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# The double well mass filter

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Various mass filter concepts based on rotating plasmas have been suggested with the specific purpose of nuclear waste remediation. We report on a new rotating mass filter combining radial separation with axial extraction. The radial separation of the masses is the result of a "double-well" in effective radial potential in rotating plasma with a sheared rotation profile.

Plasmas have long been used for separating elements, with an emphasis on isotope separation[1,2]. Among plasma techniques, rotating configurations are of particular interest since centrifugal forces offer a direct ion separation scheme based on mass[3]. This led to the development of plasma centrifuges[4,5], in which collisional diffusion produces radial separation of ions within a plasma column. Plasmas have the advantage of high rotation velocities compared to gas centrifuges.

Within the last decade, the interest for a high throughput plasma filter in the context of nuclear waste processing[6] motivated the development of alternative concepts. Waste remediation differs in particular from isotope separation in that the mass discrimination requirement is less severe. In the Ohkawa mass filter[7], ions are separated as a result of a charge to mass ratio threshold for radial confinement. Heavy ions are unconfined radially and collected at the outer wall, while light ions are radially confined and collected axially along the magnetic field lines. This collisionless separation scheme relies on a plasma solid body rotation, obtained by means of biased electrodes at each end of the device. More recently, an alternative filter concept featuring a rotating axisymmetric plasma has been proposed[8,9]. In this concept, called the Magnetic Centrifugal Mass Filter (MCMF), a particular magnetic field topology is used to provide asymmetric confinement properties at each end of the device. Ions are collected along the field lines, with light and heavy streams preferentially collected at different axial end of the device.

In this research note, we propose a new rotating plasma filter combining both radial separation and axial extraction, while only requiring a simple linear magnetic field topology. It however differs from plasma centrifuges in that ion populations of different masses will have their density peak at different positions. The filter relies on the existence, through the production of a sheared rotation profile, of two distinct effective potential wells along the radial direction for two ions of different mass and identical charge, making it what we call a double well filter. In contrast with the Ohkawa filter where only one ionic species is radially confined, both ionic species are radially confined in this filter.

Let us consider in the laboratory frame a uniform axial magnetic field $\boldsymbol{B} = B_0\hat{\boldsymbol{z}}$ and an arbitrary radial electric field $\boldsymbol{E} = E_r\hat{\boldsymbol{r}} = -\boldsymbol{\nabla}\Phi$. In a frame rotating at constant angular velocity $\boldsymbol{\Omega} = \Omega\hat{\boldsymbol{z}}$, the fields read

$$\tilde{\boldsymbol{E}} = \boldsymbol{E} + (\boldsymbol{\Omega} \times \tilde{\boldsymbol{r}}) \times \boldsymbol{B} \tag{1a}$$

$$\tilde{\boldsymbol{B}} = \boldsymbol{B}, \tag{1b}$$

where $\tilde{x}$ is the laboratory frame variable $x$ written in the rotation frame. Assuming the fields in the rotating frame are time independent, the Newton-Lorentz equation can be rewritten as

$$m\frac{\partial \tilde{\boldsymbol{v}}}{\partial t} = q(\boldsymbol{E}_\star + \tilde{\boldsymbol{v}} \times \boldsymbol{B}_\star), \tag{2}$$

with

$$\boldsymbol{E}_\star = \tilde{\boldsymbol{E}} + \boldsymbol{\nabla}(\frac{m\Omega^2\tilde{r}^2}{2q}) \tag{3a}$$

$$\boldsymbol{B}_\star = \tilde{\boldsymbol{B}} + \frac{2m}{q}\boldsymbol{\Omega}. \tag{3b}$$

Introducing

$$\Phi_\star(\tilde{r}) = \Phi(r) + \left(\frac{qB_0^2}{4m} - \frac{m\Omega^2}{2q}\right)\tilde{r}^2, \tag{4}$$

we may write $\boldsymbol{E}_\star = -\boldsymbol{\nabla}\Phi_\star$. Eq. 3b shows that there exists a particular rotation frequency value, $\Omega = -qB_0/(2m) = -\omega_c/2$, for which the magnetic field in the rotating frame cancels, in which case

$$\Phi_\star(\tilde{r}) = \Phi(r) + \frac{qB_0^2}{8m}\tilde{r}^2. \tag{5}$$

Note that since the gyro-frequency $\omega_c$ depends on the mass, the frame in which the motion of a particle appears purely electrostatic will be different for two particles having the same charge but different masses. Looking at Eq. 5, one sees that, by specifying a parabolic radial profile $\Phi$ in the lab frame, the potential $\Phi_\star$ in the rotating frame can be made concave up or concave down. More specifically, an ion will be radially confined if

$$\frac{d^2}{dr^2}\left[\Phi(r)\right] > -\frac{qB_0^2}{4m}, \tag{6}$$

and unconfined otherwise. Introducing $\Phi_d$ the potential difference between the axis and the outer wall at $r = a$, one recovers the Ohkawa filter[7] charge to mass ratio threshold

$$\frac{m}{q} = \frac{a^2B^2}{8\Phi_d} \tag{7}$$

for ion confinement. In other words, under the assumption of solid body rotation at the angular rotation frequency $\Omega = E_r/(B_0 r)$, the electric potential in the rotating frame $\Phi_\star$ has a constant curvature sign.

Considering now a sheared rotation profile, as obtained for example by means of an applied potential $\Phi$ with higher polynomial dependency in radius, one may produce a radial separation while confining both ionic species. Let us consider the simple case of the following biquadratic laboratory frame potential

$$\Phi(r) = \frac{k_b T}{e}\left[\frac{r^4}{\lambda^2 a^2} - \frac{r^2}{\lambda^2}\right], \tag{8}$$

with

$$\lambda^2 = 8\rho^2 = 8\frac{k_b T}{A^\diamond m_p {\omega_c}^{\diamond 2}} = 8\frac{k_b T A^\diamond m_p}{e^2 B^2}, \tag{9}$$

$k_b$ the Boltzmann constant, $T$ a reference ion temperature, $m_p$ the proton mass, $\rho$ the thermal ion gyro-radius, $A^\diamond$ a reference atomic mass number and ${\omega_c}^\diamond$ the corresponding gyro-frequency. Combining Eq. 8 and Eq. 5 in the rotating frame of a singly charged ion of atomic mass number $A$, one gets

$$\Phi_\star(\tilde{r}) = \frac{k_b T}{e}\left[\frac{\tilde{r}^4}{\lambda^2 a^2} + \left(\frac{A^\diamond}{A} - 1\right)\frac{\tilde{r}^2}{\lambda^2}\right] \tag{10}$$

An ion such that $A < A^\diamond$ will therefore see a monotonically increasing potential $\Phi_\star(\tilde{r})$. On the other hand, an ion such that $A > A^\diamond$ will see a potential $\Phi_\star$ decreasing close to the axis, but increasing further off axis, that is to say a potential well off axis.

In contrast to the Ohkawa filter, which relies on a collisionless plasma, the double well filter requires a collisional plasma to produce the radial ion separation. Collisional diffusion is indeed necessary in order to separate a mixture of ions introduced in a finite radius plasma column on-axis. Assuming now that the length $l$ of the device is chosen long enough for an ion population to reach equilibrium as it diffuses axially, the spatial distribution will be of the Boltzmann type, with $n = n_0 \exp[-e(\Phi_\star - \Phi_\star^{min})/(k_b T_i)]$, where $n_0$ is the ion species density, $T_i$ is the ion population temperature and $\Phi_\star^{min}$ designates the minimum of $\Phi_\star$ over $[0, a]$. Ions such that $A < A^\diamond$ will be localized in an on-axis cylindrical column, while ions such that $A > A^\diamond$ will be localized in an annular ring, as illustrated in Fig. 1. Although the aspect ratio $l/a$ required to reach such a distribution will depend on the ratios $A/A^\diamond$ considered, an estimate can be obtained as follows. Denoting $\chi = a/\rho$, and approximating the perpendicular diffusion coefficient as $D_\perp \sim \rho^2 \nu$, where $\nu$ is the collision frequency, one gets that the ion residence time $\tau$ has to satisfy $\tau \geq \chi^2/\nu$ for an ion to diffuse on a distance $R$. Introducing the ion collision cross section $\sigma$ and $n$ the ion number density, it yields $l \geq \chi^2/(\sigma n)$. The minimum length is hence a function of how strongly is an ion magnetized (through $\chi$), of the element considered (through $\sigma$) and of the density.

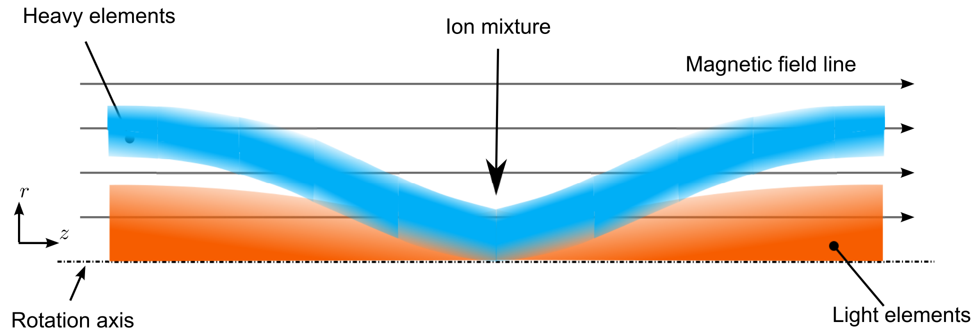
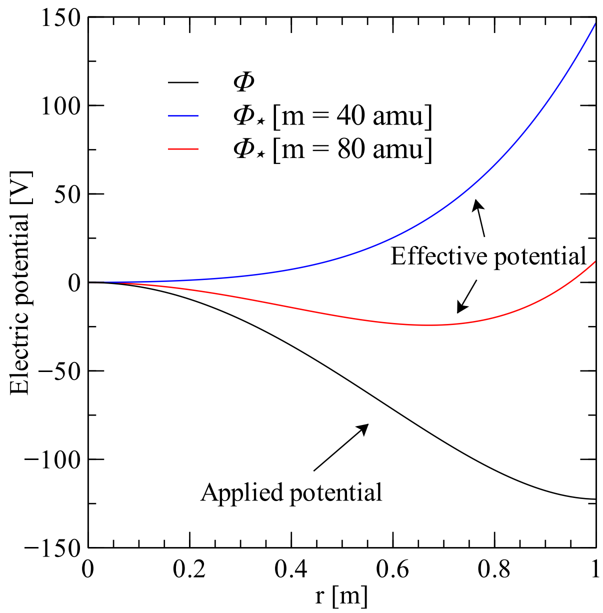


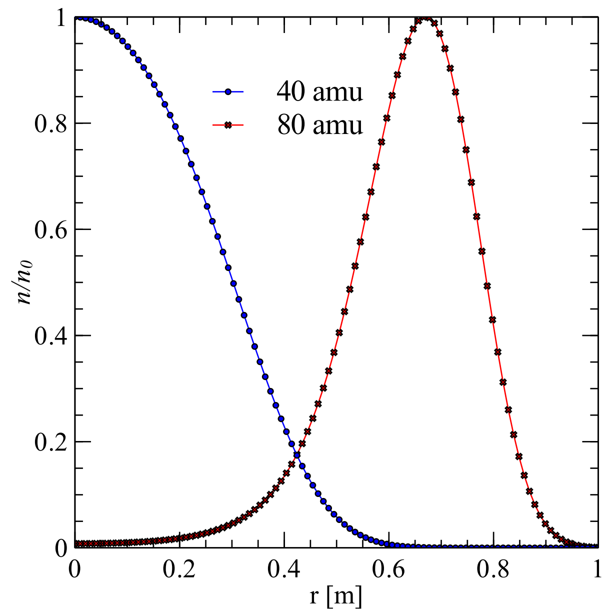

FIG. 1. Schematic of the double well filter (cut-view in the $r - z$ plane). Light elements are eventually confined in a plasma column on-axis while heavy elements are confined in an annular ring. Both species are radially confined and extracted axially.

Although the $\Phi$ form chosen in Eq. 9 conveniently illustrates the existence of an inflection point as the atomic mass number of an ion gets larger than a reference value, the lack of independent control over the second and forth degree monomial is over-constraining. By a proper choice of the laboratory potential $\Phi$ monomial coefficients, one can make the heavy ions potential well deep enough to remove the heavy ions from the axis, while maximizing the radial separation of heavy from light elements. Such a separation is illustrated in Fig. 2 for two 5 eV singly charged ion populations of 40 and 80 amu. The ratio of light to heavy elements, as plotted in Fig. 2(c), shows high separation factors close to the axis, and even higher ones further away from the axis, with less than one light ion for an hundred heavy ions for $r > 0.6$ m. In between these regions ($0.2 \leq r \geq 0.6$ in Fig.2), there exists a region where separation is much weaker and varies significantly, but the ion density is lower (Fig. 2(b)). Elements collected over this radial position range would have in principle to be processed at another time.

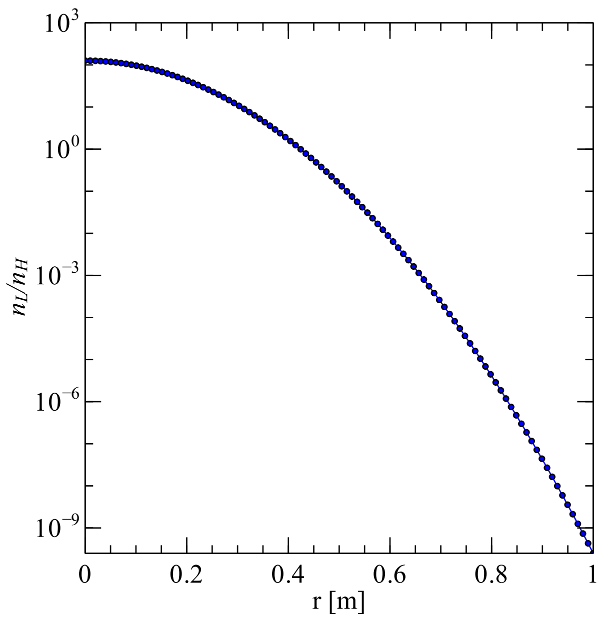
One way to avoid loosing the part of the stream where the separation is worst is simply to confine that radial region in the axial direction. This could be accomplished by pinching the fields lines within this radial zone, so as to produce a mirror force reflecting the particles towards the device mid-plane. Such a modification would be made at the expense of the magnetic field topology simplicity. Alternatively, higher degree polynomial form for the applied electric potential would allow optimizing the radial distance between the two population peaks, as well as the width of these peaks, for a given pair of ion masses and ion temperature. More complex applied potential profiles $\Phi$ are however expected to present an




(a) Electric potential




(b) Density




(c) Light to heavy element ratio

FIG. 2. Electric potential radial profile in both the laboratory ($\Phi$) and the rotating ($\Phi_\star$) frame (a), calculated Boltzmann radial distributions for two singly charged ion populations, and light (40 amu) to heavy (80 amu) elements ratio radial profile (c). Ion temperature of both population is 5 eV, $B_0 = 0.03$ T.

experimental challenge since they will require larger number of biasing electrodes to properly represent the spatial variations.

In summary, the double well filter is a new concept combining the radial separation feature offered by the Okhawa filter with the axial extraction property exhibited by the Centrifugal Mass Filter (MCMF). This is made possible through the production of a sheared rotation profile, which allows in turn setting up confining potential wells at different radial locations depending only on the ion mass for a given ion charge. Although the large heavy to light separation factors predicted at large radial positions make this concept particularly interesting for waste remediation, for which the main objective consists in removing radio-active heavy elements from a contaminated stream, the double well filter appears to be a promising filter for other challenging separation processes too, notably such as those encountered in nuclear spent fuel reprocessing.

## ACKNOWLEDGEMENTS

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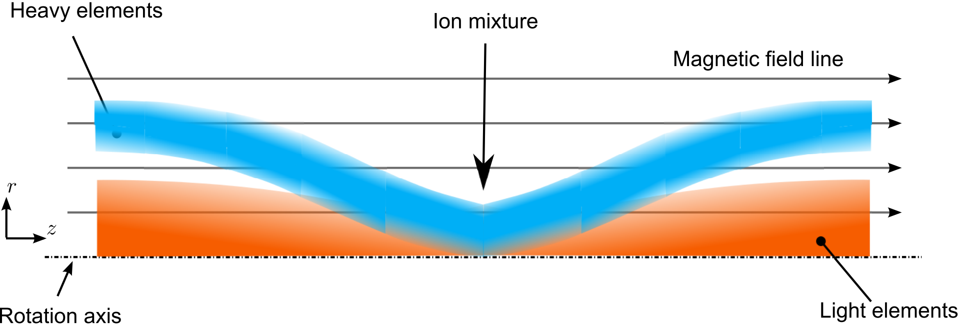
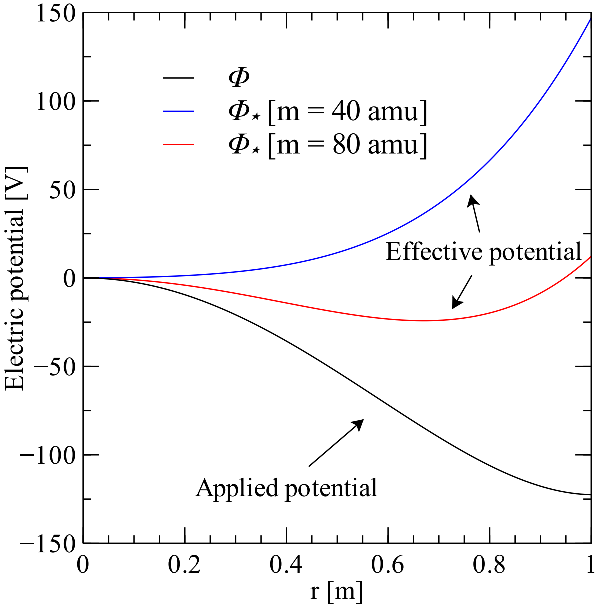
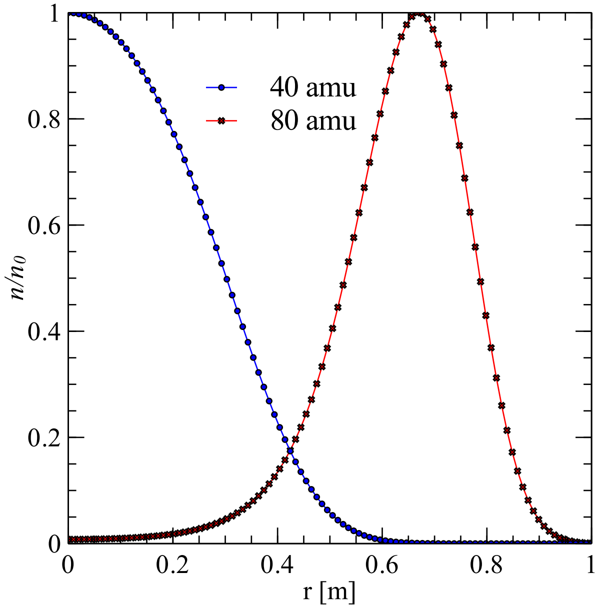
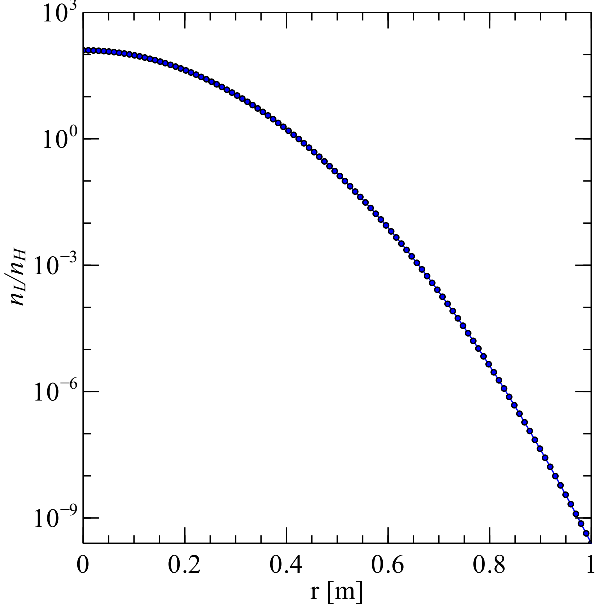

Heavy elements
Ion mixture
Magnetic field line
$r$
$z$
Rotation axis
Light elements



150
100
50
0
−50
−100
−150
Electric potential [V]
0
0.2
0.4
0.6
0.8
1
r [m]
Φ
Φ⋆ [m = 40 amu]
Φ⋆ [m = 80 amu]
Effective potential
Applied potential


40 amu
80 amu
n/n0
r [m]
0
0.2
0.4
0.6
0.8
1



$10^3$
$10^0$
$10^{-3}$
$10^{-6}$
$10^{-9}$
$n_L/n_H$
0
0.2
0.4
0.6
0.8
1
r [m]

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