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Lessons in Creativity and Courage

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Abstract. Some of the contributions that Bob Dewar made to plasma turbulence theory are recounted and put into historical perspective. Some remarks are made on the transition to turbulence and bifurcation analysis of models for the low–high confinement transition in tokamaks. However, the focus of the article is on renormalized oscillation-center theory, to which Dewar made pioneering, creative, and courageous contributions. The relationship of that approach to the formalism of Martin, Siggia, and Rose is discussed briefly.

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1. Introduction

In this article[1] I attempt to put into perspective some of Bob Dewar's contributions to plasma turbulence theory. Any serious analytical work on the theory of turbulence is courageous, as the subject is notoriously difficult and extremely challenging. Bob brought his own special, very creative insights to the problem, hence my subtitle "Lessons in Creativity and Courage." We can all profit by considering Bob's unique, deeply physical and mathematical approach.

My discussion in this paper is somewhat informal and heuristic. There are few equations, and important logical steps are omitted. My goal is to transmit the *spirit* of Bob's motivations and approach, not to teach the technical details (which one can always look up). My intended audience is students of nonlinear plasma turbulence theory. Whether young or old, we physicists must remain students throughout our careers, so the discussion should be of interest to everyone. But I hope it will have special impact on the younger generation of beginning plasma physicists. They are confronted with a relatively mature and systematized field, which can be overwhelming; it is not always easy to know how one got to the present from our confused roots. I believe that an appreciation of the early days of the field, specifically the deep physical insights and mathematical tools that Bob Dewar contributed — the "Lessons in Creativity and Courage" — can pay great dividends in paving the way for further progress.

I will begin in Sec. 2 with some general remarks on the problem of the transition to turbulence. Those will motivate consideration of states of fully developed turbulence. Bob contributed in several ways: in studies of turbulence-related bifurcations and zonal-flow generation (section 3); and in attempts to formulate a new statistical theory of plasma turbulence (section 4). Because of the time constraint for my Symposium talk and the length constraint for this paper, I will say only a few words about the former topic; the heart of the paper is on statistical turbulence theory.

2. General remarks about the transition to turbulence

The prototypical partial differential equations (PDEs) for studies of the nonlinear behavior of fluids and plasma are well known: the Navier–Stokes equation for neutral fluids; and the Vlasov equation or Landau kinetic equation for plasmas. Both of those equations are quadratically nonlinear, and it is well known that they can support stochastic, chaotic, or turbulent behavior.[2] Such equations exhibit a *transition to turbulence* as an appropriate bifurcation parameter (such as a background profile gradient) is increased. There are many "routes to chaos" (Martin, 1982), well described in textbooks and monographs such as that of Lichtenberg and Lieberman (1992).

[1] This article recounts the talk I gave at the Dewar Symposium (October 31, 2009, Atlanta), which honored the research career of Bob Dewar.

[2] Stochastic generally implies a Hamiltonian description; chaotic implies random dissipative behavior in systems with small numbers of degrees of freedom; and turbulent implies random spatio–temporal behavior in systems with many excited degrees of freedom.

For Hamiltonian systems, one may begin with an integrable Hamiltonian with good Kolmogorov–Arnold–Moser surfaces. Addition of a small, angle-dependent perturbation generally introduces tiny, nonoverlapping islands. Further increase of the amplitude of the perturbation leads to local island overlap and ultimately to global stochasticity. Dissipative systems may pass from linear stability through a supercritical bifurcation, to multiple excited modes, and ultimately to turbulence. Alternatively, linearly stable systems may suffer a subcritical bifurcation (nonlinear instability) and may become turbulent even in the face of linear stability. There are other exotic possibilities.

Near the transition to turbulence, specific techniques drawn from the theory of nonlinear dynamics (Guckenheimer and Holmes, 1983) may be useful. One can, for example, attempt to use center-manifold theory (Kuznetsov, 1998) to eliminate rapidly decaying modes. If that succeeds, one is left with a low-dimensional system for the slow modes, which one can then analyze qualitatively. An example of this kind of calculation is given by the work of Kolesnikov and Krommes (2005*b*,*a*) on the transition to collisionless ion-temperature-gradient-driven turbulence and the role of zonal flows on the so-called Dimits shift (Dimits et al., 2000).

Major problems arise, however, if many modes bifurcate at almost the same value of the bifurcation parameter. This is the case for microscale fluctuations (e.g., drift waves) in a macroscopic system such as the tokamak. Here straightforward center-manifold analysis leads to a system, containing many coupled modes, for which standard nonlinear-dynamics analysis becomes intractable. The traditional solution has been to ignore the details of the transition and consider analytical statistical techniques that predicts details of the saturated fluctuation spectrum on a wave-number by wave-number basis. Those are discussed in Sec. 4. But two other approaches have also proven fruitful: direct numerical simulations; and bifurcation analysis of equations for turbulent intensities, in which details of the wave-number spectrum are ignored. Bob Dewar contributed insights to both of those, as I describe briefly in the next section.

3. Turbulence-related bifurcations and zonal flows

It is not difficult to construct and analyze a simple predator–prey model of the interactions of drift waves and zonal flows (Diamond et al., 1994). A simple two-field model possesses a very simple bifurcation structure with two stable states, qualitatively reminiscent of the L–H transition (between low- and high-confinement states) observed in tokamak experiments. But more complicated equations, with richer physics, can be nontrivial to analyze. An example is given by the model of Sugama and Horton (1995), whose bifurcation structure was thoroughly studied by Ball and Dewar (2000) and Ball, Dewar, and Sugama (2002). Their results point to the importance of hysteresis and symmetry breaking in models of the L–H transition and support the belief "that remarkably low-dimensional models can capture and help explain essential aspects of turbulent flows."

Ultimately numerical simulation is essential even for simple paradigm models. A classic example of such a model is the system first proposed by Hasegawa and Wakatani (1983, HW), frequently advanced as a paradigm for collisional drift-wave turbulence in a tokamak edge. Numata, Ball, and Dewar (2007) used numerical simulations (on a 3D HW model that properly respected the physics of zonal flows) to support a general scenario of primary, secondary, and tertiary instabilities. That work identified the analog of the Dimits shift for collisional drift waves and showed that the ultimate onset of turbulence was "due to the disruption of zonal flows by [Kelvin–Helmholtz] instability."

If this were a proper review article, much more should be said about both the work cited in this section and similarly focused research by others. Here I have merely tried to point out that Bob has worked broadly on state-of-the-art issues relating to the development and suppression of plasma microturbulence. In the next section I will go into more depth about his courageous attempts to deal with the problem of fully developed turbulence.

4. Bob Dewar and the statistical theory of turbulence

Turbulence is inherently statistical in nature. The well-known extreme sensitivity to small changes in initial conditions means that detailed microstructure may not be observable even in principle. The use of some sort of averaging procedure seems to be inevitable (Tsinober, 2009). Consider, for example, the calculation of a turbulent transport flux, for example the flux Γ of density n due to advection by a turbulent velocity field $\widetilde{V}$. One can define an instantaneous, microscopic flux by $\widetilde{\Gamma} \doteq \delta V \delta n$, where δ denotes the fluctuation from the mean: $\delta V \doteq \widetilde{V} - \langle V \rangle$. (I use $\doteq$ for definitions.) This quantity, measured either experimentally or from computer simulation, is a random time series. The "observable" or mean flux is usually defined to be the time average of $\widetilde{\Gamma}$: $\Gamma \doteq \overline{\overline{\widetilde{\Gamma}}}$. Analytically, one usually implicitly assumes some sort of ergodic theorem and introduces an ensemble average: $\Gamma = \langle \widetilde{\Gamma} \rangle$.

Thus, by definition, turbulent fluxes or transport coefficients are inherently statistically averaged quantities. But averaging can be performed at various stages of the calculations. Given a nonlinear equation, one can either (i) measure experimentally or calculate (probably numerically) a detailed solution for the microscopic nonlinear dynamics, then average to find a mean turbulent flux; or (ii) apply a statistical averaging procedure directly to the nonlinear equation, then extract the mean flux directly from the predicted averaged quantities (the turbulent flux involves just a second-order cross correlation function). In principle, both procedures should lead to the same result. In practice, however, it is not so simple. Practical approaches to direct statistical averaging are inevitably approximate and can be very tricky. Although modern statistical closure theory is about a half-century old [and its roots date back about a century; see, for example, Taylor (1915)], it is still very difficult to assess from first principles the degree of error in an approximate calculation. Deep physical insight and powerful mathematics are equally essential. Bob Dewar provided both.

Plasma turbulence is a difficult problem! One must cope with mixtures of integrable and stochastic regions (for Hamiltonian dynamics) and the possibility of "coherent" structures embedded in a random sea of turbulence (for forced, dissipative systems). The details of the particle distribution function in velocity space may be important (for linear growth rates and kinetic dissipation), one must calculate self-consistent electromagnetic fields, and one must face up to the complications introduced by the presence of many kinds of wave-like linear eigenmodes (absent in the "simple" case of homogeneous, isotropic, incompressible neutral-fluid turbulence).

For coping with this diversity of complications, two techniques stand out. The first is *renormalization*, which for present purposes may be thought of as describing the statistical effects of nonlinear mode coupling. (I will say more about renormalization shortly.) The second technique is less often discussed and is frequently overlooked; it relates to *the optimal choice of variables*. It is, of course, well known that complicated physics can be elucidated by a proper choice of representation (and can be rendered inscrutible by a poor choice). Dewar's creative and courageous contributions relate to attempts to marry these two techniques.

4.1. Renormalization

If one wishes to truly understand renormalization, there may be no alternative to serious study of very thick and very dense books [e.g., that of Zinn-Justin (1996)]. But the basic ideas are actually quite intuitive. A variety of useful perspectives, both historical and modern, are discussed in the collection edited by Brown (1993), which [particularly the article by Dresden (1993)] have informed my following brief remarks.

Consider a sphere, of mass m_0 and radius R, accelerated from rest in an incompressible fluid of density ρ. It was already known to Stokes in 1843 that the equation of motion for the sphere is

$$(m_0 + m_{\text{fluid}})\frac{dv}{dt} = F, \tag{1}$$

where $m_{\text{fluid}} \doteq \frac{1}{2}\left(\frac{4}{3}\pi R^3\rho\right)$. (The parenthesized factor is obviously the mass of the displaced fluid; the coefficient of $\frac{1}{2}$ depends on the shape of the object.) One says that the presence of the fluid has *renormalized* the "bare" mass m_0 of the sphere to the effective mass $m_{\text{eff}} \doteq m_0 + m_{\text{fluid}}$.

Later, similar ideas were applied to the problem of the motion of charged particles through the purported ether. Thomson (circa 1881) and Lorentz considered an electromagnetic mass $m_{\text{EM}} = 2e^2/(3ac^2)$, where a is the radius of a uniformly charged sphere. (The moving sphere excites electromagnetic fields, which then act back on the sphere.) Thus the electromagnetic field *renormalizes* the mass m_0 of the particle to the total mass $m \doteq m_0 + m_{\text{EM}}$. Only the latter is experimentally observable.

Such an interpretation suffers from the indignity that $m_{\text{EM}} \to \infty$ as $a \to 0$. That implies that m_0 must also be (negatively!) infinite if the experimental mass m is to be finite. Thus renormalization may involve tricky infinities. Those are so common in quantum field theory that sometimes renormalization is equated with a program for removing infinities. But the discussion of $m_{\rm fluid}$ above, which does not involve an obvious infinity, shows that a more general interpretation is useful. It is better to think of renormalization as a way of calculating approximate expressions for *physically observable* quantities such as mass and charge. In the general theory of nonlinear PDEs, the observables are statistically averaged quantities such as the mean field and the two-point correlation and response (Green's) functions. Renormalized equations for the Navier–Stokes equation (which by definition includes viscous dissipation) are not divergent (although infinities may surface in discussions of particular asymptotic limits).

How does one deal with infinities when they do arise? A further bit of history is very relevant for understanding of the approach Dewar took to renormalized plasma turbulence theory. Dresden (1993) nicely describes a program outlined by Kramers (circa 1938) with the goal of quantizing classical electrodynamics. I paraphrase:

(i) Begin with an extended, classical charge distribution (of radius a) interacting with a given external electromagnetic field.

(ii) Construct an exact or approximate Hamiltonian for that system.

(iii) Separate the total electromagnetic field into a self field and an external field, and also separate the mass into mechanical and electromagnetic components. This leads to a dynamical Hamiltonian formulation that contains a mixture of structure-independent and structure-dependent terms.

(iv) *Eliminate the structure-dependent terms from the Hamiltonian by one or a series of canonical transformations.* This should lead to an autonomous structure-independent Hamiltonian K.

(v) Now carry out the limit $a \to 0$, which should be finite.

(vi) Finally, quantize the (presumably finite) structure-independent K.

Kramers' program was never completed; it was overtaken by the explosion of new ideas and insights relating to relativistic quantum electrodynamics as formulated by such giants as Schwinger, Tomonaga, Feynman, and Dyson.[3] But it is of great conceptual importance. In particular, the key step of transforming away the structure-dependent terms, leaving a nondivergent or "true" Hamiltonian K, anticipates Dewar's use of canonical transformations to oscillation-center coordinates, as we will see in section 4.3.

4.2. The transformative month of July, 1973

Although it is greatly tempting to linger in the challenging realm of classical and quantum electrodynamics, let us now leave the 1930's and 1940's and fast-forward to the month of *July, 1973*. In the world at large, this was not an unusually interesting month, although there were a few noteworthy events: Betty Grable died; Monica Lewinsky was born; the existence of the Nixon tapes was revealed to the Senate Watergate Committee; and novelist, philosopher, and psychonaut Robert Anton Wilson

[3] A good route to obtaining a first understanding of this exciting period in the history of physics is to peruse the scientific biographies of Schwinger (Mehra and Milton, 2000) and Feynman (Mehra, 1994).

(RAW, 2009) apparently[4] established contact with extraterrestrials from Sirius. But in the world of physics, this month stands out because of the publication of two transformative physics papers: Dewar's "Oscillation center quasilinear theory" (Dewar, 1973, see figure 1); and the work of Martin, Siggia, and Rose on the "Statistical dynamics of classical systems" (Martin et al., 1973, see figure 3). As we will see, those papers [as well as the important sequel of Dewar (1976) on "Renormalized canonical perturbation theory for stochastic propagators"] laid the foundations for two approaches to the problem of renormalization that, although superficially quite distinct, are actually closely related. I will discuss Dewar's work in the next section; I will address the MSR formalism (which introduced the concepts of "mass" and "charge" renormalization for classical systems with quadratic nonlinearity) in section 4.4.

4.3. Oscillation-center quasilinear theory: original, and renormalized

4.3.1. The original (unrenormalized) oscillation-center quasilinear theory. To understand the motivation for the work of Dewar (1973), one must back up one year, when Kaufman (1972) described his "Reformulation of quasi-linear theory." Quasilinear theory was then in a state of some confusion because of the "fake diffusion" associated with nonresonant particles and uncertainties relating to the very existence of a positive velocity-space diffusion coefficient in the presence of stable (negative growth rate) linear waves. By using the method of multiple time scales, Kaufman showed that the nonresonant particles should properly be counted as part of the wave action (thus contributing to the momentum and energy of the waves), and that the resonant particles obey the standard velocity-space diffusion equation (with a diffusion coefficient that is intrinsically positive even for decaying modes).

Dewar (1973, figure 1) then showed that Kaufman's results had a beautiful interpretation in terms of Hamiltonian canonical transformation theory. The idea was that the oscillating waves induce small wiggles on the basically straight-line motion of the nonresonant particles. Dewar removed those oscillations by a canonical change of variables to *oscillation-center coordinates*; he showed that it is the oscillation centers that obey the quasilinear diffusion equation. The sloshing motion of the nonresonant particles, tied up in the definition of the coordinate transformation, is just what is required to prove momentum and energy conservation for the combined system of resonant and nonresonant particles. A decade later, similar ideas (with some technical innovations) were used in developing the Hamiltonian formulation of gyrokinetics (Dubin et al., 1983); further elegant developments were reviewed by Brizard and Hahm (2007) and Krommes (2012).

4.3.2. Renormalized canonical perturbation theory. The heart of Dewar's attack on the plasma turbulence problem can be found in his extremely ambitious, creative, and courageous paper on "Renormalized canonical perturbation theory for stochastic

[4] I learned this striking information from a `google` search on the web. It therefore must be true. . .

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Oscillation center quasilinear theory

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A new formulation of the quasilinear theory of weakly turbulent plasmas is presented, which explicitly separates resonant and nonresonant wave-particle interactions from the outset. This is achieved by making a canonical transformation to "oscillation center variables" before attempting to solve the Vlasov equation. A systematic method of constructing the generating function to any order in the wave amplitude is presented, based on a variant of Hamilton–Jacobi perturbation theory. Momentum and energy split naturally into a wave and a particle component. The results are generalized to apply to weakly inhomogeneous plasmas, and verified by demonstrating momentum and energy conservation.

Figure 1. The header of Dewar's seminal paper on oscillation-center quasilinear theory. Reprinted with permission from Dewar (1973), copyright 1973 by the American Institute of Physics.

propagators" (Dewar, 1976). It is essentially a sophisticated application of oscillation-center theory to the description of renormalized Green's functions. Motivation can be found by considering the well-known representation of the velocity-space diffusion coefficient as a time integral of the Lagrangian (measured along the trajectories) correlation function of the acceleration $a = qE/m$:

$$D_v = \int_0^\infty d\tau\, C_{aa}(\tau). \tag{2}$$

(I consider one-dimensional motion for simplicity.) Such a correlation function, depicted in figure 2, is the net response to two distinct physical effects. First, there is the decorrelation of a particle, moving with velocity $v = v_{\mathrm{ph}}$, from the wave packet (moving with velocity v_{gr}) that accelerates it. If the spectral width of the wave packet is Δk (corresponding to an x space width of $2\pi/\Delta k$), one readily estimates the autocorrelation time to be $\tau_{\mathrm{ac}}^{\mathrm{lin}} \propto (|v_{\mathrm{ph}} - v_{\mathrm{gr}}|\,\Delta k)^{-1}$. I use the superscript lin to emphasize that $\tau_{\mathrm{ac}}^{\mathrm{lin}}$ can be calculated entirely from properties of the linear wave spectrum.

When the wave-number spectrum is discrete, as it would be for plasma confined in a periodic box, one can show [see more detailed discussion in Krommes (2002, Appendix D)] that the correlation function including only the $\tau_{\mathrm{ac}}^{\mathrm{lin}}$ effect is recurrent on a long time scale $\tau_{\mathrm{r}} \gg \tau_{\mathrm{ac}}^{\mathrm{lin}}$. The infinite-time integral of such a function is pathological, but one expects that in a stochastic regime (when the Chirikov criterion for resonance overlap is satisfied) $C(\tau)$ should decay irreversibly, not recur. The time scale for that decay can be estimated to be Dupree's diffusion time[5] $\tau_{\mathrm{d}} \propto (\overline{k}^2 D_{\mathrm{v}})^{-1/3}$, where $\overline{k}$ is a typical spectral wave number. One can show that in the stochastic regime one has $\tau_{\mathrm{ac}}^{\mathrm{lin}} \ll \tau_{\mathrm{d}} \ll \tau_{\mathrm{r}}$. Thus the effect of stochastic diffusion is to provide a decaying envelope that cuts off the recurrent peaks and gives a true correlation function that

[5] This time has the same scaling as, but is not identical to, the Lyapunov time for the exponential separation of adjacent trajectories.

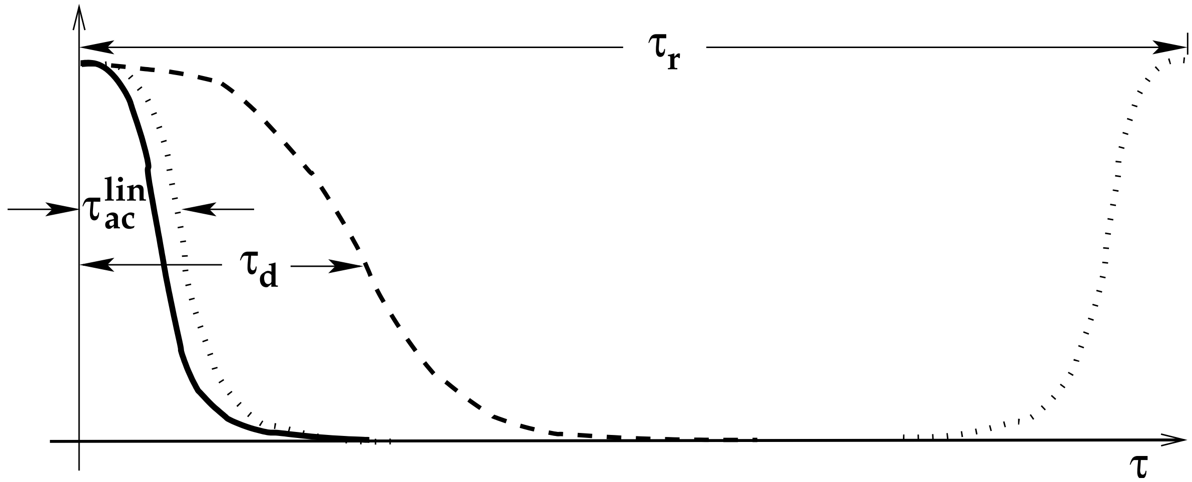

Figure 2. Schematic representation of the correlation function $C_{aa}(\tau)$, showing linear response (dotted lines, with width $\tau_{\rm ac}^{\rm lin}$) and nonlinear envelope (dashed line, with width $\tau_{\rm d}$) in the quasilinear regime. Their product is the actual response (solid line), with width $\tau_{\rm ac} \approx \tau_{\rm ac}^{\rm lin}$. The envelope prevents recurrence on the timescale $\tau_{\rm r}$. This figure is a slight correction of Fig. 12 of Krommes (2002).

decays irreversibly on the $\tau_{\rm ac}^{\rm lin}$ scale.

The stochastic diffusion gives rise to a *resonance broadening*. Consider the function

$$R(\tau) = \int_0^\infty d\tau\, e^{-i\omega\tau + ikv\tau - \tau/\tau_{\rm d}} = [-i(\omega - kv + i\tau_{\rm d}^{-1})]^{-1}, \tag{3}$$

which occurs if the τ integration is interchanged with the wave-number summation that is buried in the represention of C_{aa} in terms of Eulerian Fourier amplitudes.[6] The resonance width is thus $\Delta\omega = \tau_{\rm d}^{-1}$. This observation motivated the pioneering work of Dupree (1966) on what is now called *resonance-broadening theory*; see also Dupree (1967) for the initial application to magnetized plasmas. It underlies the work of Dewar (1976), as I will now discuss.

The basic idea of the application of Dewar's oscillation-center theory to plasma turbulence is the following. Since the presence of the stochastic envelope is essential for the proper definition of the correlation function, bring that to the fore by focusing on the *response function of the oscillation centers* (which as we know obey a diffusion equation). Linear response (associated with wave-packet decorrelation and recurrences) is encapsulated in the transformation of variables. Ultimately, the nonlinear envelope and the linear response must be appropriately multiplied together to give the total response, as depicted in figure 2.

In more detail, the goal set by Dewar (1976) was to find "a canonical transformation which removes the coherent oscillatory motion of a particle in a stochastic potential." Just as in unrenormalized oscillation-center theory, this is only possible for nonresonant particles. The extra difficulty here is that the resonances are broadened, so the concepts of "resonant" and "nonresonant" are not cleanly defined. One needs to determine the

[6] Detailed discussion of this point can be found in section D.2 of Krommes (2002).

width of the resonance self-consistently. That is, one must determine a "renormalized oscillation-centre transformation."

To accomplish this goal, Dewar first developed an operator formalism for canonical transformations. That has been quite influential in many subsequent applications unrelated to renormalized plasma turbulence; see the review article by Cary (1981) on "Lie transform perturbation theory for Hamiltonian systems." Then Dewar focused on the Dyson equation for the mean Green's function of the oscillation center. (I will define a Dyson equation in the next section.) The details are not important here, but Dewar's conclusions are worth repeating: "We have developed a canonical perturbation theory for the single particle propagator which allows systematic calculation of non-linear effects and the inclusion of resonance broadening. The use of Green functions is reminiscent of quantum field theory and statistical mechanics . . . ; conversely, the Poisson bracket structure of the perturbation theory should allow straightforward quantization." His program should be compared to Kramers's quantization program recounted in section 4.1. Clearly in both cases the core of the formalism involves a canonical transformation to remove or encapsulate unwanted physics that disguises the intrinsic features of the problem. This is intuitively plausible, and it lends itself to specific and nontrivial technical manipulations.

Dewar's approach is certainly appealing. However, it raises some difficult questions. For example, is it really correct to apply statistical methods to the oscillation-center response function? The oscillation-center transformation is both stochastic and nonlinear. If one requires a correlation function in original particle variables,[7] then it is likely that an average of the product of a stochastic response function and some function of the stochastic transformation will be required. In general, such averages do not factor due to complicated statistical dependencies. And if the results of Dewar (1976) omit certain statistical correlations (i.e., if in some sense it is a lowest-order formalism), is a systematic strategy indicated that would enable one to proceed, at least in principle, to higher order?[8]

A procedure which applies statistical methods directly to the original PDE, in the original variables, and also can be (in principle) systematically extended to include more and more statistical correlations is the formalism of Martin, Siggia, and Rose (MSR), also published in July, 1973. I discuss it briefly in the next section.

4.4. The Martin–Siggia–Rose formalism

The MSR formalism (Martin et al., 1973, figure 3) is the classical version of Schwinger's functional formalism for quantum field theory. It can be applied to any nonlinear

[7] How does one know that the original variables were in any sense appropriate? One answer is that they may have something to do with the symmetries of the problem, and one wants the ultimately calculated statistical observables to reflect those symmetries.

[8] I am not referring here to refining the OC transformation, which can be done systematically. The remark relates to improvements to the procedure of applying statistical methods to the OC response function.

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Statistical Dynamics of Classical Systems*

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The statistical dynamics of a classical random variable that satisfies a nonlinear equation of motion is recast in terms of closed self-consistent equations in which only the observable correlations at pairs of points and the exact response to infinitesimal disturbances appear. The self-consistent equations are developed by introducing a second field that does not commute with the random variable. Techniques used in the study of the interacting quantum fields can then be employed, and systematic approximations can be obtained. It is also possible to carry out a "charge normalization" eliminating the nonlinear coupling in favor of a dimensionless parameter which measures the deviation from Gaussian behavior. No assumptions of spatial or time homogeneity or of small deviation from equilibrium enter. It is shown that previously inferred renormalization schemes for homogeneous systems were incomplete or erroneous. The application of the method to classical microscopic systems, where it leads from first principles to a coupled-mode description is briefly indicated.

Figure 3. The header of the famous paper that introduced the "MSR formalism." Reprinted with permission from Martin et al. (1973), copyright 1973 by the American Physical Society.

equation or system with polynomial nonlinearity (the original paper discussed quadratic nonlinearity, which is the most common application). It is fully renormalized, meaning that it works only with statistical observables (the mean field, the two-point correlation function, the two-point response function, and some three-point vertex functions). At lowest order, in a certain well-defined sense, it recovers the direct-interaction approximation (DIA), which had been developed earlier by Kraichnan (1959) using different methodology. A lengthy review article that emphasizes the MSR formalism and contains many references is by Krommes (2002).

Although the renormalized equations of the DIA or other MSR-derived closures are sophisticated, they need not always work well in practice. In particular, they may give the wrong answer unless one already knows the qualitative physics in advance and inserts extra asystematic tweaks to keep the formalism physically on track. The most famous example is the lack of invariance of the DIA to random Galilean transformations (Kraichnan, 1964), a consequence of which is that for high-Reynolds-number turbulence it predicts the wrong inertial-range spectrum ($k^{-3/2}$ instead of Kolmogorov's $k^{-5/3}$). Also, systems with highly intermittent (non-Gaussian) statistics cannot be well described by low-order truncations of the MSR functional equations.

One prediction of the MSR formalism that may be correct is that various kinds of physical effects are mixed together in a very complicated way by the statistical averaging. The formalism does not necessarily help one in understanding that mixture. Therefore, alternate approaches such as that of Dewar have intrinsic appeal. One is reminded of the quote by Frisch (1993), "I will not rule out that renormalization methods will have a lot to say about turbulence once they are applied to the right objects, which has not been the case so far." This remark came well after Dewar's work, of which I am guessing that Frisch was not aware. But in any event his remark is very apropos. Dewar's attempt to introduce better variables into the self-consistent calculation of turbulent Green's functions is novel, creative, and courageous. It was driven by deep physical insight, aimed specifically at finding the "right objects," and synthesized two powerful techniques, namely renormalization and canonical perturbation theory. It is interesting to compare Dewar's introduction of oscillation-center variables into statistical turbulence theory with Kraichnan's development of Lagrangian statistical closures (Kraichnan, 1965). The latter cure the problem with random Galilean invariance of the DIA by introducing Lagrangian Green's functions that, essentially, follow along moving eddies. Dewar's desire to work with the oscillation centers is conceptually similar. It would be instructive to explore the connection between renormalized oscillation-center theory and Lagrangian closures at a deep mathematical level.

In spite of obvious difficulties, the appeals of the MSR formalism and the DIA were sufficiently great that they dominated formal work on statistical plasma turbulence theory for decades. The plasma DIA was studied in depth by Dubois and independently Krommes in the mid-1970's through the early 1980's. [Sample references include DuBois and Espedal (1978) and Krommes (1978); many more are given by Krommes (2002).] However, the plasma DIA is very complicated because it attempts to describe the statistical dynamics of nonlinear interactions in velocity space. An easier yet nontrivial route is to study fluid models. Pioneering work was done by Sudan and Keskinen (1977, 1979), and Krommes made various fluid-DIA-related contributions in the mid-1970's through the 1980's. Extensive study of DIA-based Markovian closures was performed by Krommes's group in the 1990's; see Krommes (2002) for references. Around 2000, attention shifted to the analytical theory of zonal flows. A seminal paper by Diamond et al. (1998) was followed by a lengthy discussion by Krommes and Kim (2000) that developed the statistical theory of zonal-flow generation in terms of the MSR formalism and Markovian statistical closures.

Those and similar works employ various versions of statistical closure methodology. They should be viewed in the context of Dewar's philosophy, which states (Dewar, 1985), "The theory [of transforming to the OC representation] is basically geometrical . . . , and we prefer not to prejudge which of the various statistical approaches to turbulence, applied *after* the transformation has been performed, would be better."

4.4.1. The relation between renormalized oscillation-center theory and the MSR formalism. Dewar's approach is, at the very least, a valiant attempt. But in assessing its fidelity, one important observation must be kept in mind: The MSR formalism is formally complete in and of itself. Given a nonlinear PDE, the formalism provides a self-consistent theory of coupled statistical observables that in principle provides a complete statistical description. This implies that the physics of oscillation centers, ponderomotive Hamiltonians, *etc.* must somehow be buried in the MSR equations — but where? It is interesting that a (partial) answer to this question was only given in 2007, some thirty years after Dewar's work on renormalized oscillation-center theory. In the proceedings of the symposium honoring Allan Kaufman's 80th birthday, the articles for which may be found online (Brizard and Tracy, 2009),[9] Krommes (2009) carefully worked out the mass operator Σ for renormalized, inhomogeneous quasilinear theory, from which he was able to extract the usual ponderomotive force. However, whereas that force follows so neatly from oscillation-center theory, it is somewhat buried in tedious algebra in the MSR approach. Part of the difficulty is that the MSR formalism proceeds in laboratory coordinates, so one must be able to somehow recognize the change of variables from those of the oscillation center to those of the particle. That was done, but it is important to remark that knowing the answer in advance was extremely helpful in reconciling the algebra! The generality and power of the MSR approach should not be denied, but it was the physical insights of the pioneers such as Dewar that provided the rock-solid foundations on which future progress can proceed.

5. Summary and conclusions

I described some of the contributions that Bob Dewar made to the theory and physical understanding of plasma turbulence, including both the transition thereto and the analytical description of the fully developed state. It is remarkable that Bob was able to make substantial, deeply insightful contributions to those problems, both of which are challenging and difficult yet represent only a small fraction of the research problems that Bob has tackled (so far!) during his career. He provides an wonderful example that we would all do well to emulate.

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[9] Kaufman's personal account of his career in plasma physics (Kaufman, 2009), which includes significant references to Bob Dewar, is of particular interest.

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