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Effects of the Spatial Extent of Multiple Harmonic Layers

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Abstract. An analytic model for single particle motion in the presence of a wave field and multiple cyclotron harmonics is developed and investigated. The model suggests that even in the absence of Doppler broadening, cyclotron harmonic layers have finite spatial extent. This allows for particles to interact with more than one harmonic layer simultaneously, provided the layers are tightly packed. The latter phenomenon is investigated in the context of the model using symplectic mapping techniques. Then the model behavior is compared with numerical simulations of neutral beam particle trajectories in NSTX using the full-orbit code SPIRAL.

Keywords: cyclotron resonance, multiple harmonic layers
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INTRODUCTION

In recent HHFW and NBI heating experiments on NSTX [1], observed fast ion distribution profiles have disagreed qualitatively with numerical simulations. This suggests that the underlying assumptions used in these simulations for the dynamics of magnetized particles interacting with wave fields should be carefully considered.

Particles moving in a magnetic confinement device heated by HHFW with short ($k_\perp \rho > \nu$) perpendicular wavelength experience large excursions in energy in those regions of the vessel where ν, the local harmonic number, is integer valued[1] [2]. This condition on ν defines 2-dimensional non-intersecting surfaces, henceforth referred to as harmonic layers. However, the situation just described is more subtle than it might seem. For instance, when the harmonic layers are "close", is it possible for a single particle to strongly interact with more than one simultaneously?

The answer to this question is "yes", and it will be shown that the harmonic layers in NSTX are sufficiently close together so that the 90 keV deuterium neutral beam ions provide a real-world example of this phenomenon. First, simple model fields are presented that are meant to resemble the fields in the vicinity of a drift surface in a tokamak. A Hamiltonian description of the single particle dynamics is then (briefly) described that exploits the assumption that the background magnetic field is slowly varying in space. After posing a perturbative solution to these Hamiltonian equations, a stationary phase argument then leads to a simple expression for the width of the interaction regions - those regions around each harmonic layer where a particle can strongly interact with the wave. Finally, after fitting the model to NSTX parameters,

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[1] Actually, this is ignoring the finite $k_\parallel$ present in any real wave heating scheme.

it is shown that the interaction regions for the neutral beam deuterons in NSTX are overlapping, thereby indicating the existence of a population of particles in NSTX that interact with more than one harmonic layer simultaneously.

MODEL FIELDS

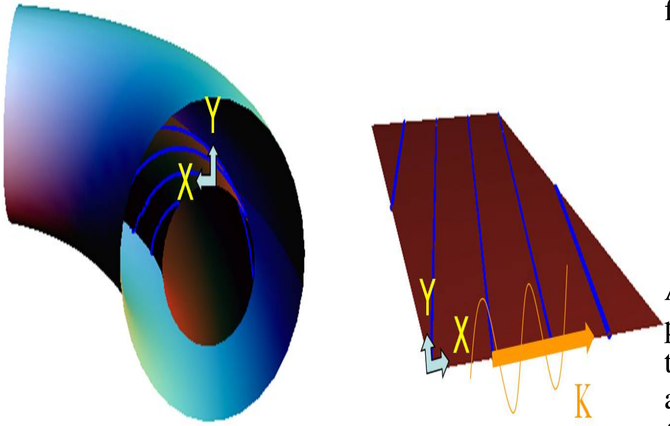

FIGURE 1. Correspondence between model fields and fields in the vicinity of a tokamak drift surface. The lined surfaces represent some drift surface, while the lines represent the magnetic field lines on that surface, which to leading order in inverse aspect ratio is the same as a flux surface. The wave propagates purely poloidally.

Consider individual charged particles moving in the electromagnetic fields given by

$$\mathbf{B} = B_z(x)e_z + B_P e_x \qquad (1)$$
$$B_z(x) = B_T(1 - \varepsilon\cos(k_b x))$$

$$\mathbf{E} = \nabla\left(\frac{E_o}{k_\perp}\sin\left(k_\perp x - \omega t\right)\right). \qquad (2)$$

After making the x-axis $2\pi/k_b$-periodic[2], these fields correspond to those found in the vicinity of a drift surface of a tokamak as follows (see also Fig. 1). B_z and B_P are the toroidal and poloidal magnetic field components, ε is the drift surface inverse aspect ratio, and $2\pi/k_b$ is the circumference of the drift surface. Finally, the electric field corresponds to a purely poloidally propagating electrostatic wave. Note that these model fields ignore the phenomena of radial and toroidal propagation, as well as electromagnetic waves. It would be a straightforward task to account for these effects in future work.

HAMILTONIAN FORMULATION OF PARTICLE DYNAMICS

The goal now is to investigate how particles will move in the above fields in the limit where the magnetic field scale length is large compared with the particle gyroradius and the wave field is weak. Because the exact dynamics are Hamiltonian, this asymptotic limit should taken in a "Hamiltonian way", as in [3], which amounts to a coordinate change to guiding center variables. After implementing this coordinate change, as well as slightly modifying the guiding center Y coordinate so as to obtain canonical equations,

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[2] Note that this forces $k_\perp/k_b$ to be integer valued.

the particle Hamiltonian in $[X, \Phi, \mu, \theta]$ coordinates is given by

$$H = \mu\Omega_T \left(1 - \varepsilon\cos(k_b X)\right) + v_D \Phi - \alpha_o \sin\left(k_\perp X + k_\perp \rho \sin\theta - \omega t\right), \tag{3}$$

where $\alpha_o = \frac{eE_o}{mk_\perp}$, $\rho = \sqrt{\frac{2\mu}{\Omega_z(X)}}$, v_D is the drift velocity, and $(\Omega_z, \Omega_T) = e/m \times (B_z, B_T)$. The "generalized coordinates" are (X, θ), guiding center x-coordinate and gyroangle, with corresponding momenta (Φ, μ), the modified guiding center y-coordinate and magnetic moment.

This Hamiltonian is naturally ordered in α_o, which is a small parameter by assumption. It is thus advantageous to use coordinates well-adapted to the zero'th order dynamics, the so-called action-angle variables $[\bar{X}, \bar{\theta}, I_{\bar{X}}, I_{\bar{\theta}}]$. These are given by

$$(\mu, \Phi, X, \theta) = \left(I_{\bar{\theta}}, I_{\bar{X}} + \varepsilon\frac{\Omega_T}{v_D}\cos(k_b\bar{X})I_{\bar{\theta}}, \bar{X}, \bar{\theta} - \varepsilon\frac{\Omega_T}{k_b v_D}\sin(k_b\bar{X})\right). \tag{4}$$

In these coordinates the zero'th order Hamiltonian is simply

$$H_o = I_{\bar{\theta}}\Omega_T + I_{\bar{X}} v_D, \tag{5}$$

and the perturbation is

$$H_1 = -\sum_{m\in\mathbb{Z}} J_m(k_\perp \rho)\sin\left(k_\perp \bar{X} + m\left[\bar{\theta} - \varepsilon\frac{\Omega_T}{k_b v_D}\sin(k_b\bar{X})\right] - \omega t\right), \tag{6}$$

where each term in the sum may be identified with a different cyclotron harmonic. In particular, note that only those terms with $k_\perp \rho > m$ are important because Bessel functions become very small when their argument is greater than their index.

PREDICTION OF THE WIDTH OF THE RESONANT REGIONS AND COMPARISON WITH SPIRAL

With the Hamiltonian in this convenient form, it is now simple to apply an iterative perturbation technique (Neumann series) as an attempt to determine the effect of the wave perturbation on the zero'th order trajectories. As is readily checked, this effect is determined by the integral of the perturbing potential along the zero'th order trajectories. However, for fixed α_o,this integral will accurately represent the effect of the wave only when its value is approximately 0. Because each term in the potential is rapidly varying except near those times where the phase of the corresponding sin function is stationary, it follows that the integral will indeed nearly vanish provided the time of integration does not come near a stationary phase point of any term in the potential. The temporal width of the interaction region for the m'th harmonic is therefore determined by the second derivative of that harmonic's phase via

$$\Delta t = \sqrt{\frac{2}{|\phi_m''(t_o)|}} = \frac{1}{k_b v_D}\sqrt{2\frac{k_b v_D}{\Omega_T}}\left(\varepsilon^2 m^2 - \left(m - \frac{\omega - k_\perp v_D}{\Omega_T}\right)^2\right)^{-1/4}, \tag{7}$$

where $\phi_m(t)$ is the phase of the m'th sin function evaluated along a zero'th order trajectory, and t_o is a stationary phase point.

Now consider 90 keV deuterons in NSTX interacting with the waves generated by the HHFW antenna. Using G. J. Kramer's full-orbit Lorentz force solver SPIRAL [4], it is a simple matter to determine the orbit parameters necessary to fit (roughly) the above model to these particles. After doing so, it is possible to predict using (7) the times during a circulation orbit that a deuteron interacts with some cyclotron harmonic. An example calculation is shown in Fig. 2 for a particle with pitch angle of $1/2$. The height of the boxes denotes harmonic number while the width gives the temporal extent of the corresponding interaction region. The overlapping of the interaction regions is evident.

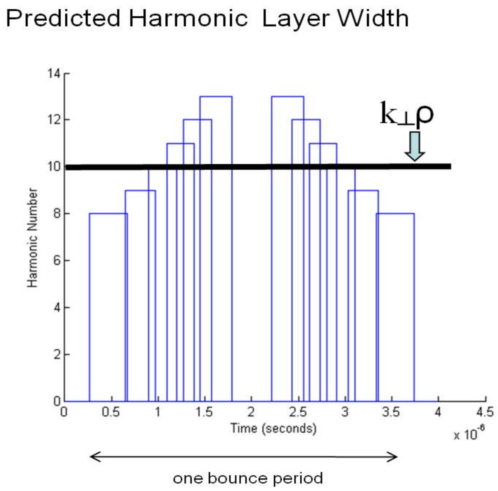

FIGURE 2. Model prediction of temporal width of interaction regions for 90 keV deuteron in NSTX

Having identified the existence of this resonance overlap phenomenon in NSTX, it is now necessary to investigate its implications experimentally and theoretically. This will be carried out in forthcoming publications.

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