Princeton Plasma Physics Laboratory

PPPL- 4722

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Current Drive In Expanding and Compressing Plasma

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January, 2012

Prepared for the U.S. Department of Energy under Contract DE-AC02-09CH11466.

Current drive in expanding and compressing plasma

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(Dated: January 16, 2012)

A magnetized plasma preseeded with an initially undamped Langmuir wave is shown to transition suddenly to a collisionless damping regime upon expansion of the plasma perpendicular to the background magnetic field. The collisional relaxation of the resulting modified, anisotropic particle distribution then generates electrical current. The current drive efficiency of this effect in nonstationary plasmas is shown to depend on the rate of expansion of the plasma. Subsequent re-compression of the plasma enhances this current drive effect by reducing the collision rate of the current-carrying electrons.

PACS numbers: 52.35.Fp, 47.10.ab, 52.25.-b, 52.65.Rr

Introduction.— Time-varying plasma with embedded waves [1] or dust [2] can often exhibit radically different behavior from steady state plasma. Here, a new current drive scheme is predicted in nonstationary plasma, whereby an initially undamped monochromatic wave, embedded in a magnetized plasma and propagating in one direction parallel to the magnetic field, is induced into wave-particle resonance with plasma particles due to magnetic expansion perpendicular to the wavevector. The sudden, collisionless damping causes the wave to transfer its energy anisotropically onto the co-moving high-energy tail of the resonant particle distribution, while subsequent anisotropic collisional relaxation of the modified distribution results in a net current. As a paradigmatic example, embedded Langmuir waves are considered, though other waves may prove more suitable for specific applications. The peak attainable current densities occur for expansion rates, η, comparable to the electron collision rate, ν_c. However, expansion rates significantly faster than the collision rate lead to more prolonged current as a result of enhanced electron trapping by the wave, which carries more electrons to superthermal velocities and, hence, reduces their effective collisionality. Interestingly, the current can be prolonged by magnetically compressing the plasma following the collisionless damping, which increases particle perpendicular velocities, thereby lowering the collisionality of the current-carrying electrons.

Before presenting the results of the numerical simulations, the basic current drive mechanism will be explained briefly. For slow variation of external forces, the Langmuir wave dispersion relation obeys the eikonal equation [3]: $\omega^2 = \omega_p^2(t) + 3k^2 v_{T\|}^2(t)$, where ω_p is the plasma frequency and $v_{T\|}$ is the electron thermal velocity in the direction parallel to the wavevector $\mathbf{k}$, with $|\mathbf{k}| \equiv k$. This is just the normal dispersion relation for plasma oscillations, but with ω_p and $v_{T\|}$ varying slowly in time. Assume there exists a homogeneous magnetic field $\mathbf{B} \parallel \mathbf{k}$ that is sufficiently strong to magnetize both the ions and the electrons. Then, the plasma number density $n \propto |\mathbf{B}| \equiv B(t)$ by magnetic flux-freezing, as B is slowly varied with time. Additionally, the conservation of the magnetic moment, $\mu = mv_\perp^2/2B$, on time scales short compared to ν_c^{-1} leads to the reduction of perpendicular velocities for magnetic expansion, i.e., $dB/dt < 0$.

Consider, for simplicity, a torus of plasma in a toroidal magnetic field. The torus is high aspect ratio, so any small toroidal segment appears cylindrical, and expansion is presumed to occur in the minor radius only, while the major radius is fixed. Initially, the phase velocity of an embedded Langmuir wave $v_{\rm ph} = \omega/k \gg v_{T\|}$, with $k = $ const, and $\omega \approx \omega_p \propto n^{1/2}$ for perpendicular expansion. Neglecting anisotropy-driven electromagnetic instabilities (addressed in the *Discussion* section), $v_{T\|}$ is decoupled from $v_{T\perp}$ when $\nu_c \ll \eta$, and the wave can be made to satisfy the condition $v_{\rm ph} \sim \mathcal{O}(v_{T\|})$ via magnetic expansion, at which point Landau damping of the wave initiates [1]. Even in the limit $\nu_c \sim \mathcal{O}(\eta)$, where the temperature variation scales with the system volume adiabatically, i.e., $TV^{2/3} = $ const, one still finds $v_T \propto n^{1/3}$. Thus, in this limit the ratio $v_{\rm ph}/v_{T\|} \propto n^{1/6}$, and the Landau damping criterion still can become satisfied.

As opposed to the case of compression parallel to $\mathbf{k}$ by walls, where only heating occurs [1], here the Landau damping of a wave traveling in one direction forms an anisotropic high-energy electron tail, with collisions leading to electric current [4]. Since particle collision rates exhibit a $1/v^3$ dependence, the high velocity tail relaxes more slowly than the rest of the distribution, producing a net current. However, as opposed to steady state current drive [4], here the evolution of current carried by suprathermal particles will be impacted significantly by the rarefaction or densification of the plasma [5].

Current drive in expanding plasma.—To describe this current, a novel particle-in-cell (PIC) simulation was developed that treats electrostatic fields with mobile electrons and ions in one spatial dimension, while particle speeds *perpendicular* to the simulation domain are tracked along with longitudinal velocities. Periodic boundary conditions are imposed to allow the current loop to close, and the parallel and perpendicular velocities are initialized as Maxwellian with a sinusoidal per-

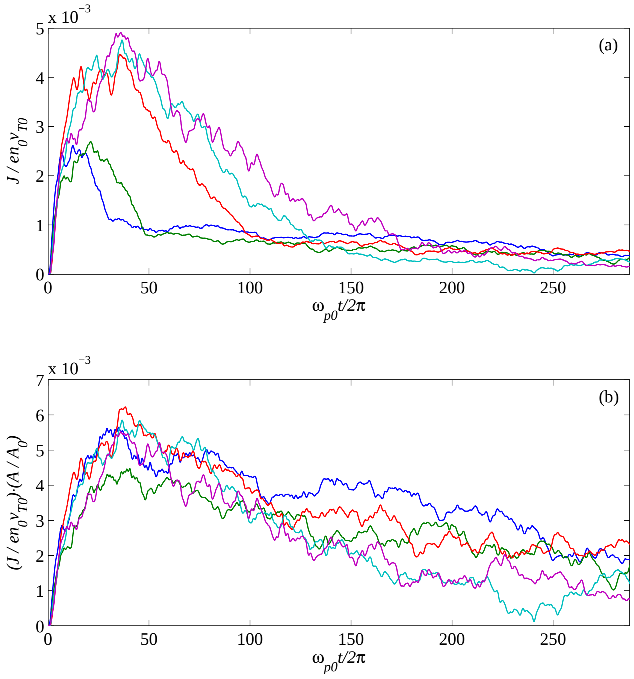


FIG. 1: (color online) (a) Dimensionless current density for expansion time $T = 30$ (blue), 50 (green), 100 (red), 150 (cyan), 250 (magenta), time measured in units τ_{p0}. (b) Dimensionless total flux tube current, same color scheme.

turbation for the plasma wave. Perpendicular velocities are coupled to the change in n (and thus $\mathbf{B}$) by enforcing conservation of the magnetic moment, $\Delta\mu = 0$, at each time step. Variations in density due to perpendicular compression and expansion are simulated by scaling both the PIC particle charge, Q, and the PIC particle mass, M, proportionally to the normalized density $\tilde{n}_j = n_j/n_0$, while holding the ratio Q/M fixed. This can be visualized as the redistribution of charge density across the perpendicularly homogeneous sheets of charge modeled by a 1D PIC code. Finally, a continuous, time-explicit PIC collision algorithm for fully three-dimensional (3D) collisions was used based on the approach derived in Ref. [6]. The advantage of this method is that the 2D effects of magnetic expansion and collisional relaxation can be retained while simulating motion in only one spatial dimension, allowing for more particles to be simulated per cell, and, hence, drastically decreasing the statistical noise.

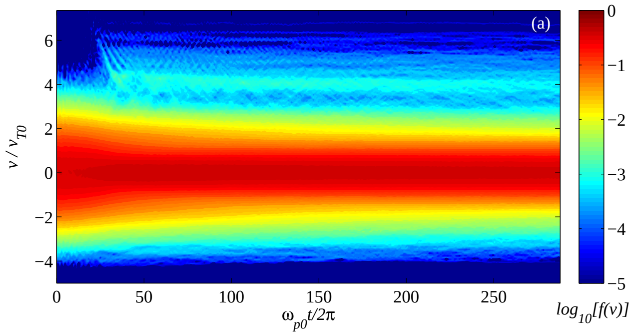
Any time-varying current will induce an electric field, whose strength will depend on the L/R time, τ. The total current then can be described [7] by: $\tau dI/dt = I_{\rm rf}(t) - I$, where $I_{\rm rf}(t)$ is the current source that arises from the effects considered here. Since the number of fast current carriers is small compared to the bulk current carriers, the induced electric field affects primarily bulk electrons, producing current with classical resistivity. What is addressed and simulated here is the wave-generated current, $I_{\rm rf}(t)$, for which the induced field can be ignored.

To illustrate the new current-drive effect, consider plasma parameters characteristic of magnetized liner fusion experiments [8]; for instance, take the initial number density $n = 10^{22}$ cm^3 and the initial isotropic temperature $T = 750$ eV. To this hydrogen plasma, we add a preseeded Langmuir wave of wavelength $\lambda = 87$ nm and amplitude $E = 4 \times 10^6$ statvolts/cm. The plasma is expanded until it is 20% of its original density at a number of different linear rates with brief, smooth ramp-up and ramp-down periods. Following expansion, the plasma is allowed to evolve further at the fixed reduced density.

The wave is initialized such that the trapping width $v_{\rm tr}/v_{\rm T} = 3$, and the phase velocity $v_{\rm ph}/v_{\rm T} = 7$, where $v_{\rm tr} = 2\sqrt{eE/mk}$, and e/m is the electron charge-to-mass ratio. Particles are not initialized beyond $v = 4v_{\rm T}$, so the resonant region of the wave initially lies just beyond the fastest particles in the simulation, and no damping occurs if collisions are excluded and the density is held fixed. The electron collisionality relative to the wave frequency ω initially goes like $\omega/2\pi\nu_c = 94$, so that an optimal expansion rate η may exist in the range $\omega \gg \eta \gtrsim \nu_c$, where sufficient collisionless damping can occur before the wave energy is lost to collisional damping.

The current response of the plasma subject to a number of different compression times, $T \sim 1/\eta$, is shown in Fig. 1. Figure 1(a) shows the normalized current density, $J/en_0v_{\rm T0}$, versus time for $T = 30$, 50, 100, 150, and 250, where T has units of plasma periods, $\tau_{p0} = 2\pi/\omega_{p0}$, and the subscript '0' signifies an initial value. Since the PIC code utilizes a random collision routine, the data in Fig. 1 represents the mean of many identically initialized simulations for each value of T, with an ω_p-scale smoothing filter applied to remove fast oscillations. The statistical error in the data is on the order of $\pm 1 \times 10^{-4}$ for Fig. 1(a), approximately a 2% to 5% error at peak current density. To maximize the peak current density, there is clearly a benefit to slower expansion, though the peak current density appears to level off for $T \gtrsim \mathcal{O}(\nu_c^{-1})$ ($\nu_c^{-1} \approx 100\tau_{p0}$ initially in these simulations). Of course, for $\eta \ll \nu_c$, the wave damps promptly without generating current.

Figure 1(b) shows the total normalized flux tube current. The peak current is far less sensitive to T than the peak current density, though Fig. 1(b) shows that *faster* expansion generally leads to a more prolonged current profile. Thus, another figure of merit might be the total time-integrated current. In this case, optimization of time-integrated current also points to faster expansion rates. Figure 2(a) shows the behavior of the normalized electron (parallel) velocity distribution function $f(v)$, with $\int f(v)dv = 1$, for $T = 30\tau_{p0}$. At this expansion rate, enhanced particle trapping by the wave modifies significantly the particle distribution in the vicinity of the resonance [9], accelerating high energy electrons to higher parallel velocities by amounts comparable to $v_{\rm tr}$. On the other hand, Fig. 2(b) shows that slower expansion leads to less particle trapping, thus limiting the



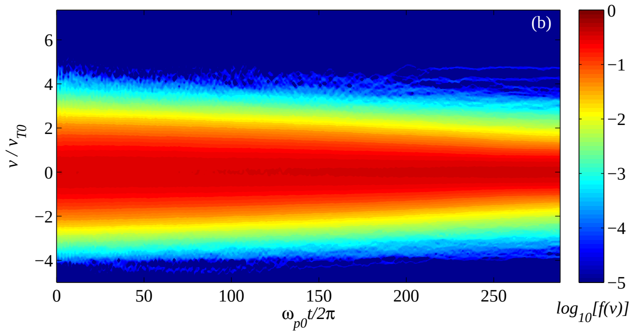


FIG. 2: (color online) Plot of $\log_{10}[f(v,t)]$, with $f(v,t)$ the electron parallel velocity distribution function, for (a) $T = 30\tau_p$ and (b) $T = 250\tau_p$.

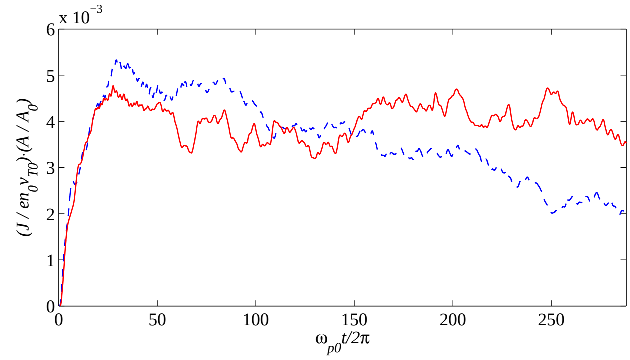


FIG. 3: (color online) Normalized flux tube current for the original $T = 30\tau_p$ expansion scenario (dashed line), and where the plasma is compressed immediately back to its original density with an identical ($T = 30\tau_p$) linear profile (solid line).

production of high energy electrons. In phase space, the advancement of the trapped orbit wave separatrix along the velocity axis toward the particle distribution, i.e., $d/dt|v_{\rm ph}/v_{\rm T\|}| \equiv \alpha < 0$, is counterbalanced by the recession of the separatrix due to the shrinking of the trapping width, i.e., $d/dt(v_{\rm tr}/v_{\rm T\|}) \equiv \gamma < 0$, due initially to plasmon conservation and later to resonant wave-particle energy exchange. Then, when $|\alpha| \gg |\gamma|$, as is the case with fast system expansion, a phase space bubble is dragged into the plasma, and resonant particles experience an impulse of $\mathcal{O}(v_{\rm tr})$ [10]. In the slower expansion case, where $|\alpha| \approx |\gamma|$, the wave damps appreciably as it penetrates the distribution, limiting the size of the phase space bubble. Consequently, the current carriers in the slower expansion cases are significantly lower energy relative to the high energy current carriers produced in the faster expansion cases, resulting in quicker collisional damping of the associated current.

Enhancement of current drive with re-compression.— The lifetime of the driven current can be extended significantly through magnetic re-compression of the plasma following collisionless damping of the wave. Figure 3 shows the normalized flux tube current for the original $T = 30\tau_{p0}$ case from Fig. 1(b) as well as the case where, after $t = T$, the plasma is then compressed back to its original density at the same linear rate. The current in the re-compressed plasma possesses a longer decay time, since the compression perpendicularly heats the current-carrying particles, whose collisionality exhibits a $1/v^3$ velocity dependence. Generally, simulations reveal that faster re-compression leads to more prolonged current.

Discussion.— The use of lower dimensional models to simulate wave-particle interactions often offers benefits and has been implemented recently in other systems [11]. The number of particles N required in an S-dimensional PIC simulation scales like N^S, owing to the necessary condition for weakly-coupled plasma, $N(\lambda_{\rm D}/L)^S \gg 1$, where $\lambda_{\rm D}$ is the Debye length and L is the characteristic length of the simulation domain. Furthermore, the lower dimensionality of the code allows N to be increased beyond the minimum requirements for weak coupling so that the noise, scaling like $1/\sqrt{N}$, may be reduced further without loss of computational tractability.

The simulation does not treat electromagnetic interactions and, thus, does not capture temperature anisotropy-driven instabilities [12]. For $a \equiv T_\perp/T_\|$, the relevant electron instabilities, the firehose ($a < 1$) and whistler ($a > 1$) instabilities, have maximum growth rates determined by two parameters: parallel beta, $\beta_e = 8\pi n T_\|/B^2$, and the anisotropy parameter a. The systems modeled in Fig. 1 traverse this parameter space as indicated in Fig. 4. Initial conditions correspond to the lower right side of the plot, and given enough time, all trajectories eventually come back to $a = 1$ from collisional isotropization. The dashed line shows the limit of negligible collisions, i.e., $\eta \gg \nu_c$. In this limit, $\beta_e \propto n^{-1}$, and $a \propto n$, so the (reversible) paths are along lines of constant $a\beta_e$.

Figure 4 can be compared directly with Fig. 5 of Ref. [12]-b. For the current drive effect produced by expansion, which generates $a < 1$, the firehose instability generally can be avoided by picking a sufficiently small initial β_{e0}. For the initial temperature and density chosen here, and assuming a background field $|\mathbf{B}| \sim \mathcal{O}(100\ \text{MG})$, which is typical near stagnation in magnetized liner fusion [8], one has $\beta_{e0} \sim \mathcal{O}(10^{-2})$, and the firehose instability is avoided easily. On the other hand, the whistler instability may occur under the re-compression if $a > 1$, limiting the extent to which plasma compression may prolong the current.

While the Langmuir wave serves as a paradigmatic example to illustrate the current drive mechanism, in practice, other waves may be more useful in contemporary experimental geometries, such as lower frequency waves or possibly even nonlinear waves [13]. The Langmuir wave is heavily damped in very dense plasma, so the expansion time η cannot easily be made as short as the collision time ν_c. For example, for magnetized liner parameters [8], a cylindrical plasma with radius 0.2 cm, $T_0 = 750$ eV, and initial density, $n_0 = 10^{22}$ cm^{-3}, the collision rate $\nu_c \sim \mathcal{O}(10^{13}$ s$^{-1})$ would exceed any subsonic expansion rates. However, for a more tenuous plasma, say $n_0 = 10^{16}$ cm^{-3}, then $\nu_c \sim \mathcal{O}(10^7$ s$^{-1})$, so that more easily realizable expansion rates of the order of hundreds of nanoseconds would be sufficient to induce Langmuir waves into resonance before significant collisional damping occurs. In such plasmas, lightly damped Langmuir modes exist with frequencies at several hundred GHz and sub-millimeter wavelengths. A transition from stagnation to expansion at these rates and initial densities is possible on modern Z-pinch devices [14]. Expansion rates of the order of 100 ns also require only subsonic or sub-Alfvénic expansion velocities, allowing for relatively uniform expansion throughout the column.

The analysis here describes Langmuir waves propagating along field lines of a toroidally magnetized annulus of plasma, which is allowed to expand uniformly about its toroidal axis. Such a plasma could be created using a single current-carrying wire running along the central axis of a Z-pinch, which produces an azimuthal magnetic field within the plasma surrounding the wire. The mean-free paths of the high energy electrons produced for $n_0 = 10^{16}$ cm^{-3} and $T_0 = 750$ eV are >50 cm, meaning these electrons circulate the wire many times before slowing down. An 87 μm Langmuir wave satisfies the initial relations $v_{\rm tr}/v_{\rm T} = 3$, and $v_{\rm ph}/v_{\rm T} = 7$, like the simulated wave, when the electrostatic energy density is roughly 3% the thermal energy density of the hydrogen plasma. Assuming the Langmuir wave fills the entire interior region of the plasma torus, a resulting wave-driven current peaking at about 10 kA is predicted by the simulations. However, at these temperatures and densities, a plasma torus consistent with typical Z-pinch dimensions, with minor radius 0.5 cm and major radius 3 cm, exhibits an L/R time of tenths of a second. Thus, as the fast particle current rises, a loop voltage peaking around 8 mV is induced, which drives an Ohmic counter-current that nearly cancels the wave-driven current on the relatively short expansion time scales of 100 ns.

In summary, magnetic expansion of plasma can induce sudden collisionless damping of an embedded Langmuir wave with phase velocity in one direction. The collisional relaxation of the resulting anisotropic particle distribution then generates current in the expanding plasma. The efficiency of this current drive process varies with the expansion rate of the plasma, with faster rates leading to longer-lasting currents due to greater particle trapping by the wave. This current drive effect is enhanced further by the subsequent re-compression of the plasma.

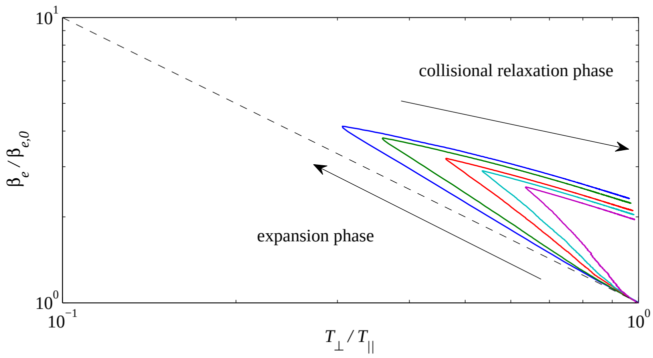


FIG. 4: (color online) Plot of system trajectories in a–β_e space (parameters defined in text). Same color scheme as Fig. 1. Dashed line shows the quasi-reversible pathway for $\eta \gg \nu_c$.

This work was supported by the U.S. DOE under Contract No. DE-AC02-09CH11466 and through the NNSA SSAA Program by DOE Research Grant No. DE-FG52-08NA28533.

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