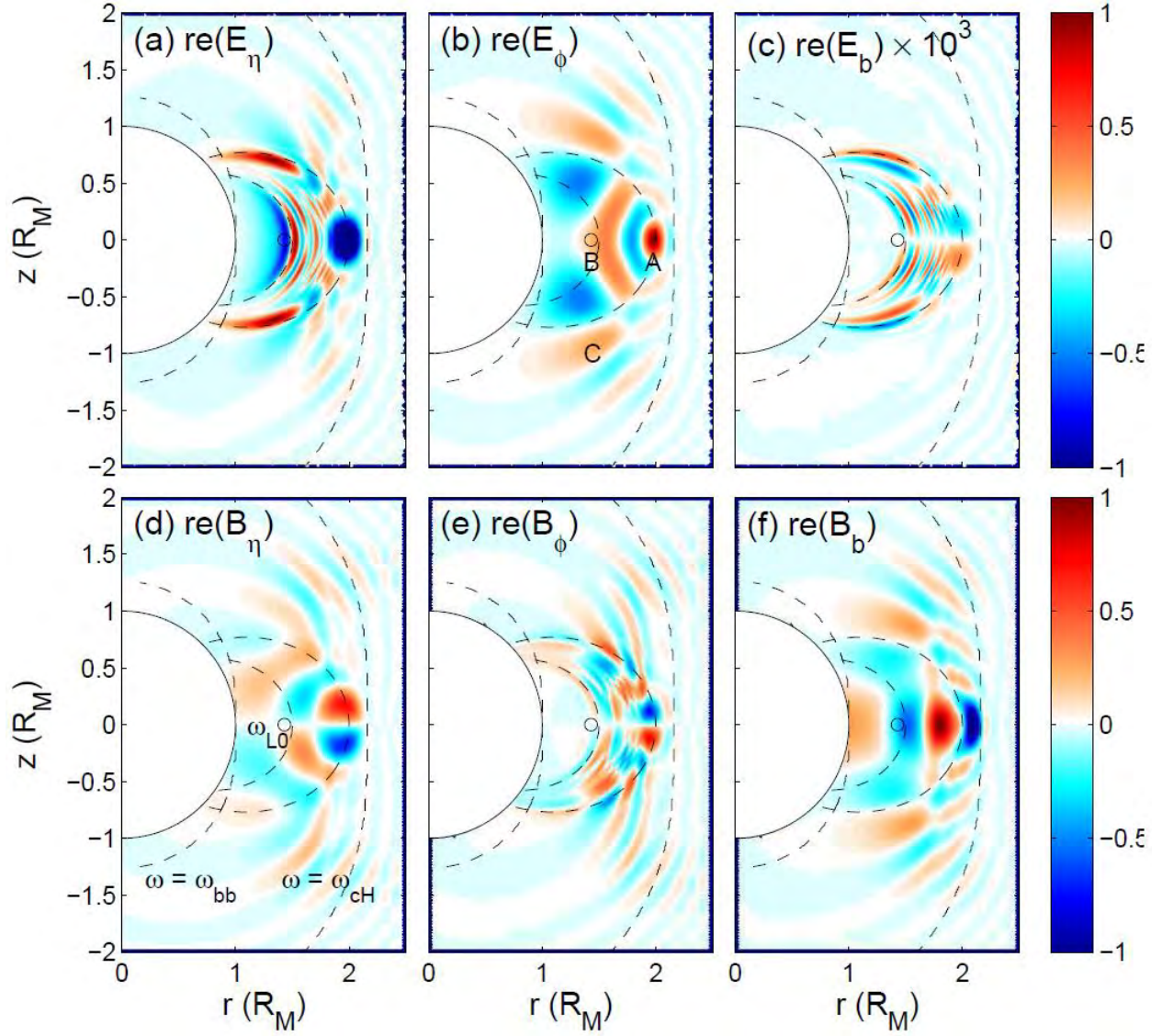
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384 Figure 1. The ambient magnetic field (B_0) in logarithm scale. Here, two solid lines are the proton
 cyclotron resonance location where $\omega = \omega_{ci}$ and ω_{ci} is proton gyro frequency, and Buchsbaum
 386 resonance location where $\omega = \omega_{bb}$ and ω_{bb} is Buchsbaum resonance frequency. ω_{L0} represent the
 FW wave cutoff locations at the magnetic equator.

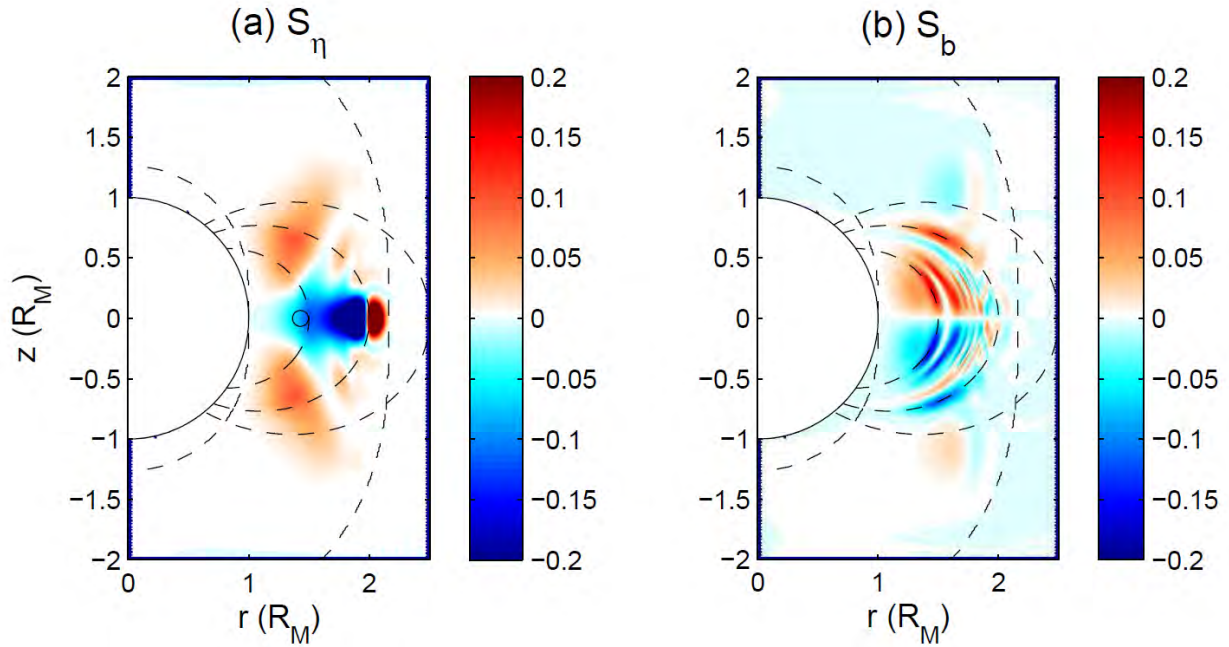
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Figure 2. Wave solutions if (a-c) electric field and (d-f) magnetic field. Here, $\hat{\mathbf{b}}$ is along the
 392 magnetic field line from north to south, $\hat{\boldsymbol{\phi}}$ is the usual azimuthal direction and $\hat{\boldsymbol{\eta}}$ is normal to the
 field line pointing outward ($\hat{\boldsymbol{\eta}} = \hat{\mathbf{b}} \times \hat{\boldsymbol{\phi}}$), with the assumption of $\mathbf{B}_0 \cdot \hat{\boldsymbol{\phi}} = 0$. The dashed lines are
 394 where $\omega = \omega_{ci}$ and $\omega = \omega_{bb}$, or field line at $L_M = 1.5$ and 2.0, respectively. The circle represent $\omega = \omega_{L0}$,
 the FW wave cutoff locations at the magnetic equator.

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400 Figure 3. The Poynting flux S of wave (a) S_η . The positive direction is outward across the
 magnetic field line; and (a) S_b . The positive direction is south to north direction. The circle
 402 represent $\omega = \omega_{L0}$, the FW wave cutoff locations at the magnetic equator. In this figure, spatial
 structure of S_η are same as the FW structure shown in E_η and E_b , on the other hand, S_b clearly
 404 shows the fine structure of field-aligned IHH wave modes.

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412 Media 1. Time dependent wave solution of $E(r, z, t) = E(r, z) \exp(-i\omega t)$; (a) E_η and (b) E_φ .

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