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Response to Comment on "Geometric Phase of the Gyromotion for Charged Particles in a Time-dependent Magnetic Field

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Response to "Comment on 'Geometric phase of the gyromotion for charged particles in a time-dependent magnetic field'" [Phys. Plasmas XX, xxxx (2012)]*

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The reformulation of our analysis on the geometric phase of the gyromotion [J. Liu and H. Qin, Phys. Plasmas 18, 072505 (2011)] in terms of spatial angles presented in the comment by Brizard and Guillebon is interesting and correct. The subtlety of whether the adiabatic term associated with the long term average of the variation of pitch angle completely disappears after the gyrophase average is related to where valid approximations are applied. But it has no impact on the main conclusions.

We welcome that comment by Brizard and Guillebon on our paper on the geometric phase of the gyromotion for charged particles in a time-dependent magnetic field [1]. We believe that the reformulation of our analysis in terms of spatial angles presented in their comment [2] is correct and interesting.

We intentionally avoided using the coordinates and techniques associated with the gyrokinetic theory (guiding center theory) because of the extra assumptions and subtleties that might be introduced when the method of gyrokinetic theory is applied. One of such subtleties is whether the adiabatic term associated with the long term average of the variation of pitch angle completely disappears after the gyrophase average. Though the resulting difference, a higher-order term, is very small and has no impact on the main conclusions, we analyze the reason of this small difference as follows.

When considering the equation of motion for the pitch angle , Brizard and Guillebon applied gyroaverage to Eq. (18) in Ref. [2]:

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} = \sin\zeta\frac{\mathrm{d}\theta}{\mathrm{d}t} + \cos\zeta\sin\theta\frac{\mathrm{d}\varphi}{\mathrm{d}t} \equiv \omega_2\ . \quad (1)$$

Brizard and Guillebon concluded that $\langle \mathrm{d}\lambda/\mathrm{d}t\rangle = 0$ since $\mathrm{d}\theta/\mathrm{d}t$ and $\mathrm{d}\varphi/\mathrm{d}t$ are both gyrophase independent, where $\langle\cdots\rangle$ denotes the gyrophase average. In our analysis, the gyrophase average is equivalent to a time average which averages out the fast gyromotion:

$$\langle\cdots\rangle = \int_0^{2\pi}\cdots\,\mathrm{d}\zeta = \int_t^{t+T}\cdots\,\mathrm{d}t'\ . \quad (2)$$

That is to say that the gyrophase ζ and time t cannot be taken as two independent variables when applying the gyrophase average. Though θ and φ do not depend on gyrophase explicitly, they both may change slowly with time. In this sense, $\mathrm{d}\theta/\mathrm{d}t$ and $\mathrm{d}\varphi/\mathrm{d}t$ cannot be taken as constants during the averaging procedure. Thus the relation $\langle \mathrm{d}\lambda/\mathrm{d}t\rangle = 0$ doesn't hold rigorously. Similar gyrophase average appears when treating the adiabatic phase term $\langle\cot\lambda\,\frac{\partial}{\partial\zeta}\frac{\mathrm{d}\lambda}{\mathrm{d}t}\rangle$, which doesn't equal zero exactly. That's why the third term in Eq. (19) of Ref. [2] disappears after the gyrophase average while this term was proved to be higher-order term compared with the geometric term in our analysis [1]. However, we emphasize that this subtle difference is not of significance. It merely reflects the fact that valid approximations are made at different places.

In their comment, Brizard and Guillebon also gave a brief remark on the gyrophase of a charged particle in a general space-time-dependent magnetic field. This is indeed an interesting problem. Spatial inhomogeneity brings in new physics. The term $\frac{1}{2}\frac{\mathrm{d}\chi}{\mathrm{d}t}$ in Eq. (29) of Ref. [2] arises from the adiabatic term, but it is no longer a higher-order term compared with the geometric term. It results from the coupling between the inhomogeneity of the magnetic field and the gyromotion of charged particles. This effect also has an interesting geometric origin and should be addressed in follow-up studies.

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[1] J. Liu and H. Qin, Phys. Plasmas **18**, 072505 (2011).
[2] A. J. Brizard and L. Guillebon, Phys. Plasmas **19**, xxxx (2012).

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