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PPPL- 4543

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August, 2010

Prepared for the U.S. Department of Energy under Contract DE-AC02-09CH11466.

Princeton Plasma Physics Laboratory
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SXR continuum radiation transmitted through metallic filters: An analytical approach to fast electron temperature measurements*

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June 3, 2010

Abstract

A new set of analytic formulae describes the transmission of soft X-ray (SXR) continuum radiation through a metallic foil for its application to fast electron temperature measurements in fusion plasmas. This novel approach shows good agreement with numerical calculations over a wide range of plasma temperatures in contrast with the solutions obtained when using a transmission approximated by a single-Heaviside function [S. von Goeler, Rev. Sci. Instrum., 20, 599, (1999)]. The new analytic formulae can improve the interpretation of the experimental results and thus contribute in obtaining fast teperature measurements in between intermittent Thomson Scattering data.

*Contributed paper published as part of the Proceedings of the 18th Topical Conference on High-Temperature Plasma Diagnostics, Wildwood, New Jersey, May, 2010.
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1 Motivation.

Soft X-ray (SXR) diagnostics are common in high temperature plasma research providing information on plasma temperature ($T_{e,i}$), density (n_e) and impurity content (n_Z). The majority of space- and time-resolved density and temperature diagnostics are not capable of responding to the broad range of magneto-hydrodynamic (MHD) fluctuating phenomena ($10^2 - 10^5$ Hz) and thus an independent inference of local density and temperature fluctuations from high time-resolution SXR measurements is highly desirable. An adequate candidate for providing fast profile measurements is the multi-energy SXR method which consists in measuring the broadband SXR emission in multiple energy ranges and using the intensity profiles and their ratios to determine the "slope" of the SXR continuum radiation constrained solely by $T_e(R,t)$. An example of such diagnostic is the tangential ME-SXR array installed at the National Spherical Torus Experiment (NSTX) [1] at the Princeton Plasma Physics Laboratory (PPPL). This system provides a simultaneous "*multi-energy* image" of the plasma midplane, subject to a simple 1D Abel reconstruction [2]-[7]. The accuracy and resolution of the SXR-based temperature measurement can be greatly improved by including an intermittent normalization to the $T_e(R,t)$ profiles from the discrete multi-point Thomson Scattering (MPTS) diagnostic. Both the normalization and the further calculation of temperature profiles rely on an accurate understanding of the continuum radiation transmitted through each of the metallic foils; thus the new analytic solutions presented here will improve the fast $T_e(R,t)$ measurements in between intermittent MPTS time points. In section 2 we review the formalism behind the the determination of the transmission function for metallic foils and its applications for measuring the SXR local radiated power and photon emissivities. The analytic solutions of the SXR integrals and summary are left for section 3.

2 Background.

The energy-dependent transmission of X-rays through a slab of thickness "τ" and "N" number of atoms per unit volume is given by equation (1), where the atomic photo-absorption cross section μ_A can be obtained from the imaginary part of the atomic scattering factor ($f = f_1 + if_2$) as indicated in equation (2); r_0 is the classical electron radius and $\lambda = hc/E$ is the photon wavelength [8].

$$\mathcal{T}(E) = \exp\left(-N\mu_A\tau\right) \tag{1}$$

$$\mu_A(E) = 2r_0\lambda f_2(E) \tag{2}$$

Here thereafter we will refer to the properties of Beryllium filters used in the first ME-SXR prototype [2]-[7]; the Beryllium atomic scattering factors are presented in Figure 1-a). A fit of a functional form of f_2 given by $\sim k^\alpha/E^\alpha$ was obtained by minimizing the chi-square statistic error [see Figure 1-a)] and the result obtained was a constant of proportionality $k = 12.13$ and a value of α of the order of -2.073; both parameters have a 1-σ uncertainty estimate of 0.05 and 0.006, respectively. For SXR energies of interest in the range of 200 eV and 30 keV, the energy-dependent atomic scattering factors f_2 can thus be approximated by $f_2(E) \approx k^2/E^2$, resulting in a monotonic profile for the filer transmission function. The resultant X-ray transmission function approximation is shown in equation (3) and a comparison between the latter and the real transmission function [described by equations (1) and (2)], for the three thicknesses of choice, is shown in Figure 1-b). The residual errors are approximately 10% for transmission values of the same order, while at higher photon energies the deviations of the analytical approximation are negligible. The relationship between the filter characteristic energy $E_0 = (2Nr_0hc\tau k^2)^{1/3}$ and the usually referred as the filter cutoff-energy for a transmission of 50% is $E_0 = (\ln 2)^{1/3}\, E_{C,50\%}$.

$$\mathcal{T}(E) \approx \exp\left(-\frac{2Nr_0hc\tau k^2}{E^3}\right) = \exp\left(-\frac{E_0^3}{E^3}\right) \tag{3}$$

The plasma continuum radiation on the other hand, can be described by equation (4); P_X is the X-ray radiated power, E is the SXR photon energy, and T_e, n_e and Z_{eff} are the local electron temperature, electron density and the average plasma ion charge [9]; the factor $\gamma(T_e)$ is the enhancement of the radiated power over Bremsstrahlung emission from free-bound recombination, arising predominantly from the ionized carbon impurities sourced from the carbon-tile walls in NSTX [9]. The SXR power emitted by a local volume-element, and later filtered by a metallic foil with a transmission function $\mathcal{T}(E)$ and absorbed by the detector of choice is given by equation (5); the number of photons impinging in the detector is calculated in a similar way by using equation (6). Here thereafter we will assume that all the photons of interest will be absorbed by the detector of choice. The procedure followed to numerically evaluate $\mathcal{I}(T_e)$

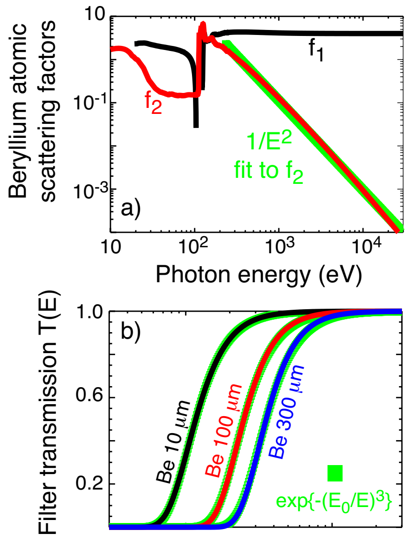

Figure 1: a) Atomic scattering factors f_1 and f_2 for a Be foil and the $1/E^2$ fit to f_2 from ∼0.2 to 30 keV. The real and approximated transmission functions are shown in b).

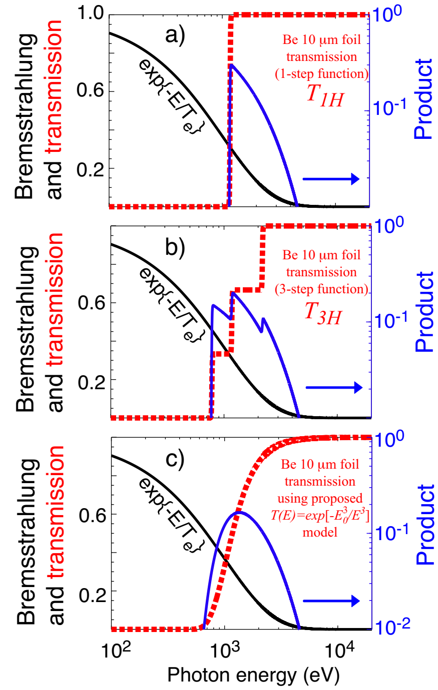

Figure 2: Analytic options for the transmission of a 10μm Be foil: the models based on one- and three-step functions are shown in a) and b); the analytical model based on the $1/E^2$ fit of f_2 is shown in c).

and $\mathcal{II}(T_e)$ uses a tabulated set of data on a closed interval using a five-point Newton-Cotes integration formula. The numerical results for each of the SXR integrals will be shown below.

$$\frac{d\mathcal{P}_X}{dE} = \mathcal{C}\frac{\gamma(T_e)Z_{eff}n_e^2}{\sqrt{T_e}}\exp\{-E/T_e\} \tag{4}$$

$$\mathcal{P}_X \approx \frac{\mathcal{C}\gamma(T_e)Z_{eff}n_e^2}{\sqrt{T_e}}\underbrace{\int_0^\infty \exp\{-E/T_e\}\mathcal{T}(E)dE}_{\mathcal{I}(T_e)} \tag{5}$$

$$\mathcal{N}_X \approx \frac{\mathcal{C}\gamma(T_e)Z_{eff}n_e^2}{\sqrt{T_e}}\underbrace{\int_0^\infty \frac{\exp\{-E/T_e\}}{E}\mathcal{T}(E)dE}_{\mathcal{II}(T_e)} \tag{6}$$

3 Analytic solutions.

3.1 Heaviside approximations.

The transmission function can be represented in a first approximation by a single Heaviside function evaluated at a 50% photon transmission through the SXR metallic foil [see Figure 2-a)]. Traditionally, this has been the transmission function of choice [9] due to its symmetric characteristics around the 50% cutoff energy; the cutoff energies of interest for the three filters of choice are listed in Table 1. The result from the analytic integrations using a single-Heaviside function are given by equations (7) and (8); E_1 is the exponential integral of order one [9].

$$\mathcal{I}_{1\mathcal{H}}(T_e) = T_e \exp\{-E_{C,50\%}/T_e\} \tag{7}$$

$$\mathcal{II}_{1\mathcal{H}}(T_e) = E_1\left(E_{C,50\%}/T_e\right) \tag{8}$$

A comparison between this first 'zero-*th* order' analytical solution (I_{1H}) and the numerical integration with the appropriate atomic scattering factors (I_{Num}) is shown in Figure 3-a). The deviations from the numerical solution for the thin filter are underestimated by $\sim$ 20% (see data in black) at low plasma temperatures and overestimated by $\sim$ 8% at higher temperatures. The intensities obtained for the thicker Beryllium 100 μm and 300 μm foils on the other hand (data in red and blue), show an underestimate of more than 100% at low energies with better matching at higher plasma temperatures. The temperature-dependent intensity ratios depicted in Figure 3-b) show also strong deviations from the numerical integration at low plasma temperatures with better matching at plasma temperatures

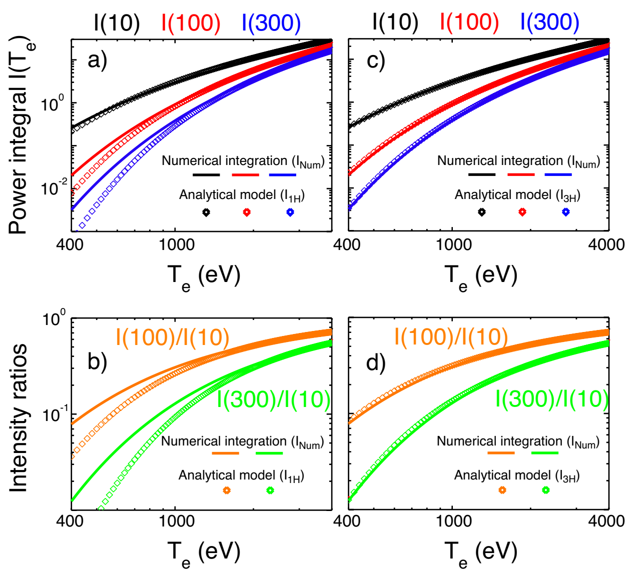

Figure 3: Comparison between the numerical integration ($\mathcal{I}_{Num}$) and the one- and three step functions analytical model ($\mathcal{I}_{1H}$ and $\mathcal{I}_{3H}$) for both the power integral and the temperature-sensitive SXR ratios.

Foil (μm)	$E_{c,10\%}$	$E_{c,50\%}$	$E_{c,90\%}$
Be 10	780.64	1170.2	2181.6
Be 100	1690.0	2496.8	4557.7
Be 300	2416.1	3550.1	6492.6

Table 1: Metallic Be foils used in this analysis and approximate cutoff energies [eV] for SXR transmissions (T) of 10%, 50% and 90%.

higher than 1.0 keV. This solution introduced first by S. von Goeler [9] offers a qualitative description of the high time resolution electron temperature measurements that can be obtained by calculating the ratio of the SXR emissivities measured over two or more energy ranges. An assessment of the contribution to the emissivity ratios by SXR line-emission from metallic high-Z impurities is also underway [10].

A better assessment of the SXR integrals can be formulated by describing the transmission function as a superposition of three-Heaviside ($3\mathcal{H}$) functions evaluated at transmissions of 10%, 50% and 90% [see Figure 2-b)]; the solutions for the SXR integrals are shown in equations (9)-(10). Although this intuitive correction lacks analytical accuracy it offers a much better quantitative representation of the SXR integrals both at low and high plasma temperatures in comparison to the overly simplified one-Heaviside function approach discussed before. A comparison between this second approximation (I_{3H}) and the numerical integration (I_{Num}) is shown in Figure 3-c). The deviations from the numerical solution have been reduced to $\sim 2-10\%$ for plasma temperatures from 0.4-4 keV, for the three filters considered. The temperature-dependent ratios also show a maximum deviation of about 10% from the numerical integration.

$$\mathcal{I}_{3\mathcal{H}}(T_e) = \frac{T_e}{3}\sum_{i=1}^{3}\exp\{-E_{C,T_i\%}/T_e\} \tag{9}$$

$$\mathcal{II}_{3\mathcal{H}}(T_e) = \frac{1}{3}\sum_{i=1}^{3}E_1\left(E_{C,T_i\%}/T_e\right) \tag{10}$$

3.2 Asymptotic integration.

It is also possible to re-write de SXR radiated power transmitted through a metallic foil using the approximate analytic transmission functions developed in section 2 as,

$$\mathcal{P}_X \approx \frac{\mathcal{C}\gamma Z_{eff} n_e^2}{\sqrt{T_e}} \underbrace{\int_0^\infty \exp\left[\overbrace{-\frac{E}{T_e}-\frac{E_0^3}{E^3}}^{\phi(E)}\right] dE}_{\mathcal{I}(T_e)} \tag{11}$$

The integrand in equation (11) has an extreme at $E_m = (3E_0^3T_e)^{1/4}$; the second derivative of such function is negative, so the contribution to the integral is thus maximum. This introduces a first analytic correction to the one-Heaviside model described above; using this new formalism, the photon energy for maximum contribution to the SXR integral is $E_m \approx 1.2E_{C,50\%}^{3/4}T_e^{1/4}$ instead of $E_m \approx E_{C,50\%}$ as assumed with the single step-function model. The SXR integral can be then approximated by the saddle-point asymptotic integration (SPI) leading to equation (12). The small deviations with respect to the numerical integration shown in Figure 4-a) are of the order of -13%, -4% and +3% at low temperatures, while at high temperatures the SXR integral is overestimated by approximately 100%, 40% and 25% for the low, medium and high energy filters, respectively. These departures from the numerical solution at high temperatures could be explained by the higher order correction terms assumed to be negligible ($T_e/2\Delta E << 1$ where $\Delta E = \left(\frac{E_mT_e}{2}\right)^{1/2}$) in the Taylor expansion of $\phi(E)$.

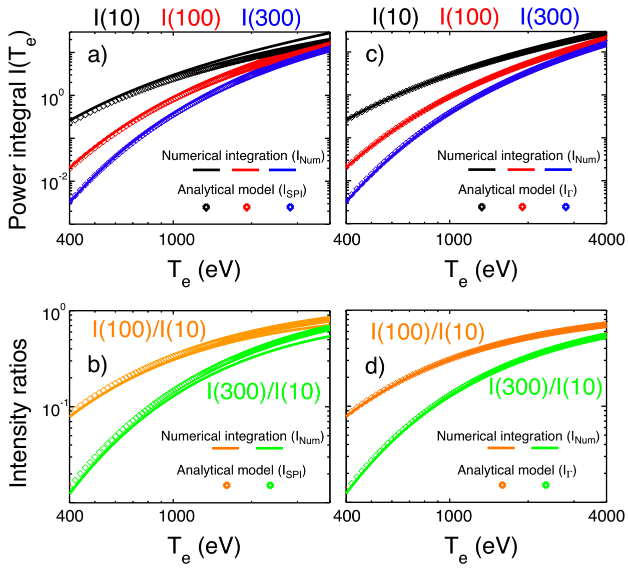

Figure 4: Comparison between the numerical integration (I_{Num}) and the the saddle-point-integration (I_{SPI}) and gamma-function (I_Γ) solutions.

$$\mathcal{I}_{SPI}(T_e) = \sqrt{\pi}\Delta E \exp\left(-\frac{4}{3}\frac{E_m}{T_e}\right) \quad (12)$$

A more rigorous analytical solution can be obtained, not by Taylor expanding $\phi(E)$, but by using a Taylor sum for the analytical model describing the transmission function. The analytic transmission function $\mathcal{T}(E) \approx \exp\left[-E_0^3/E^3\right]$ is an example of an infinitely differentiable function whose Taylor series converge, but is not equal to $\mathcal{T}(E)$; for instance, all its derivatives at $x = E_0/E = 0$ are zero, so the Taylor series of $\mathcal{T}(E)$ at zero is zero everywhere, even though the function is non-zero for every $x \neq 0$. The numerical Taylor expansion divergence for $E_0/E < 1$, calls for a careful truncation by considering for instance, a high and even number of terms in the series (e.g. n=20) and by setting $\mathcal{T}(E < 0.6E_0) \approx 0$ and $\mathcal{T}(E \geq 0.6E_0) \approx \sum_{k=0}^{n=20}\left(-\frac{E_0^3}{E^3}\right)^k \frac{1}{k!}$. After a slight re-arrangements of terms and change of variables, the SXR integrals describing both the transmitted radiated power and photon emissivities can be written as in equations (13) and (14), where Γ is the incomplete gamma function defined as $\Gamma(1-p, x) = \int_x^\infty \exp(-u)u^{-p}du$. The comparison between the numerical integration (I_{Num}) and the analytical solution using the gamma function formalism (I_Γ) is shown in Figure 4-c); the deviations at low temperatures are as small as -1%, 2% and 5%, while at high temperatures the analytical solution shows a remarkable agreement with the numerical integration; this good agreement is also observed for the temperature-sensitive SXR ratios depicted in Figure 4-d).

$$\frac{\mathcal{I}_\Gamma(T_e)}{T_e} = \sum_{k=0}^{n=20} \frac{(-1)^k}{k!}\left(\frac{E_0}{T_e}\right)^{3k} \Gamma\left(1-3k, \frac{3}{5}\frac{E_0}{T_e}\right) \quad (13)$$

$$\mathcal{I}\mathcal{I}_\Gamma(T_e) = \sum_{k=0}^{n=20} \frac{(-1)^k}{k!}\left(\frac{E_0}{T_e}\right)^{3k} \Gamma\left(-3k, \frac{3}{5}\frac{E_0}{T_e}\right) \quad (14)$$

In summary, a new set of analytic formulae have been derived to describe the transmission of SXR continuum radiation through a metallic foil for its application to fast electron temperature measurements in fusion plasmas. The analytic solution based on a limited Taylor expansion and the incomplete gamma function formalism is the most accurate representation of the SXR integrals at both low and high plasma temperatures. This work was supported by the United States Department of Energy (DOE) grants No. DE-FG02-86ER52314ATDOE and DE-AC02-76-CH0-3073.

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