Princeton Plasma Physics Laboratory

PPPL- 4558

PPPL- 4558

Advances and Current Challenges in the Theory of Zonal-Flow Generation

John A. Krommes

September, 2010

PPPL
PRINCETON PLASMA
PHYSICS LABORATORY

Prepared for the U.S. Department of Energy under Contract DE-AC02-09CH11466.

Advances and Current Challenges in the Theory of Zonal-Flow Generation

John A. Krommes

Plasma Physics Laboratory, Princeton University,
P.O. Box 451, M.S. 28, Princeton, NJ 08543–0451 USA

Abstract. Some remarks are made about the use of modern statistical formalism in the calculation of the zonal-flow growth rate and the backreaction of zonal flows on drift waves.

Keywords: zonal flows, drift waves, gyrokinetics, convective cells, Casimir invariants, random refraction
PACS: 52.35.Ra, 52.30.Gz, 52.35.Mw

INTRODUCTION

It is a great pleasure and significant honor to participate in this much-deserved celebration of the research career of Prof. Predhiman Kaw. Through my own career, Predhiman has been an important role model, and it is truly impressive how his research on diverse topics has influenced in a very substantial way the fields of both basic and applied plasma and fusion research.

When I first arrived at the Princeton Plasma Physics Laboratory (PPPL) in 1971 to begin my graduate training, Predhiman was already something of a mythic figure. He had first come to PPPL in 1967 (see Fig. 1) as a brilliant post-doc with already several tens of published papers under his belt. He left in 1971 (now as a Lecturer with Rank of Assistant Professor) to become an Associate Professor (soon to be promoted to Professor) at Ahmedabad's Physical Research Laboratory, but returned to Princeton as a Principal Research Physicist (the top rank) and Lecturer with Rank of Professor in 1975, just in time to participate in my Final Public Oral Exam (the last hurdle in defending my dissertation before receiving my PhD). My research focused on the theory of convective cells in 2D, strongly magnetized, thermal equilibrium plasmas, and it required making a fundamental distinction between classical and "anomalous" transport coefficients. (One cannot say "turbulent" coefficients here because the convective-cell transport was being studied in the special case of thermal equilibrium.) At one point I was vague about which kind of coefficient I was referring to, and I can still clearly remember Predhiman asking, "Now is that the classical coefficient or the anomalous cefficient?" I guessed that it was the anomalous one, and that must have sounded correct since Predhiman and his fellow examiners ultimately agreed that I had passed the exam.

I have continued to work on aspects of anomalous transport throughout my career, and have always been on the lookout for contributions from Predhiman and his colleagues on that and related topics. In the last decade an important area of interest for the field at large has been the implications of zonal flows for the theory and practical consequences

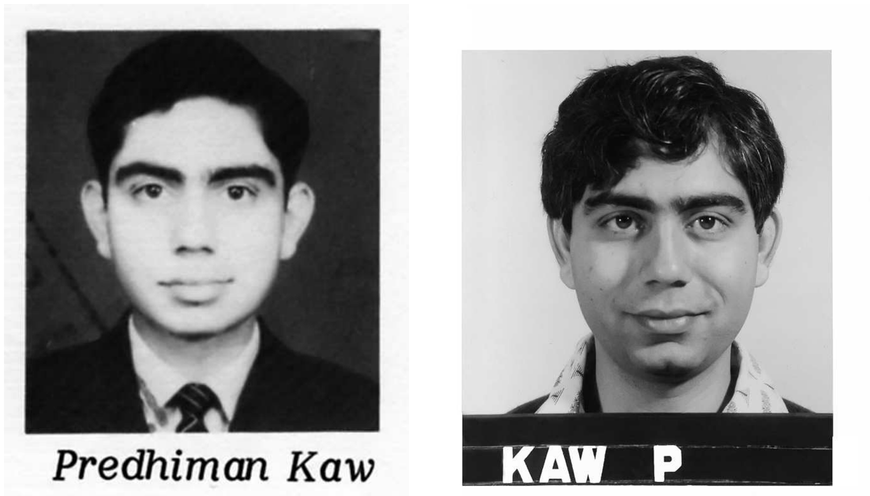

FIGURE 1. Predhiman Kaw at the Princeton Plasma Physics Laboratory. Left: circa 1967 (around the time he first came to PPPL. Right: in 1975 (when he served as a member of the Examining Committee for the Final Public Oral PhD Exam of J. A. Krommes).

of microturbulence in fusion plasmas. Not surprisingly, Predhiman and his coworkers have been right there with incisive contributions that nicely blend the complementary approaches of physical intuition, numerical simulation, and systematic analysis. Here are just a few examples drawn from a much larger list:

- Kaw, Singh, and Diamond, "Coherent nonlinear structures of drift wave turbulence modulated by zonal flows"[1];
- Bisai, Das, Deshpande, Jha, Kaw, Sen, and Singh, "Formation of a density blob and its dynamics in the edge and the scrape-off layer of a tokamak plasma"[2];
- Singh, Tangri, Kaw, and Guzdar, "Coupled drift-wave–zonal flow model of turbulent transport in the tokamak edge"[3].

In the remainder of this paper I am going to discuss some fundamental aspects of the drift-wave–zonal-flow paradigm. Most of the work I will describe is not new, but there remain confusions in the literature and in general it is important to think about these problems with an adequate and sufficiently broad perspective. Perhaps these remarks will help. In particular, I consider the question, "Does the 'new' drift-wave–zonal-flow paradigm totally invalidate all of the earlier voluminous literature on drift-wave turbulence?" The answer is no if one formulates the turbulence problem in a sufficiently general way. By using ideas of statistical turbulence theory and renormalization, one can demonstrate a nice continuity to past research, resolve some paradoxes, and also obtain a methodology for situations where intuition fails. Techniques from Hamiltonian field theory are also useful. I will comment (very briefly) on these interrelated threads, beginning in the next section with some discussion of the underlying formalism of nonlinear gyrokinetics.

BACKGROUND: NONLINEAR GYROKINETICS

If one is to discuss the interactions of drift waves and zonal flows, one needs to know what they are and what equations they obey. Since we will be concerned only with low-frequency fluctuations with $\omega \ll \omega_{ci}$, the nonlinear gyrokinetic formalism is a good place to start. The technical details have been thoroughly reviewed by Brizard and Hahm [4] and an introductory tutorial is by Krommes [5]; here I will need only the basics. In a constant magnetic field, the gyrokinetic equation is

$$\frac{\partial F(\boldsymbol{X}, v_\parallel, \mu, t)}{\partial t} + v_\parallel \nabla_\parallel F + \boldsymbol{V}_E \cdot \boldsymbol{\nabla} F + \frac{q}{m} E_\parallel \frac{\partial F}{\partial v_\parallel} = 0. \tag{1}$$

I consider only the electrostatic approximation, so one requires the gyrokinetic Poisson equation (written here in the quasineutral approximation)

$$-\rho_s^2 \widehat{\nabla}_\perp^2 \delta\Phi = \frac{\delta n_i^G}{\overline{n}_i} - \frac{\delta n_e^G}{\overline{n}_e}. \tag{2}$$

Here $\Phi \doteq e\varphi/T_e$, the superscript G denotes a gyrocenter quantity, and $\widehat{\nabla}_\perp$ is an operator (dependent on T_i) that reduces at $T_i = 0$ to the ordinary perpendicular Laplacian. The right-hand side of (2) is the gyrocenter charge, which the left-hand side is the negative of the ion polarization charge.

The simplest paradigm for the interaction of drift waves and zonal flows emerges by considering the limit $T_i = 0$. That removes finite-Larmor-radius terms from (1) and replaces $\widehat{\nabla}_\perp^2$ by $\nabla_\perp^2$. For modes with $k_\parallel \neq 0$, one may assume adiabatic electron response $\delta n_e/\overline{n}_e = \delta\Phi$ provided that one ignores wave–particle interactions. Such response is prohibited when $k_\parallel = 0$ because the electrons are in the fluid regime. Since the electron polarization drift is very small, one may simply set the electron response to zero for $k_\parallel = 0$ modes: $\delta n_e/\overline{n}_e = \widehat{\alpha}\delta\Phi$, where $\widehat{\alpha} = 0$ for $k_\parallel = 0$ and $\widehat{\alpha} = 1$ for $k_\parallel \neq 0$. The density moment of the ion GKE for $T_i = 0$ leads to the continuity equation for ion gyrocenter density:

$$\partial_t n_i^G + \boldsymbol{\nabla} \cdot (\boldsymbol{u_E} n_i^G) + \nabla_\parallel (u_\parallel n_i^G) = 0. \tag{3}$$

For simplicity, I ignore the last term (related to ion sound waves). Write $n_i^G = \langle n_i \rangle + \delta n_i$ and assume a constant background density gradient with $L_n^{-1} \doteq -\partial_x \ln\langle n_i \rangle$. Finally, upon replacing δn_i^G by the remaining terms in the GK Poisson equation, one obtains the *generalized Hasegawa–Mima equation*:

$$(\widehat{\alpha} - \rho_s^2 \nabla_\perp^2) \frac{\partial \delta n_i}{\partial t} + \widehat{\alpha} V_* \frac{\partial \delta\Phi}{\partial y} + \boldsymbol{u_E} \cdot \boldsymbol{\nabla}[(\widehat{\alpha} - \rho_s^2 \nabla_\perp^2)\delta\Phi] = 0. \tag{4}$$

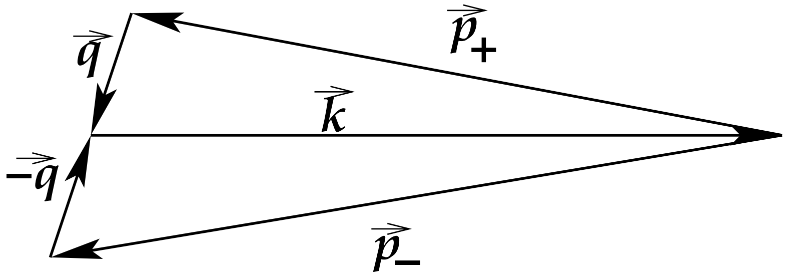

FIGURE 2. Drift-wave ($\boldsymbol{k}$) sidebands ($\boldsymbol{p}_+$ and $\boldsymbol{p}_-$) couple to drive convective cells ($\boldsymbol{q}$).

This is equivalent to the following two equations for the DW and the CC components:

$$(1-\rho_s^2\nabla_\perp^2)\frac{\partial \delta n_i^{\mathrm{DW}}}{\partial t} + V_* \frac{\partial \delta\Phi^{\mathrm{DW}}}{\partial y} + \boldsymbol{u}_{\boldsymbol{E}}^{\mathrm{DW}} \cdot \boldsymbol{\nabla} ??? + \boldsymbol{u}_{\boldsymbol{E}}^{\mathrm{CC}} \cdot \boldsymbol{\nabla}[(1-rs^2\nabla_\perp^2)\delta\Phi^{\mathrm{DW}}] = 0, \tag{5a}$$

$$\partial_t \varpi^{\mathrm{CC}} + \overline{\boldsymbol{u}_{\boldsymbol{E}}^{\mathrm{DW}} \cdot \boldsymbol{\nabla}\varpi^{\mathrm{DW}}} + \cdots = 0. \tag{5b}$$

That is, the convective cells are driven by DW–DW interactions, and the convective cells advect (modulate) the DWs. This basic picture was emphasized by Diamond et al. [6].

DISPARATE-SCALE EXPANSION AND THE CONVECTIVE-CELL GROWTH RATE

Two basic methodologies have been used to discuss CC and ZF generation: modulational instability, and statistical turbulence theory. Both begin with the basic wave-number triad interactions depicted in Figure 2, Here the triangle relation $\boldsymbol{k}+\boldsymbol{p}+\boldsymbol{q}=\boldsymbol{0}$ is a consequence of the quadratic nonlinearity of the gyrokinetic equation. For modulational instability, the amplitudes of the fundamental drift wave ($\boldsymbol{k}$) and its sidebands ($\boldsymbol{p}_+$ and $\boldsymbol{p}_-$) are held fixed while the evolution of mode $\boldsymbol{q}$ is calculated. This approximation holds only for short times and does not conserve energy. Alternatively, one can postulate a steady state of turbulence and consider the predictions of standard Markovian statistical closure, including the self-consistent back-reaction of the CCs on the DWs. That theory does conserve total energy, so it is the sensible one to use in discussing a turbulent soup of DWs and ZFs.

In general, Markovian statistical closure of a scalar amplitude equation leads (under the assumption of homogeneous statistics) to a spectral evolution equation of the form

$$\partial_t C_{\boldsymbol{k}} + 2\,\mathrm{Re}\,\eta_{\boldsymbol{k}} C_{\boldsymbol{k}} = 2F_{\boldsymbol{k}}. \tag{6}$$

Here $\eta_{\boldsymbol{k}} \doteq i\Omega_{\boldsymbol{k}} - \gamma_{\boldsymbol{k}} + \eta_{\boldsymbol{k}}^{\mathrm{nl}}$, $\Omega_{\boldsymbol{k}}$ is the real frequency, $\gamma_{\boldsymbol{k}}$ is the linear growth rate, $\eta_{\boldsymbol{k}}^{\mathrm{nl}}$ is a complex nonlinear damping rate, and $F_{\boldsymbol{k}}$ is the variance of internal nonlinear noise. This equation can be written for arbitrary wavevector, including the $\boldsymbol{q}$ of the convective cells. By definition, the CC growth rate is $\gamma_{\boldsymbol{q}} \doteq -\mathrm{Re}\,\eta_{\boldsymbol{q}}$. More specifically, the CC growth rate due to interactions with the DWs is $\gamma_{\boldsymbol{q}} = -\operatorname{Re}\eta^{\mathrm{nl}}_{\boldsymbol{q}}$, where only the triad interactions that couple $\boldsymbol{q}$ to drift waves are included in the calculation of $\eta^{\mathrm{nl}}_{\boldsymbol{q}}$. In general, that calculation is nontrivial for arbitrary $\boldsymbol{k}$ and $\boldsymbol{q}$.

Analytical progress can be made by introducing the assumption of *disparate scales*, i.e., one takes $\varepsilon \doteq q/k \ll 1$ and expands $\eta^{\mathrm{nl}}_{\boldsymbol{q}}$ in ε. That expansion was performed by Krommes and Kim [7] for the generalized Hasegawa–Mima equation. The calculation is quite tedious because $\eta^{\mathrm{nl}}_{\boldsymbol{q}}$ involves a sum over all $\boldsymbol{k}$'s and $\boldsymbol{p}$'s that sum to a fixed $\boldsymbol{q}$. The fact that $\boldsymbol{k}$ and $\boldsymbol{p}$ do not rotate around the same center means that unusually shaped integration domains are involved [see Fig. 6 of Krommes and Kim [7]], so the calculation becomes quite tedious. Nevertheless, in the end one is led to a definite result:

$$\gamma^{\mathrm{nl}}_{\boldsymbol{q}} = -2\frac{q^4}{\overline{q}^2}\sum_{\boldsymbol{k}}\frac{k_y^2 k_x}{\overline{k}^4}\theta_{\boldsymbol{q},\boldsymbol{k},-\boldsymbol{k}}\,\widehat{q}\cdot\frac{\partial \mathscr{N}_{\boldsymbol{k}}}{\partial \boldsymbol{k}}, \tag{7}$$

where the x direction is parallel to $\widehat{q}$, $\theta_{\boldsymbol{k},\boldsymbol{p},\boldsymbol{q}}$ is an appropriate triad interaction time (more about that later),

$$\mathscr{N}_{\boldsymbol{k}} \doteq \frac{1}{2}\overline{k}^4\langle|\delta\varphi_{\boldsymbol{k}}|^2\rangle, \tag{8}$$

and $\overline{k}^2 \doteq \widehat{\boldsymbol{a}}_{\boldsymbol{k}} + k^2$. ($\widehat{\boldsymbol{a}}_{\boldsymbol{k}} = 1$ for the DWs. $\widehat{\boldsymbol{a}}_{\boldsymbol{q}} = 0$ for ZFs and is unity for other long-wavelength fluctuations like streamers.) This result has a clear physical interpretation, as will be discussed in the next section. Moreover, one can also use the expansion to calculate the effects of the CCs on the DWs; a Fokker–Planck equation in $\boldsymbol{k}$ space results, although space precludes describing the details here.

The discussion thus far has been conceptually straightforward: one assumes that the CCs are long wavelength relative to the DWs and calculates the consequences. But there is a crucial and enormously instructive connection between the disparate-scale calculation and the well-known statistical theory of eddy viscosity given by Kraichnan [8]. Working with homogeneous and isotropic turbulence, Kraichnan partitioned $\boldsymbol{k}$ space into resolved or explicit modes ($k \le k_m$ for some constant k_m) and unresolved modes ($k > k_m$). He then defined "an *effective eddy viscosity* acting on modes of wavenumber k due to dynamical interaction with wavenumbers $> k_m \ldots$ by

$$\nu(k \mid k_m) = -T(k \mid k_m)/[2k^2E(k)], \quad k < k_m.\text{"} \tag{9}$$

Here $T(k \mid k_m)$ is the energy transfer into wave number k due to triads such that $k < k_m$ and p and/or $q > k_m$. This definition is totally general, and for arbitrary wave numbers the value of $\nu(k \mid k_m)$ depends on both the $\eta_{\boldsymbol{k}}$ and $F_{\boldsymbol{k}}$ terms. However, for $k \ll k_m$ Kraichnan showed that $\nu(k \mid k_m)$ is determined dominantly by $\eta_{\boldsymbol{k}}$, and he calculated $\nu(k \mid k_m)$ to leading order in ε for both 3D and 2D turbulence. In both cases, ν approaches a constant in that limit; it is negative in 2D.

Brief reflection now shows that the disparate-scale calculation of the CC growth rate is actually effectively *identical* to Kraichnan's leading-order calculation of the eddy-damping rate in 2D, except for a sign change: $\gamma_{\boldsymbol{q}} = -q^2\nu$. Effectively, the DWs (short scale in our approximation) are unresolved modes from the point of view of the CCs. In the end, the only essential technical difference is that Krommes and Kim allowed for an anisotropic DW spectrum whereas Kraichnan invoked the (major) simplification of isotropic unresolved modes. There does exist an important physical difference between the evolution of drift waves and of 2D neutral fluids: in the drift-wave problem, adiabatic electron response is responsible for the factor of 1 in the DW term $\overline{k}^2 \doteq 1+k^2$. Thus the mode-coupling coefficients differ for the two problems. However, if the closure theory is formulated in terms of the quantity $\overline{k}^2 \doteq \widehat{\boldsymbol{a}}+k^2$, it is easy to reduce the general result of the DW–CC expansion to Kraichnan's simply by setting $\widehat{\boldsymbol{a}}=0$ for all modes (and assuming an isotropic spectrum so one can average over angle). Upon replacing the $\boldsymbol{k}$ sum by a 2D integral, integrating over polar angle, and dividing by $-q^2$ to turn growth rate into eddy viscosity, one obtains the 2D Navier–Stokes eddy viscosity

$$\nu_{\mathrm{NS}}(q \mid k_{\min}) = \frac{\pi}{4}\int_{k_{\min}}^{\infty} dk\,\theta_{q,k,k}\frac{dk^2U}{dk}, \tag{10}$$

where $E_{\boldsymbol{k}} \doteq \frac{1}{2}(2\pi/L)^2U(k)$. This agrees completely with Eq. (4.6) of Kraichnan [8].

I will return to Kraichnan's analysis after discussing in the next section the wave-kinetic algorithm that is motivated by the disparate-scale expansion just described.

WAVE KINETICS AND CASIMIR INVARIANTS

Diamond et al. [6] proposed that the ZF generation problem be addressed by the use of wave kinetic methods borrowed and generalized from the well-known problem of ray propagation in weakly inhomogeneous random media. This important insight is essentially correct. However, just what wave kinetic equation to use is unclear. In generalizing from standard linear wave theory, several questions arise: (i) What is the appropriate action density? (ii) What role, if any, does the linear drift-wave frequency play in this intrinsically nonlinear problem? (iii) How does one introduce the triad interaction time (which does not arise in linear theory at all)? Ultimately, such questions can only be definitively answered by *deriving* a wave kinetic algorithm from fundamental principles, i.e., by performing the disparate-scale expansion as described above. For example, the form of (7) strongly suggests that the appropriate action density is the $\mathscr{N}_{\boldsymbol{k}}$ defined in (8). Note that this quantity is *not* the linear drift-wave action density, which is proportional to the derivative of the dielectric function with respect to frequency and therefore involves a k_y; neither is it the energy invariant $\mathscr{E}_{\boldsymbol{k}} \doteq \frac{1}{2}(1+k^2)\langle|\delta\varphi_{\boldsymbol{k}}|^2\rangle$ divided by the linear mode frequency. Why, therefore, does this particular quantity appear? Smolyakov and Diamond [9] proved that $\mathscr{N}_{\boldsymbol{k}}$ is conserved by the generalized Hasegawa–Mima equation under modulation by the CCs; however, their method did not lead to physical understanding. A more general proof was given by Krommes and Kolesnikov [10] in terms of a (functional) Hamiltonian description of $\boldsymbol{E}\times\boldsymbol{B}$ advection. They showed that within the disparate-scale expansion the relevant action is a *Casimir invariant* (a quantity conserved because of the structure of the Poisson bracket that generates the nonlinearity, independent of the form of the Hamiltonian functional), for which a general formula was provided. It is probably the case that further insights will follow by a closer examination of the physical significance of Casimir invariants.

To complete the derivation of the wave kinetic equation, one needs to know how drift-wave packets propagate under convective-cell modulation. As in linear wave theory, a certain frequency plays the role of a Hamiltonian conserved under the motion. That is not the linear wave frequency, however, but rather is the nonlinear advection frequency (of the drift waves by the CCs)

$$\widetilde{\Omega} \doteq \boldsymbol{k} \cdot \widetilde{\boldsymbol{V}}_{E,\boldsymbol{q}}^{\mathrm{CC}}. \tag{11}$$

One then has the random ray equations

$$\frac{d\boldsymbol{X}}{dt} = \frac{\partial \Omega}{\partial \boldsymbol{k}}, \tag{12a}$$

$$\frac{d\boldsymbol{k}}{dt} = -\frac{\partial \Omega}{\partial \boldsymbol{X}}. \tag{12b}$$

In particular, the last equation describes the process of *random refraction* — the same phenomenon responsible for the twinkling of starlight.

The change in wave number under the random long-wavelength modulation changes the spectral energy distribution of the drift waves (while conserving the Casimir invariant). Because in a self-consistent theory total energy is conserved by the collection of drift waves and CCs, energy leaving the drift waves shows up as CC energy. Thus the CC growth rate can be calculated from the drift-wave energetics, which in turn are governed by the wave kinetic equation. That can be constructed from the random ray equations, and is

$$\frac{\partial \widetilde{\mathscr{N}_{\boldsymbol{k}}}}{\partial t} - \{\widetilde{\Omega}, \widetilde{\mathscr{N}_{\boldsymbol{k}}}\} = 0, \tag{13}$$

where

$$\{\widetilde{\Omega}, \widetilde{\mathscr{N}_{\boldsymbol{k}}}\} \doteq \frac{\partial \widetilde{\Omega}}{\partial \boldsymbol{X}} \cdot \frac{\partial \widetilde{\mathscr{N}_{\boldsymbol{k}}}}{\partial \boldsymbol{k}} - \frac{\partial \widetilde{\Omega}}{\partial \boldsymbol{k}} \cdot \frac{\partial \widetilde{\mathscr{N}_{\boldsymbol{k}}}}{\partial \boldsymbol{X}}. \tag{14}$$

From (13) one can construct the evolution equation for the energy $\mathscr{E}_{\boldsymbol{k}} \doteq \mathscr{N}_{\boldsymbol{k}}/\overline{k}^2$:

$$\frac{\partial \widetilde{\mathscr{E}}_{\boldsymbol{k}}}{\partial t} - \{\widetilde{\Omega}, \widetilde{\mathscr{E}}_{\boldsymbol{k}}\} = 2\widetilde{\gamma}^{(1)}\widetilde{\mathscr{E}}_{\boldsymbol{k}}, \tag{15}$$

where

$$\widetilde{\gamma}^{(1)} \doteq \frac{1}{2}\{\widetilde{\Omega}, \ln(\overline{k}^2)\}. \tag{16}$$

The $\widetilde{\gamma}^{(1)}$ term describes the random change of drift-wave energy content due to the modulation. ($\widetilde{\gamma}^{(1)}$ is of first order in the fluctuations because $\widetilde{\Omega} \propto \widetilde{\boldsymbol{V}}_{E}^{\mathrm{CC}}$.) The term $\widetilde{\gamma}^{(1)}\widetilde{\mathscr{E}}_{\boldsymbol{k}}$ averages to zero when calculated with the unmodulated drift-wave energy, but contributes coherently at second order, i.e., when the first-order change to $\widetilde{\mathscr{E}}_{\boldsymbol{k}}$ due to the modulation is included. That is most conveniently calculated from (13), which because the background is taken to be spatially homogeneous reduces to

$$\frac{\partial \widetilde{\mathscr{N}}_{\boldsymbol{k}}^{(1)}}{\partial t} = \frac{\partial \widetilde{\Omega}^{(1)}}{\partial \boldsymbol{X}} \cdot \frac{\partial \widetilde{\mathscr{N}}_{\boldsymbol{k}}^{(0)}}{\partial \boldsymbol{k}}. \tag{17}$$

We must integrate this equation in time to find $\widetilde{\mathscr{N}}_{\boldsymbol{k}}^{(1)}$, construct $\widetilde{\mathscr{E}}_{\boldsymbol{k}}^{(1)}$, then average the right-hand side of (15) to find the coherent energy loss from the drift waves. In doing so, the time integral of a correlation function between CC and DW quantities appears, and this must be somehow approximated. It is at this point that a purely heuristic theory of wave kinetics fails. The algorithm can be completed consistently only by making it compatible with systematic statistical closure. In Markovian closure, the effective interaction time is the triad interaction time θ. Following Krommes and Kim [7], I thus assert that it is algorithmically correct to replace $\partial \widetilde{\mathcal{N}_{\boldsymbol{k}}}/\partial t$ by $\theta^{-1}\widetilde{\mathcal{N}_{\boldsymbol{k}}}$. The remainder of the calculations are straightforward, and they lead to formula (7).

A formula similar to (7) was written by the authors of [1], the difference (in the present context) being that the triad interaction time was replaced by the function

$$R \doteq \frac{1}{\Omega_{\boldsymbol{q}} - \boldsymbol{q}\cdot\boldsymbol{v}_{\mathrm{gr}} + i\gamma_{\boldsymbol{k}}}, \tag{18}$$

where $\Omega_{\boldsymbol{q}}$ is an eigenvalue of the long-wavelength motion. This formula, which also appears in various other references that employ a heuristic wave kinetic algorithm, is problematical in several respects, as discussed by Krommes and Kim in [7]. (The authors of [1] were apparently unaware of that reference.) An issue regarding the group-velocity term is too subtle to reiterate here; see Sec. III C of [7]; in any event, Kaw etal. discarded $\Omega_{\boldsymbol{q}} - \boldsymbol{q}\cdot\boldsymbol{v}_{\mathrm{gr}}$ in their discussion of large-scale instability. But more importantly, a proper theory must be invariant to random Galilean transformations, and (18) is not. In the systematic derivation of $\gamma_{\boldsymbol{q}}$ from Markovian closure theory, random Galilean invariance is ensured by using Kraichnan's Test Field Model (TFM)[11, 8] or something similar to calculate the relevant θ. When that is done, the resulting theory has the following properties[8]. "If the effective shear actingon the small scales represented by $U(k)$ in [Eq. (10)] is dominated by wavenumbers $\ll k$, then θ_{qkk} is found from the TFM equations to be approximately the eddy-circulation time or the correlation time of the large-scale straining motion, whichever is shorter. On the other hand, if the shear and rotation acting on motions of wavenumber k are due primarily to interactions that are local in wavenumber, or scale size, then θ_{qkk} is the order of the eddy-circulation time or correlation time of the motions of scale $1/k$." (To conform with the present notation, k and q were interchanged in the previous quotation.) These properties do not follow from the appearance of $\gamma_{\boldsymbol{k}}$ in (18).

TRUNCATED GYROKINETICS AND LONG-WAVELENGTH FLOWS

Since this paper focuses on zonal-flow generation, I would like to describe very briefly, without presenting technical details, on one of the current topics of interest, namely the relevance of truncated gyrokinetics to long-wavelength flow generation (of relevance to momentum transport and plasma rotation). This issue has been raised in a series of recent papers by Parra and Catto. They correctly point out that since gyrokinetics is developed as an asymptotic expansion, it must be somehow truncated before it is practically used in, say, a computer simulation. They are concerned that standard truncations do not properly conserve momentum, for example by introducing spurious momentum sources.

In fact, several kinds of approximations are involved in the usual practical implementations of gyrokinetics. Most fundamental is what can be called the "gyrokinetic closure." As described, for example, in the tutorial article by Krommes [5], the derivation of gyrokinetics begins merely with a series of variable transformations. No physics content is lost in that process; information is just packaged in a possibly unusual way. But ultimately information about high-frequency physics is deliberately removed in order to obtain a new nonlinear dynamical system that describes only low-frequency motions. This is done by replacing, in the representation of charge or current, a gyrophase-dependent distribution function by its gyrophase average. For collisionless physics, this is completely justifiable, as shown by Dubin et al. [12]. When collisions are important, this is an approximation that misrepresents some collisional effects (which might be important in neoclassical theory, for example). Here I will be concerned only with collisionless gyrokinetics.

Even after the gyrokinetic closure is made, there remains the issue of properly truncating the asymptotic development of the transformation between particle coordinates and gyrocenter coordinates. Truncations must be done in two places: in the gyrokinetic equation, through the expressions for the gyrocenter drifts; and in Maxwell's equations, *via* the pullback transformation that expresses charge or current in terms of the gyrokinetic distribution. An important question is how to make those truncations consistent. Parra and Catto apparently consider nth-order truncations in which n is the same (e.g., $n = 2$) in both the gyrokinetic equation and the Maxwell equations. This can be shown-Parra and Catto [13] to lead to a spurious momentum source in the evolution equation for zonal flow. However, Brizard [14] has shown that gyrokinetics can be obtained from a variational formulation. Given an appropriate gyrokinetic action functional, variation with respect to the distribution function generates the kinetic equation, while variation with respect to the potential(s) generates the gyrokinetic Maxwell equations. The important point here is that if a potential φ (consider electrostatics for simplicity) is of the order of ε, where ε is the gyrokinetic expansion parameter, then variation of a nonlinear functional of φ *lowers its order by 1*. For example, $\delta\varphi^n/\delta\varphi = O(\varepsilon^{n-1})$. Thus, if the drifts in the kinetic equations are truncated to $O(\varepsilon^n)$, then the pullback transformation in the gyrokinetic Maxwell equations must be truncated to $O(\varepsilon^{n-1})$. I assert that this removes difficulties with spurious momentum sources. Scott [15] has explicitly demonstrated momentum conservation for a version of gyrokinetics with $n = 2$.

DISCUSSION

ACKNOWLEDGMENTS

This work was supported by U.S. Dept. of Energy Contract No. DE–AC02–09CH11466.

REFERENCES

1. P. Kaw, R. Singh, and P. H. Diamond, *Plasma Phys. Control. Fusion* **44**, 51–9 (2002).
2. N. Bisai, A. Das, S. Deshpande, R. Jha, P. Kaw, A. Sen, and R. Singh, *Phys. Plasmas* **12**, 102525 (2005).

3. R. Singh, V. Tangri, P. Kaw, and P. N. Guzdar, *Phys. Plasmas* **12**, 092307 (2005).
4. A. J. Brizard, and T. S. Hahm, *Rev. Mod. Phys.* **79**, 421–68 (2007).
5. J. A. Krommes, *Phys. Scripta* (2009), in press.
6. P. H. Diamond, M. N. Rosenbluth, F. L. Hinton, M. Malkov, J. Fleischer, and A. Smolyakov, "Dynamics of zonal flows and self-regulating drift-wave turbulence," in *17th IAEA Fusion Energy Conference*, International Atomic Energy Agency, Vienna, 1998, pp. 1421–8, IAEA–CN–69/TH3/1.
7. J. A. Krommes, and C.-B. Kim, *Phys. Rev. E* **62**, 8508–39 (2000).
8. R. H. Kraichnan, *J. Atmos. Sci.* **33**, 1521–36 (1976).
9. A. I. Smolyakov, and P. H. Diamond, *Phys. Plasmas* **6**, 4410–3 (1999).
10. J. A. Krommes, and R. A. Kolesnikov, *Phys. Plasmas* **11**, L29–32 (2004).
11. R. H. Kraichnan, *J. Fluid Mech.* **47**, 513–24 (1971).
12. D. H. E. Dubin, J. A. Krommes, C. R. Oberman, and W. W. Lee, *Phys. Fluids* **26**, 3524–35 (1983).
13. F. Parra, and P. Catto, *Plasma Phys. Control. Fusion* (2009), submitted.
14. A. J. Brizard, *Phys. Plasmas* **7**, 4816–22 (2000).
15. B. Scott (2010), private communication.

The Princeton Plasma Physics Laboratory is operated
by Princeton University under contract
with the U.S. Department of Energy.

Information Services
Princeton Plasma Physics Laboratory
P.O. Box 451
Princeton, NJ 08543

Phone: 609-243-2245
Fax: 609-243-2751
e-mail: pppl_info@pppl.gov
Internet Address: http://www.pppl.gov