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On the nature of kinetic electrostatic electron nonlinear (KEEN) waves

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An analytical theory is proposed for the kinetic electrostatic electron nonlinear (KEEN) waves originally found in simulations by Afeyan *et al* [arXiv:1210.8105]. We suggest that KEEN waves represent saturated states of the negative mass instability (NMI) reported recently by Dodin *et al* [Phys. Rev. Lett. **110**, 215006 (2013)]. Due to the NMI, trapped electrons form macroparticles that produce field oscillations at harmonics of the bounce frequency. At large enough amplitudes, these harmonics can phase-lock to the main wave and form stable nonlinear dissipationless structures that are nonstationary but otherwise similar to Bernstein-Greene-Kruskal modes. The theory explains why the formation of KEEN modes is sensitive to the excitation scenario and yields estimates that agree with the numerical results of Afeyan *et al.* A new type of KEEN wave may be possible at even larger amplitudes of the driving field than those used in simulations so far.

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Introduction. — As originally shown in Ref. [1], collisionless plasmas can support stationary nonlinear waves, commonly known today as Bernstein-Greene-Kruskal (BGK) modes. Resonant particles in such modes are trapped and phase-mixed, so Landau damping is suppressed [2]. On the other hand, trapped particles are known to be responsible for a number of instabilities [3], so BGK waves are not necessarily attractor states, and, as such, are not always easily accessible [4]. It was shown in Refs. [5, 6] that, when excited by a strong enough force, plasma oscillations can instead saturate in the form of structures that, unlike BGK modes, are nonstationary in any frame of reference and yet are undamped too. Such modes are believed to have no fluid or linear analogs and, in one-dimensional electron plasmas (to which our discussion will be limited for clarity), were termed kinetic electrostatic electron nonlinear (KEEN) waves [5, 6].

KEEN waves were numerically observed near the branch of the dispersion relation corresponding to the electron-acoustic waves (EAW), i.e., at frequencies close to $\omega_{\mathrm{EAW}} \approx 1.31 k v_T$; here k is the wave number, and v_T is the electron thermal speed [7–10]. (Albeit strongly damped in Maxwellian plasma, and thus rarely taken into account, EAW can be nondissipative if the particle distribution is flat at velocities close to ω_{EAW}/k. This occurs naturally when plasma is driven externally at frequency $\omega \approx \omega_{\mathrm{EAW}}$ for a long enough time.) However, KEEN modes are qualitatively different from EAW, as they contain multiple pronounced phase-locked harmonics. The advanced numerical modeling reported recently in Refs. [11–13] corroborate that such a spectrum is a robust feature of KEEN waves. In particular, it was proposed in Ref. [11] that KEEN waves represent essentially a superposition of BGK-like structures. However, the physical nature of these structures, as well as the sensitivity of KEEN waves to the excitation scenario and the driver amplitude [5], are yet to be understood in detail.

The purpose of this brief note is to offer a qualitative explanation of these issues by pointing to the connection between KEEN waves and the negative mass instability (NMI) that was recently identified for BGK-like waves in Ref. [14]. In essence, the NMI causes trapped electrons to bunch into macroparticles, which then produce sideband oscillations of the wave field, shifted from the main wave by, roughly, integers of the bounce frequency. These sidebands survive in the long run only if they are phase-locked to the main wave. This requires, for parameters at which KEEN waves have been studied yet, that the bounce frequency be somewhat higher than half of ω_{EAW}. Below, we explain this in detail.

Physical mechanism. — Suppose, as in Ref. [5], that electron oscillations are excited by an external driving force with some frequency ω, wave number k, and amplitude that is spatially homogeneous. Assuming that the driver is turned on slowly, both trapped and passing particles conserve certain adiabatic invariants that can be expressed in terms of their actions, J. The action is defined as the appropriately normalized [15] phase space area encircled by the particle trajectory in the frame traveling at the driver phase velocity, $u = \omega/k$, termed here the driver rest frame. For a trapped particle, the invariant is J itself, whereas for a passing particle the invariant is the oscillation-center canonical momentum, $P = mu + kJ\,\mathrm{sgn}\,(v-u)$, where m is the electron mass, and v is the electron velocity [16, 17].

Let us assume that both ω and k are constant; then conservation of P implies conservation of J for passing particles too [18]. But J, if normalized appropriately [15], is conserved also when a particle crosses the separatrix, albeit with worse-than-exponential accuracy [19–21]. Therefore, the action distribution, $F(J)$, is conserved throughout the entire process of the wave excitation. This gives [15, 22]

$$F(J) = (k/m)[f_0(u + kJ/m) + f_0(u - kJ/m)], \qquad (1)$$

where $f_0(v)$ is the initial velocity distribution. The separatrix action is $J = (4/\pi) m\Omega_0/k^2$ [15], where $\Omega_0 = (eEk/m)^{1/2}$ is the characteristic bounce frequency, and

E is the amplitude of the total electric field, including both the driver and the induced field. (We assume, for clarity, that $eE > 0$ and $k > 0$.) Hence, assuming E is small enough, the trapped distribution can be approximated with the second-order Taylor expansion of Eq. (1),

$$F_t(J) \approx 2kf_0(u)/m + J^2 k f_0''(u)/m. \tag{2}$$

For $\omega \approx \omega_{\rm EAW}$ in Maxwellian plasma assumed here, one has $f_0''(u) > 0$, so the action distribution of trapped particles is inverted, $F_t'(J) > 0$. (This is also seen directly in simulations. In addition to the already mentioned papers, see, e.g., Refs. [23, 24].) As shown in Ref. [14], such distributions can be unstable due to the particle bounce frequency $\Omega(J)$ being a decreasing function of J.

The instability is explained as follows. Consider a pair of electrons bouncing in the wave potential, i.e., rotating in phase space around a local equilibrium. Through Coulomb repulsion (strictly speaking, via collective fields), the leading particle increases its energy; then it moves to an outer phase orbit and slows down its phase space rotation (as $\Omega' < 0$), whereas the trailing particle moves to a lower orbit and speeds up, correspondingly. This way, mutually repelling electrons can undergo phase-bunching, or condensation, as if they had negative masses. The condensation may or may not eventually saturate in the form of a stable macroparticle, but its very formation constitutes a fundamental instability in itself. By analogy with similar effects in accelerators [25] and ion traps [26, 27], the term NMI was coined for this instability in Ref. [14].

Now consider the effect of macroparticles as production of *sidebands* of the wave field. As the driver continues to feed the instability, these sidebands grow and initiate stochastization of electron orbits in the resonance region. (One can view this as an effect akin, if not identical, to quasilinear diffusion.) This stochastization tends to flatten the trapped distribution and thus eventually suppresses the NMI. Most particles then phase-mix (cf. Ref. [28]), so a standard, albeit non-sinusoidal [29], BGK mode is formed. However, the scenario is different when the sidebands are in approximate resonance with the main wave (and, thus, with the driver too). In that case, the system is close to periodic, so one can expect formation of invariant tori in the particle phase space, even for a relatively strong driver. Then the system can sustain large stable macroparticles and the corresponding well-pronounced sidebands phase-locked to the main wave; cf. Refs. [28, 30, 31]. Once phase-locked, the wave should also be able to tolerate moderate variations of the wave amplitude from the exact resonance, as in a typical autoresonance [32], without abrupt modifications of the spectrum; i.e., one can expect that its nonlinear features are *robust*. (But, of course, large enough variations of the wave parameters destroy the resonance.)

Phase-locking conditions. — To figure out when the phase-locking occurs, suppose that, in each trapping island, N macroparticles are formed with about the same J and are distributed equidistantly in bounce phases. They will hence experience rotation in phase space at the same bounce frequency, $\Omega(J)$. On the other hand, assuming all macroparticles are identical, their induced field oscillates at frequency $\pm N\Omega$ in the driver rest frame. Let us also assume that these oscillations are in phase for all trapping islands (as can be dictated also by specific boundary conditions adopted for simulations [14]), so the sidebands have the same wave number as the main wave and the driver. For the laboratory frame, this gives sideband harmonics with frequencies $\ell\omega \pm N\Omega$, where $\ell = 1, 2 \ldots$. Phase-locking is hence possible if $\Omega = N'\omega/N$, where N' is any natural number. On the other hand, interactions at resonances with $N' > 1$ are weak, so actually mattering are only those resonances that correspond to $N' = 1$.

This suggests the following picture for wave excitation in initially-quiescent plasma. There, Ω grows from zero, so it passes infinitely many resonances of the type

$$\Omega = \omega/N. \tag{3}$$

As the bounce frequency is J-dependent, Eq. (3) can be satisfied for more than one N for a given driver. On the other hand, resonances unavoidably compete when they enter the nonlinear stage. What survives is always the strongest resonance, i.e., the one that has the lowest order allowed by Eq. (3), $N_m = \omega/\Omega_0$. Using the dimensionless variables $\kappa = kv_T/\omega_p$ and $a = eE/(m\omega_p v_T)$, where ω_p is the plasma frequency, one can express N_m as follows:

$$N_m = (u/v_T)(\kappa/a)^{1/2}. \tag{4}$$

For $u/v_T = 1.31$ and $\kappa = 0.26$, which are typical for KEEN-wave simulations, Eq. (4) becomes $N_m = 2(a/a_c)^{-1/2}$, where $a_c \approx 0.11$. This shows that, at $a > a_c$, phase-locking is possible into a resonance with $N = 2$, which corresponds to two macroparticles per island. In contrast, at $a < a_c$, phase-locking is possible only at $N = 3$, which corresponds to three macroparticles per island. In the latter case, the macroparticle size is much smaller, so one can expect an abrupt modification of the wave spectrum at $a \approx a_c$. This is indeed what is seen in simulations [5]. Moreover, the typical KEEN mode shown in Fig. 1 of Ref. [5] clearly shows the presence of exactly two macroparticles in a trapping island.

One can also anticipate a similar threshold at $a \sim 0.5$, when Eq. (4) predicts $N_m = 1$. A single macroparticle can form then and bounce resonantly to the main wave. At such large amplitudes, however, the electron quiver speed becomes comparable to v_T, so the above estimates (which rely on the weak-interaction model and the EAW dispersion being linear) may lack quantitative accuracy.

Conclusions. — In this brief note, we propose, for the first time, a basic semi-quantitative theory of KEEN waves. We argue that key to the KEEN mode formation is a specific instability, the NMI [14], that produces macroparticles out of trapped electrons. These macroparticles can, under certain conditions, become phase-locked to the main wave. For parameters typical for KEEN-wave simulations reported in literature, this requires that the bounce frequency be higher than half of $\omega_{\rm EAW}$, imposing a lower limit on the driver amplitude. This picture readily explains why the formation of KEEN modes is sensitive to the excitation scenario; e.g., pre-flattening of the resonant distribution would eliminate the source of the NMI, so macroparticles would not form, and the wave would remain in the linear regime. We also propose numerical estimates that agree with existing simulation results and argue that a new type of KEEN waves may be possible at even larger amplitudes of the driving field than those tried in simulations so far.

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