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Current Density and Plasma Displacement Near Perturbed Rational Surface

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Current density and plasma displacement near perturbed rational surfaces

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The current density in the vicinity of a rational surface of a force-free magnetic field subjected to an ideal perturbation is shown to be the sum of both a smooth and a delta-function distribution, which give comparable currents. The maximum perturbation to the smooth current density is comparable to a typical equilibrium current density and the width of the layer in which the current flows is shown to be proportional to the perturbation amplitude. In the standard linearized theory, the plasma displacement has an unphysical jump across the rational surface, but the full theory gives a continuous displacement.

A magnetic field whose field lines lie on toroidal magnetic surfaces, $\vec{B}\cdot\vec{\nabla}\psi_t = 0$ can be represented as

$$2\pi\vec{B} = \vec{\nabla}\psi_t \times \vec{\nabla}\theta + \iota(\psi_t)\vec{\nabla}\varphi \times \vec{\nabla}\psi_t. \quad (1)$$

The toroidal magnetic flux enclosed by a surface is ψ_t, the poloidal angle is θ, the toroidal angle is φ, and the rotational transform is $\iota(\psi_t)$, which is the average number of poloidal transits of a field line per toroidal transit. If such a magnetic field is subjected to a perturbation $\delta\vec{B}$ such that $b \equiv \delta\vec{B}\cdot\vec{\nabla}\psi_t/\vec{B}\cdot\vec{\nabla}\varphi$ has a resonant Fourier coefficient at a rational surface $\iota(\psi_t = \psi_r) = N/M$, then the topology of the magnetic field is changed by the opening of what is known as a magnetic island. A resonant Fourier coefficient means a term such as $b_{MN}\sin(M\theta - N\varphi)$. Changes in the topology of magnetic field lines require an electric field parallel to the magnetic field lines. Such electric fields are very small in a low collisionality plasma and are by definition zero in an ideal magnetic evolution.

In the simplest ideal model, the magnetic field is force free, which means $\vec{j}\times\vec{B} = 0$, so Ampere's law takes the form $\vec{\nabla}\times\vec{B} = \mu_0(j_{||}/B)\vec{B}$. The divergence gives $\vec{B}\cdot\vec{\nabla}(j_{||}/B) = 0$, which implies $j_{||}/B = \kappa_0(\psi_t) + \sum \kappa_{MN}\delta(\iota - M/N)\cos(M\theta - N\varphi)$ plus a similar sinusoidal term, where $\delta(\cdots)$ is the Dirac delta function. An appropriate choice of the κ_{MN}'s can ensure the resonant Fourier components of $b \equiv \delta\vec{B}\cdot\vec{\nabla}\psi_t/\vec{B}\cdot\vec{\nabla}\varphi$ are zero on the rational surfaces, so it is natural to assume such delta-function currents arise in an ideal equilibrium. However, it will be shown that delta-function currents at the resonant rational surfaces cannot by themselves prevent the breaking of the magnetic topology on a spatial scale δ_r, Equation (6), which is proportional to the perturbation amplitude.

In 1942 Hannes Alvén introduced the concept of an ideal magnetic evolution and the term magnetohydrodynamics (MHD) [1], and in 1955 he pointed out that currents in MHD systems can be so spatially localized that they should be called line currents [2]. In 1967 Harold Grad noted that a singular current density naturally arises on rational surfaces [3]. However, the classic paper on the existence of delta-function currents in force-free equilibria is the 1973 paper of Rosenbluth, Dagazian, and Rutherford [4]. That paper developed a technique for studying perturbations at rational surfaces but applied the technique approximately. Waelbroeck [5] extended their analysis by including the effect of resistivity to explain the rapid growth of the $m = 1, n = 1$ instability.

In this paper, the Rosenbluth, Dagazian, and Rutherford technique [4] is applied without approximations made in their paper. The solution for the current density is found to have a smooth as well as a delta function distribution with the two distributions having a comparable contribution to the island shielding.

To obtain the non-singular part of the current density, the solution to the ideal evolution equations must include the effects of non-linearity. In contrast, for example, the solution for a perturbed magnetic field found by Hahm and Kulsrud [6] evolves toward a pure delta-function current profile at the rational surface when the resistivity is zero. They used an equation that is equivalent to Equation (4) but with the $x\xi + \xi^2/2$ replaced by $x\xi$, where x is the distance from the rational surface and ξ is the displacement of the magnetic surfaces by the perturbation. The actual width of the current distribution is roughly the $|\xi|$ of the linearized theory. If $x\xi + \xi^2/2$ replaced by $x\xi$, the plasma displacement ξ has a non-physical discontinuity at $x = 0$. The full non-linear equations give a continuous displacement.

The width of the current distribution near rational surfaces is needed to determine the required resolution of codes that study magnetic island shielding and opening. The approximate half-width of the current channel is δ_r, Equation (6). In many cases of interest to the magnetic fusion program, this width can be comparable to $\rho_s \equiv C_s/\omega_c$, where the speed of sound $C_s \equiv \sqrt{(T_e + T_i)/m_i}$ depends on the electron and ion temperatures and the ion mass. The ion cyclotron frequency is $\omega_c = qB/m_i$, where q is the ion charge. Plasmas provide little shielding for islands that are narrower than ρ_s, so δ_r and ρ_s are competing scales for setting the width of the current channel [7]. Reference [8] is a recent discussion of the importance of ρ_s to fast reconnection.

Slab model—Equation (1) can be generalized to obtain an island at the rational surface $\iota(\psi_r) = N/M$ by writing $2\pi\vec{B} = \vec{\nabla}\psi_t \times \vec{\nabla}(\theta - N\varphi/M) + \vec{\nabla}\times(A\vec{\nabla}\varphi)$, where $A = A_0(\psi_t) + \delta A(\psi_t, \theta - N\varphi/M)$ and $A_0 =$

$-(d\iota/d\psi_t)_r(\psi_t-\psi_r)^2/2$. Then, $\vec{B}\cdot\vec{\nabla}A=0$ and $\iota(\psi_t)=N/M+(d\iota/d\psi_t)_r(\psi_t-\psi_r)$.

When the inverse aspect ratio of the resonant surface, $\epsilon=a/R_0$ is small, this representation of a magnetic field with an island can be simplified to a slab model,

$$\vec{B}=B_0\hat{Z}-\hat{Z}\times\vec{\nabla}A(X,Y), \tag{2}$$

where B_0 is a constant and $Z=R_0\varphi$. The coordinate X is the distance from the rational surface, $d\psi_t/dX=2\pi B_0 a$, and the coordinate Y is defined by $kY=M\theta-N\varphi$, where $k=M/a$ is a wavenumber. Since $\vec{B}\cdot\vec{\nabla}A=0$, the magnetic field lines lie in surfaces of constant $A(X,Y)=A_0(X)+\delta A(X,Y)$, where $A_0=-\epsilon\iota' B_0X^2/2$ and $\iota'\equiv d\iota/dX$.

The slab equilibrium is assumed to be force free, so the current density $\vec{j}$ is parallel to the magnetic field lines. In the vicinity of the rational surface at $X=0$, the constraint $\vec{B}\cdot\vec{\nabla}j_{||}/B=0$ is equivalent to $\vec{j}=j(A)\hat{Z}$. Ampere's law then implies $\nabla^2A=-\mu_0 j(A)$. In the unperturbed equilibrium, $\nabla^2A_0=-\mu_0J_0$ where $J_0\equiv\epsilon\iota' B_0/\mu_0$ is a constant background equilibrium current density. That is

$$A_0=-\frac{\mu_0J_0}{2}X^2, \quad J_0=const. \tag{3}$$

However, the physical interpretation of J_0 in general differs from that in a slab. For example consider a cylindrical equilibrium in which $|B_\theta/B_Z|<<1$, then J_0 is the spatially averaged current density in the region enclosed by the rational surface. The current itself can be zero in the vicinity of a rational surface.

The displacement ξ of the magnetic surfaces in the slab model is defined by letting $A(X-\xi)=A_0(X)$. The spatial coordinate $x\equiv X-\xi$ is constant along the perturbed magnetic surfaces. The relation between the perturbed vector potential and the displacement is

$$A=-\frac{\mu_0J_0}{2}x^2, \text{ and } \delta A=\mu_0J_0\left(x\xi+\frac{\xi^2}{2}\right). \tag{4}$$

Residual islands—A zero of the resonant magnetic perturbation b_{MN} at the rational surface $\iota=N/M$ does not eliminate topology breaking in the vicinity of the rational surface when $db_{MN}/d\psi_t$ is non-zero. This property of Hamiltonian systems was demonstrated in his thesis by Ilon Joseph [9] and its importance to toroidal plasmas was discussed in Section III of Reference [7]. This section reproduces the results of References [9], and [7] that are required to understand this paper and gives the form of the plasma displacement in the slab model near a rational surface on which $b_{MN}=0$ but $db_{MN}/d\psi_t\neq0$. An appreciation of this form is required to understand the solution for the displacement.

Before showing the inadequacy of a delta-function for preserving magnetic topology in the vicinity of a rational surface, the slab model will be used to calculate the width of magnetic islands when a curl-free magnetic perturbation $\delta A=A_\infty\exp(kX)\cos(kY)$ is applied at $X=\infty$. The X component of the magnetic perturbation is $\delta B_X=-B_\infty\exp(kX)\sin(kY)$, where $B_\infty\equiv kA_\infty$. When $|kX|<<1$, the identity $\cos\theta=1-2\sin^2(\theta/2)$ implies $2A/\mu_0J_0=-X^2-\delta_I^2\{\sin^2(kY/2)-1/2\}$, where $\delta_I^2\equiv4A_\infty/\mu_0J_0$. Since A is constant along the field lines, the magnetic field line trajectories are $X(Y)=\pm\sqrt{X_0^2-\delta_I^2\sin^2(kY/2)}$, where X_0 is a constant. For $X_0>\delta_I$, each magnetic field line extends over the full range of Y. However for $X_0<\delta_I$, each magnetic field line extends only over a limited range of Y, which is a different topology from that of the magnetic field lines of the unperturbed system in which $\delta_I=0$. The quantity δ_I is the half-width of the magnetic islands.

Now consider the current-free perturbation

$$\delta A=\delta A_\infty(e^{kX}-e^{-kX})\cos(kY). \tag{5}$$

This perturbation can represent (1) the effect of perturbations applied at $X=\pm\infty$ or (2) the effect for $X>0$ of a perturbation applied at $X=\infty$ but with shielding at the rational surface. The first case is logically simpler, so it will be assumed. Since $\delta A=0$ on the rational surface, $X=0$, the rational surface is neither split to form an island nor does its position vary in X as a function of Y. However, this perturbation breaks the topology of the magnetic field lines by limiting the range in Y of some of the magnetic field lines. Let

$$\delta_r\equiv\frac{2k}{\mu_0J_0}\delta A_\infty=\frac{1}{2}k\delta_I^2, \tag{6}$$

where δ_I is the half width of the island that would be produced by the perturbation in the absence of shielding. When $|kX|<<1$, the vector potential can be written as $2A/\mu_0J_0=-X^2+2\delta_rX\cos kY$. This expression for A has maxima at $X=\pm\delta_r$, when $\cos kY=\pm1$, and saddle points at $X=0$, when $\cos kY=0$. The implication is that a break in the topology has occurred, Figure (1), in a region of width $2\delta_r$ on either side of the rational surface, which is itself preserved. A delta-function current at $X=0$ cannot shield out the residual islands. Instead, a current layer that has a half-width of approximately $2\delta_r$ is required to eliminate these islands. For small perturbations the residual islands are very narrow compared to the magnetic islands discussed in the previous paragraph, $\delta_r=k\delta_I^2/2$. In the regions on either side of the rational surface in which $1/k>>|X|>>\delta_r$, the displacement is a function of Y alone $\xi=\delta_r\cos(kY)$ and is the same on the two sides of the rational surface, Figure (1).

The scale δ_r is also the distance from a rational surface at which the linearized equation for the ideal evolution of a magnetic field, $\delta\vec{B}=\vec{\nabla}\times(\vec{\xi}\times\vec{B})$ must fail. The vector $\vec{\xi}$ is the displacement of a field line. The X component of the ideal evolution equation is $\delta B_X=\vec{B}\cdot\vec{\nabla}\xi$, where $\xi(X,Y)\equiv\xi_X$. Since $\vec{B}\cdot\vec{\nabla}\xi=\mu_0J_0X\partial\xi/\partial Y$ and $\delta B_X=-\delta B_\infty(e^{kX}-e^{-kX})\sin(kY)$, the displacement is $\xi=\delta_r\cos(kY)$. This equation has two important impli-

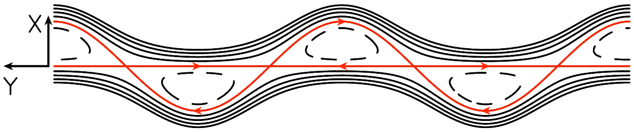

FIG. 1: Although the curl-free perturbation $\delta A = A_\infty(e^{kX} - e^{-kX})\cos(kY)$ drives no primary islands at the location of the unperturbed rational surface, it does drive residual islands on either side of the rational surface. The figure gives computed trajectories of the perturbed magnetic field lines near the $X = 0$, $A = 0$ magnetic surface.

cations for the region near a rational surface: (1) the linear ideal-evolution equations cannot be valid closer than a distance δ_r from the rational surface for otherwise the plasma could cross itself at the rational surface and (2) the plasma displacement is a constant for $|X| >> \delta_r$ but $k|X| << 1$.

Solution near a rational surface—The technique developed by Rosenbluth, Dagazian, and Rutherford [4] and used by Waelbroeck [5] can be extended to resolve two paradoxes associated with the ideal evolution equations near a rational surface and to obtain a solution for the current density in a force-free equilibrium. These paradoxes are (1) the unphysical discontinuity in the plasma displacement ξ across the resonant surface in linearized theory and (2) the breaking of the magnetic surfaces by residual islands if the shielding current is restricted to a delta function.

Since $k\delta_r << 1$, the X variation is far more rapid than the Y variation in the current layer of width $\sim \delta_r$ about the rational surface, and $\nabla^2 A$ can be approximated as d^2A/dX^2 with Y held constant. If the equation $d^2A/dX^2 = -\mu_0 j(A)$ is multiplied by dA/dX, one finds $(dA/dX)_Y^2 = (\mu_0 J_0)^2 \{f(A) + g(Y)\}$, where $g(Y)$ is an integration "constant," which is set by the form of the magnetic perturbation imposed on the system, and

$$f(A) \equiv -2\frac{\int j(A)dA}{\mu_0 J_0^2}. \tag{7}$$

As $|X| \to\infty$, $f(A) \to -2A/(\mu_0 J_0) \to X^2$ and $dA/dX \to -J_0 X$. These relations imply that the appropriate sign choice for $(dA/dX)_Y$ on the two sides of the resonant surface, which means $x = X + \xi$ positive or negative, is

$$\begin{aligned}\left(\frac{dA}{dX}\right)_Y &= -\mu_0 J_0\sqrt{f(A)+g(Y)} \qquad x>0;\\ &= +\mu_0 J_0\sqrt{f(A)+g(Y)} \qquad x<0. \end{aligned}\tag{8}$$

If (x, Y) coordinates are used instead of (X, Y), where $X = x + \xi$ and $A = -\mu_0 J_0 x^2/2$, then $(dA/dX)_Y = (dA/dx)/(dX/dx) = (-\mu_0 J_0 x)/(1 + d\xi/dx)$. This equation and Equation (8) give expressions for the displacement:

$$\begin{aligned}\frac{d\xi}{dx} &= +\frac{x}{\sqrt{f(x)+g(Y)}} - 1 \text{ for } x>0;\\ &= -\frac{x}{\sqrt{f(x)+g(Y)}} - 1 \text{ for } x<0. \end{aligned}\tag{9}$$

Although the function $f(x)$ could in principle depend on whether x is positive or negative, the form of the solution that will be found implies that it does not.

As discussed above, the displacement is a function of Y alone in the regions on either side of the rational surface in which $1/k >> |x| >> \delta_r$. In these regions the displacement is written as $\xi_+(Y)$ or $\xi_-(Y)$ depending on whether x is positive or negative. Equation (9) implies

$$\xi_+(Y) = \xi_0 + \xi_\infty \quad \text{and} \quad \xi_-(Y) = \xi_0 - \xi_\infty. \tag{10}$$

The displacement of the rational surface is $\xi_0(Y)$. The displacement $\xi_\infty(Y) = \bar{\xi}_\infty(Y)\delta_0$, where $\delta_0 \sim \delta_r$ is a length scale,

$$x = \bar{x}\delta_0, \quad f = \bar{f}\delta_0^2, \quad g = \bar{g}\delta_0^2, \text{ and} \tag{11}$$

$$\bar{\xi}_\infty(Y) \equiv \int_0^\infty \left(\frac{\bar{x}}{\sqrt{\bar{f}(\bar{x})+\bar{g}(Y)}} - 1\right) d\bar{x}. \tag{12}$$

Boundary conditions determine the scale length δ_0 and the function $g(Y)$ and give a unique solution for the displacement. The boundary conditions are the displacements $\xi_+(Y)$ and $\xi_-(Y)$. Equation (10) implies that the scale length δ_0 is given by $\bar{\xi}_\infty(Y)\delta_0 = \{\xi_+(Y) - \xi_-(Y)\}/2$ and that the sum of the two displacements gives the displacement of the rational surface, $\xi_0(Y) = \{\xi_+(Y) + \xi_-(Y)\}/2$. The Y dependence of $\bar{g}(Y)$ must be varied until the Y dependence of $\xi_+(Y) - \xi_-(Y)$ is obtained.

The magnetic flux enclosed by a constant-A surface cannot be changed by an ideal perturbation. Near the rational surface at $x = 0$, this is equivalent to $\oint \xi(x, Y)dY = 0$, which requires

$$S(\bar{f}) \equiv \frac{1}{\left\langle \frac{1}{\sqrt{\bar{f}(\bar{x})+\bar{g}(Y)}}\right\rangle} = \pm\bar{x}, \tag{13}$$

where $\langle\cdots\rangle$ is an average over a period in Y. Equation (13) determines $\bar{f}(\bar{x})$ once the function $\bar{g}(Y)$ is given. Since the argument of the square root must be positive, $\bar{f}(\bar{x}) \geq -\bar{g}_{min}$. As $\bar{x} \to 0$, which means near the rational surface, $\left\langle 1/\sqrt{\bar{f}(\bar{x})+\bar{g}(Y)}\right\rangle$ must go to infinity, so $\bar{f}(\bar{x} \to 0) = \bar{g}_{min}$. Equation (13) also implies $\bar{f}(\bar{x} \to \infty) = \bar{x}^2$, which ensures the convergence of the integral for $\bar{\xi}_\infty$, Eq. (12), as $\bar{x} \to \infty$.

The shielding current—The shielding current in the layer around the rational surface, $I'_\infty(Y)$ is related to the jump in the displacement across the region by

$$I'_\infty(Y) \equiv \int_{-\infty}^{\infty} (j - J_0)dX = -[\xi]J_0 = -2\delta_0\bar{\xi}_\infty J_0, \tag{14}$$

which coupled with the condition that $\oint \xi dY = 0$ implies $\oint I'_\infty(Y)dY = 0$. This expression for $I'_\infty(Y)$ follows from $\mu_0 j = -d^2A/dX^2$, so $\mu_0 \int j dX = -[dA/dX]$. Now $A = -\mu_0 J_0 (X-\xi)^2/2$, consequently $(dA/dX)/(\mu_0 J_0) = -X+\xi+(X-\xi)d\xi/dX$, but $d\xi/dX$ goes to zero far from the rational surface.

At the rational surface dA/dX has a jump, Equation (8), which implies a surface current

$$I'_\delta = 2J_0\sqrt{f(0)+g(Y)}. \quad (15)$$

The current between the rational surface and a constant-x surface can be found using Equation (7) for f, and $A(x) = -\mu_0 J_0 x^2/2$, which together give the current density

$$j(x) = \frac{J_0}{2}\frac{1}{x}\frac{df}{dx}. \quad (16)$$

Since $dX/dx = x/\sqrt{f+g}$ for $x>0$,

$$\int_0^x j(x)\frac{dX}{dx}dx = J_0\left(\sqrt{f(x)+g} - \sqrt{f(0)+g}\right) \quad (17)$$

using $dx = df/(df/dx)$. Since $dX/dx = -x/\sqrt{f+g}$ for $x<0$,

$$\int_{-|x|}^0 j(x)\frac{dX}{dx}dx = J_0\left(\sqrt{f(x)+g} - \sqrt{f(0)+g}\right). \quad (18)$$

The shielding current that lies between the surfaces at $x = \pm|x|$ is

$$I'(x,Y) \equiv \int_{-|x|}^x (j-J_0)\frac{dX}{dx}dx + I'_\delta. \quad (19)$$

Equations (15), (17), and (18) imply $I'(x,Y) = 2J_0\sqrt{f(x)+g(Y)} - J_0[X]^x_{-|x|}$. Using $X = x+\xi$, the shielding current is

$$I'(x,Y) = 2J_0\left(\sqrt{f(x)+g(Y)} - |x|\right) - J_0[\xi]^x_{-|x|}, \quad (20)$$

which in the limit as $|x| \to\infty$ becomes Equation (14).

Requirement for a non-analytic $g(Y)$—In order to have an analytic displacement $\bar{\xi}_\infty(Y)$ and shielding current $I'_\infty(Y)$, the function $g(Y)$ must be non-analytic as $Y \to 0$. This is illustrated in Figure (2) for various choices of $g(Y)$. If $\xi_\infty(Y)$ is an even function, $g(Y)$ must also be even, so $g(Y) = sin^{2p}(kY)$ represents the general analytic function as $Y \to 0$. For this $g(Y)$ and for $p = 1, 2$ and 3, the Fourier series for $\xi_\infty(Y)$ is seen to converge approximately as $1/m^{(p-1)}$, which means derivatives of $\xi_\infty(Y)$ greater than the $p-2$ derivative have divergent Fourier series. The implication is that if $\xi_\infty(Y)$ is to be analytic, then $g(Y \to 0)$ must be non-analytic. The non-analytic function

$$\bar{g}(Y) = e^{c\left(1-\frac{1}{|\sin(kY/2)|}\right)}, \text{ where } c = 5.5, \quad (21)$$

gives an exponentially convergent Fourier series for $\bar{\xi}_\infty(Y)$, Figure (2), with Fourier coefficients that are bounded by $e^{-m/2.5}$.

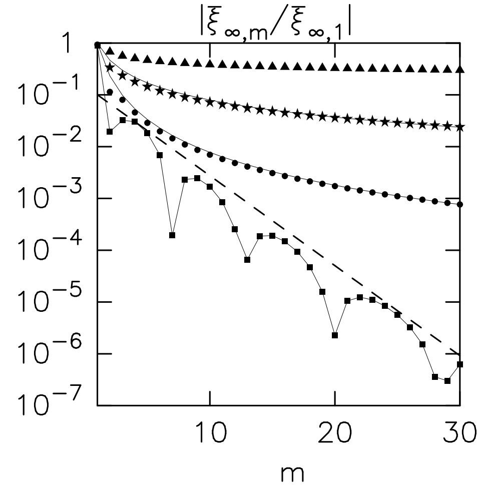

FIG. 2: The convergence of the Fourier series for $\xi_\infty(Y)$ is illustrated for various $g(Y)$. When $g(Y) = sin^{2p}(kY)$ with $p = 1$ (▲), $p = 2$ (★) and $p = 3$ (●), the Fourier series for $\xi_\infty(Y)$ are well represented by the solid curves, which are proportional to $1/m^{(p-1)}$. The Fourier series (■) for $\xi_\infty(Y)$ given by the non-analytic $g(Y)$ of Equation (21) is bounded from above by the dashed line, $0.15e^{-m/2.5}$. The Fourier series of this $\xi_\infty(Y)$ is exponentially convergent as required for an analytic function.

The non-analytic $g(Y)$ of Equation (21) lies in the range $0 \le g(Y) \le 1$ for any value of c, but $c = 5.5$ gives an almost cosinusoidal $\bar{\xi}_\infty(Y)$, Figure (2) and (3a). The Fourier expansions of $\bar{\xi}_\infty$ and $\sqrt{\bar{g}(Y)}$ are $\bar{\xi}_\infty = 0.905\cos(kY) + 0.019\cos(2kY) + \cdots$ and $\sqrt{\bar{g}(Y)} = 0.408 - 0.527\cos(kY) + 0.092\cos(2kY) + \cdots$, Figure (3a). The current density $j(x)$, Figure (3b), in the layer of width $\sim 2\delta_0$ gives a current $I'_\infty = -2\delta_0 J_0\{0.905\cos(kY) + 0.019\cos(2kY) + \cdots\}$ while the delta-function current is $I'_\delta = 2J_0\delta_0\{0.4 - 0.527\cos(kY) + 0.092\cos(2kY) + \cdots\}$. The non-singular part of the current in the layer is the difference between the two, $I'_{ns} = -2J_0\delta_0\{0.408 + 0.378\cos(kY) + 0.111\cos(2kY) + \cdots\}$, so 42% of the shielding current is in the smooth distribution and 58% is in the delta function.

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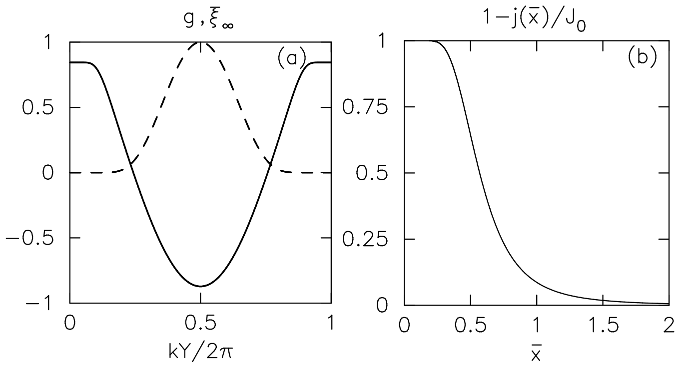

FIG. 3: The normalized displacement $\bar{\xi}_\infty(Y)$, (—), Equation (12), is shown in (a) together with the function $g(Y)$, (- - -), of Equation (21) that produces it. The deviation in the current density on each magnetic surface $j(x)$ from its unperturbed value J_0 if shown in (b).

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