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On Higher-order Corrections to Gyrokinetic Vlasov-Poisson Equations in the Long Wavelength Limit

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On higher-order corrections to gyrokinetic Vlasov-Poisson equations in the long wavelength limit*

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In this paper, a simple iterative procedure is presented for obtaining the higher order $\mathbf{E}\times\mathbf{B}$ and $d\mathbf{E}/dt$ (polarization) drifts associated with the gyrokinetic Vlasov-Poisson equations in the long wavelength limit of $k_\perp\rho_i \sim o(\epsilon)$ and $k_\perp L \sim o(1)$, where ρ_i is the ion gyroradius, L is the scale length of the background inhomogeneity and ϵ is a smallness parameter. It can be shown that these new higher order $k_\perp\rho_i$ terms, which are also related to the higher order perturbations of the electrostatic potential ϕ, should have negligible effects on turbulent and neoclassical transport in tokamaks regardless of the form of the background distribution and the amplitude of the perturbation. To address further the issue of a non-Maxwellian plasma, higher order finite Larmor radius terms in the gyrokinetic Poisson's equation have been studied and shown to be unimportant as well. On the other hand, the terms of $o(k_\perp^2\rho_i^2)$ and $k_\perp L \sim o(1)$ can indeed have an impact on microturbulence, especially in the linear stage, such as those arising from the difference between the guiding center and the gyrocenter densities due to the presence of the background gradients. These results will be compared with a recent study questioning the validity of the commonly used gyrokinetic equations for long time simulations.

Since the first derivations of the gyrokinetic Vlasov-Poisson equations [1, 2], tremendous progress has been made over the years in formulating and understanding these equations [3–8] for use in magnetic fusion research. They have also become the basis for the many simulation codes for turbulence and neoclassical transport studies in the community. In the electrostatic limit, this set of nonlinear equations keeps only the linear response of the perturbed potential ϕ and conserves energy to the order of ϕ^2. It has been pointed out by a recent paper [9] that this version of the gyrokinetic Poisson's equation (also known as the gyrokinetic quasineutralilty condition) based on the linear ϕ may not be adequate for long wavelength modes, which, in turn, limits its application in describing correct physics on transport time scales. Specifically, they argued for the presence of higher order terms in ϕ in the gyrokinetic Poisson's equation due to the fact that the background distribution is Maxwellian only to the lowest order. In view of the recent highly interesting investigations based on the original gyrokinetic Vlasov-Poisson equations (using the linear ϕ) for studying the nonlinear interactions between microturbulence and the long wavelength zonal flow modes [10–12], we will re-examine these equations in the present paper from the point of view of the drift kinetic approximation of the original Vlasov-Poisson system and the associated guiding center dynamics. Starting from a simple procedure as described in Ref. [13], we will show, without invoking the assumption related a Maxwellian background, that the nonlinear terms in ϕ both in the gyrokinetic Vlasov equation and the gyrokinetic Poisson's equation are higher order in $k_\perp \rho_i$. As such, they can be ignored based on the $k_\perp \rho_i \ll 1$ ordering. This is also true even if we keep the the finite Larmor radius (FLR) terms through the Bessel function expansion for a non-Maxwellian plasma in the gyrokinetic Poisson's equation as originally given in Refs. [1, 2, 14]. The effects on microinstabilities in the presence of a Maxwellian background for an inhomogeneous plasma due to the difference between gyrocenter and guiding center densities for $k_\perp L \sim o(1)$ will also be addressed. Comparisons will be made with those from Ref. [9].

It is well-known that the single particle motion in an electric field, $\mathbf{E}$, perpendicular to a magnetic field, $\mathbf{B}$ with charge q and mass m, is given by the Lorentz force of the form

$$\frac{d\mathbf{v}_\perp}{dt} = \frac{q}{m}\left(\mathbf{E}_\perp + \frac{1}{c}\mathbf{v}_\perp \times \mathbf{B}\right),$$

Upon taking the second time derivatives of the equation of motion, we obtain, in simple slab limit,

$$\frac{d^2\mathbf{v}_\perp}{dt^2} + \Omega^2\mathbf{v}_\perp = \frac{q}{m}\frac{d\mathbf{E}_\perp}{dt} + \frac{1}{c}(\frac{q}{m})^2\mathbf{E}_\perp \times \mathbf{B}.$$

Assuming that gyrofrequency $\Omega[\equiv qB/mc]$ is much higher than any frequency of interest, i.e., $\omega^2 \ll \Omega^2$, we get the approximate solution for the perpendicular motion of the guiding center as

$$\mathbf{v}_\perp \approx \frac{q}{m\Omega^2}\frac{d\mathbf{E}_\perp}{dt} + \frac{c}{B^2}\mathbf{E}_\perp \times \mathbf{B}. \tag{1}$$

Here, the convective derivative for the electric field can be written as

$$\frac{d\mathbf{E}_\perp}{dt} = \frac{\partial\mathbf{E}_\perp}{\partial t} + \mathbf{v}\cdot\frac{\partial\mathbf{E}_\perp}{\partial\mathbf{x}} \approx \frac{\partial\mathbf{E}_\perp}{\partial t} + \mathbf{v}_\perp\cdot\frac{\partial\mathbf{E}_\perp}{\partial\mathbf{x}_\perp}, \tag{2}$$

by assuming that $k_\parallel \ll k_\perp$. To the lowest order, we can use

$$\frac{d\mathbf{E}_\perp}{dt} \approx \frac{\partial\mathbf{E}_\perp}{\partial t},$$

which, in turn, gives the lowest order for the perpendicular velocity as

$$\mathbf{v}_\perp \approx \frac{q}{m\Omega^2}\frac{\partial\mathbf{E}_\perp}{\partial t} + \frac{c}{B^2}\mathbf{E}_\perp \times \mathbf{B}. \tag{3}$$

Here, the first term on the RHS of Eq. (3) is of higher order than the second for $\omega/\Omega \sim o(\epsilon)$, where ω is the frequency of interest and $\epsilon \ll 1$. It is well-known that the first term in Eq. (3) is the polarization drift and the second the $\mathbf{E}\times\mathbf{B}$ drift. Substituting Eq. (3) into Eq. (2) and then using the resulting equation in Eq. (1), we can write the next order corrections for the perpendicular velocity as

$$\mathbf{v}_\perp \approx \left(\underline{\underline{\mathbf{I}}} - \rho_t^2 \frac{\partial}{\partial \mathbf{x}_\perp}\frac{\partial}{\partial \mathbf{x}_\perp}\frac{q\phi}{T}\right)\cdot\left(\frac{q}{m\Omega^2}\frac{\partial \mathbf{E}_\perp}{\partial t} + \frac{c}{B^2}\mathbf{E}_\perp \times \mathbf{B}\right), \tag{4}$$

where $\underline{\underline{\mathbf{I}}}$ is a unit tensor, $\rho_t = \sqrt{T/m}/\Omega$ is the thermal velocity and $\mathbf{E}_\perp = -\nabla_\perp \phi$. The correction term is of $o(\epsilon^2)$ for $k_\perp \rho_t \sim o(\epsilon)$ and $q\phi/T \sim o(1)$. Thus, the correction to the polarization drift is even of higher order.

Let us first use Eq. (3) and follow a simple procedure [13] to derive the lowest-order gyrokinetic Vlasov-Poisson equations. From the leading term in Eq. (3), i.e., $\mathbf{v}_\perp \approx (c/B^2)\mathbf{E}_\perp \times \mathbf{B}$ in the original Vlasov equation, we can write the resulting drift kinetic equation as

$$\frac{\partial F}{\partial t} + \left[\mathbf{v}_\parallel + \frac{c}{B^2}\mathbf{E}_\perp \times \mathbf{B}\right]\cdot\frac{\partial F}{\partial \mathbf{x}} + \frac{q}{m}\mathbf{E}_\parallel \cdot \frac{\partial F}{\partial \mathbf{v}} = 0, \tag{5}$$

where

$$\mathbf{E}_\parallel = -\nabla_\parallel \phi = -\frac{\mathbf{B}}{B}\cdot\nabla\phi.$$

Next, we take the zeroth order moment of the same Vlasov equation, which yields the continuity equation as

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial \mathbf{x}}\cdot\int \mathbf{v} F d\mathbf{v} = 0,$$

with

$$n = \int F d\mathbf{v}. \tag{6}$$

Upon substituting the polarization drift part of Eq. (3) of $\mathbf{v}_\perp \approx (q/m\Omega^2)(d\mathbf{E}_\perp/dt)$ into the continuity equation, and using the resulting polarization density as a part of the ion number density in the original Poisson's equation, we then arrive at

$$\nabla^2 \frac{e\phi}{T_i} + \nabla\cdot\frac{\omega_{pi}^2}{\Omega_i^2}\nabla_\perp \frac{e\phi}{T_i} = -\frac{4\pi e^2}{T_i}(n_i - n_e), \tag{7}$$

which is the gyrokinetic Poisson's equation in the long wavelength limit. Since

$$\frac{\omega_{pi}^2}{\Omega_i^2} = \frac{\rho_s^2}{\lambda_{De}^2} \gg 1$$

for tokamak plasmas, the first term on the LHS of Eq. (7) can be neglected, thereby, to achieve quasi-neutrality, where $\rho_s \equiv \sqrt{T_e/T_i}\rho_i$ and λ_{De} the electron Debye length. Equations (5) and

(7) are the usual gyrokinetic Vlasov-Poisson equations [1] in the long wavelength limit in slab geometry. This set of equations conserves total energy, i.e.,

$$\left\langle m_i\int v^2F_i d\mathbf{v} + m_e\int v^2F_e d\mathbf{v} + n_iT_i\rho_i^2\nabla_\perp\frac{e\phi}{T_i}\cdot\nabla_\perp\frac{e\phi}{T_i}\right\rangle_{\mathbf{x}}$$
$$= const., \tag{8}$$

where $\langle\cdots\rangle_{\mathbf{x}}$ represents spatial averaging. The ion temperature variable denoting by T_i in Eqs. (7) and (8) is assumed to be spatially constant. The FLR version of Eqs. (5), (7), and (8) for $k_\perp\rho_i \sim o(1)$ is given in Ref. [1, 2, 14].

Following the same procedure but now using Eq. (4) for the $\mathbf{E}\times\mathbf{B}$ and polarization drifts, the higher order gyrokinetic Vlasov-Poisson equations in the long wavelength drift kinetic limit can then be written as

$$\frac{\partial F}{\partial t} + \left[\mathbf{v}_\parallel + \left(\underline{\underline{\mathbf{I}}} - \rho_t^2\nabla_\perp\nabla_\perp\frac{q\phi}{T}\right)\cdot\frac{c}{B^2}\mathbf{E}_\perp\times\mathbf{B}\right]\cdot\frac{\partial F}{\partial\mathbf{x}}$$
$$+\frac{q}{m}\mathbf{E}_\parallel\cdot\frac{\partial F}{\partial\mathbf{v}} = 0, \tag{9}$$

and

$$\nabla\cdot\frac{\omega_{pi}^2}{\Omega_i^2}\left(\underline{\underline{\mathbf{I}}} - \rho_i^2\nabla_\perp\nabla_\perp\frac{e\phi}{T_i}\right)\cdot\nabla_\perp\frac{e\phi}{T_i} = -\frac{4\pi e^2}{T_i}(n_i - n_e), \tag{10}$$

where, for simplicity, we have assumed that higher order terms are weakly time-dependent in the formulation in Eq. (10), and n is given by Eq. (6) . In comparison with the linear version of these equations, Eqs. (5), and (7), these additional terms in Eqs. (9) and (10) are obviously of higher order in the long wavelength limit of $k_\perp^2\rho_i^2 \ll 1$ even if the perturbation amplitude, $|q\phi/T_i|$, is of the order of unity. Therefore, they are all small and ignorable. It should be mentioned here that the number density in Eq. (6) is for an arbitrary F in the velocity space in the long wavelength limit and does not need non-Maxwellian corrections. Moreover, the additional nonlinear terms in the gyrokinetic Vlasov equation, Eq. (9), are related to the nonlinear term associated with $\partial/\partial\mu$ in Eq. (19) of the paper by Dubin et al. [2], where μ is the magnetic moment, while the higher order correction terms in the gyrokinetic Poisson's equation, Eq. (10), resemble the nonlinear $\partial/\partial\mu$ and $\partial^2/\partial\mu^2$ terms in Eq. (20) of the same reference. Furthermore, the higher order energy conservation as given by Dubin et al [2] included only the higher order terms in the gyrokinetic Vlasov equation, but not those in the gyrokinetic Poisson's equation, in agreement with our earlier comments on Eqs. (3) and (4) regarding the higher order nature of the polarization drift. Thus, the energy conservation for Eqs. (9) and (10) can also be approximated by Eq. (8), consistent with the long wavelength limit of Eq. (25) in Ref. [2]. In all, the results presented here agree with the prevailing understanding on nonlinear gyrokinetics.

To shed some light on the issue of the FLR effects on finite ion pressure, let us re-write the number density of Eq. (6) in the gyrokinetic limit for $k_\perp \rho_i \sim o(1)$ as [1, 2]

$$N = \left\langle \int \left[F_{gc} + \frac{q}{m}(\phi - \bar{\phi})\frac{\partial F_{gc}}{\partial \mu} \right] \delta(\mathbf{R} - \mathbf{x} + \boldsymbol{\rho}) d\mathbf{R} dv_\parallel d\mu \right\rangle_\varphi , \tag{11}$$

where N is the total number density, $F_{gc}(\mathbf{R}, \mu, v_\parallel, t)$ is the gyrocenter distribution, $\mathbf{x} = \mathbf{R} + \boldsymbol{\rho}$, $\mu \equiv v_\perp^2/2$, $\mathbf{x}$ represents the particle coordinates, $\mathbf{R}$ denotes the gyrocenter coordinates, $\rho \equiv v_\perp/\Omega$ is the particle gyroradii, $v^2 = v_\perp^2 + v_\parallel^2$, $\mu \equiv v_\perp^2/2$, $\langle \cdots \rangle_\varphi$ is the gyrophase averaging,

$$\bar{\phi}(\mathbf{R}) = \langle \phi(\mathbf{x}) \rangle_\varphi = \sum_{\mathbf{k}} \phi(\mathbf{k}) J_0(k_\perp v_\perp/\Omega) exp(i\mathbf{k} \cdot \mathbf{R}),$$

and the Bessel function J_0, which can be calculated numerically in the configuration space [14], is related to the phase averaging process through

$$\langle \delta(\mathbf{R} - \mathbf{x} + \boldsymbol{\rho}) \rangle_\varphi = \sum_{\mathbf{k}} \langle e^{i\mathbf{k}\cdot(\mathbf{R}-\mathbf{x}+\boldsymbol{\rho})} \rangle_\varphi / V,$$

$$= \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot(\mathbf{R}-\mathbf{x})} J_0(k_\perp v_\perp/\Omega)/V.$$

The (second) phase averaging on Eq. (11) yields

$$N = \bar{n}(\mathbf{x}, t) + \frac{q}{m} \sum_{\mathbf{k}=\mathbf{k}'+\mathbf{k}''} \int \phi(\mathbf{k}') \frac{\partial}{\partial \mu} F_{gc}(\mathbf{k}'') e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\times \left[J_0(k''_\perp v_\perp/\Omega) - J_0(k'_\perp v_\perp/\Omega) J_0(k_\perp v_\perp/\Omega) \right] dv_\parallel d\mu, \tag{12}$$

where

$$\bar{n}(\mathbf{x}, t) = \sum_{\mathbf{k}} \int F_{gc}(\mathbf{k}, t) J_0(k_\perp v_\perp/\Omega) e^{i\mathbf{k}\cdot\mathbf{x}} dv_\parallel d\mu$$

is the gyrocenter density. For the assumptions of $\mathbf{k}''_\perp \approx 0$ and $\partial F_{gc}/\partial \mu \approx -(m/T) F_{gc}^M$, where $F_{gc}^M = (n/\sqrt{2\pi})(m/T)^{3/2} exp(-mv^2/2T)$ denotes Maxwellian, Eq. (12) can be approximated as [1, 2]

$$N = \bar{n}(\mathbf{x}, t) + \frac{q}{T}(\int F_{gc}^M d\mu dv_\parallel) \sum_{\mathbf{k}} \phi(\mathbf{k}) \left[1 - \Gamma_0(\frac{k_\perp^2 T}{m\Omega^2}) \right] e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{13}$$

which is valid for arbitrary $k_\perp$ and is commonly used in the community, where $\Gamma_0(b) \equiv I_0(b) e^{-b}$ and I_0 is the modified Bessel function. For $k_\perp v_\perp/\Omega \ll 1$, $N \approx \bar{n}(\mathbf{x}, t) - (qn/T)[\rho_i^2 \nabla_\perp^2 \phi + \rho_i^4 \nabla_\perp^4 \phi/4]$.

However, in this limit, the Maxwellian approximation is not needed. By expanding J_0 in Eq. (12), we then obtain

$$N \approx \bar{n}(\mathbf{x},t) + \frac{q}{m}\left[\frac{1}{\Omega^2}\nabla\cdot n\nabla_\perp\phi + \frac{3}{4m\Omega^4}\nabla_\perp^2 p_\perp \nabla_\perp^2\phi + o(k_\perp^4)\right], \tag{14}$$

where $n = \int F_{gc} d\mu dv_\parallel$ and $p_\perp = m\int \mu F_{gc} d\mu dv_\parallel$. The gyrokinetic Poisson's equation, to the order of $k_\perp^4$, then becomes

$$\nabla^2\frac{e\phi}{T_i} + \nabla\cdot\frac{\omega_{pi}^2}{\Omega_i^2}\nabla_\perp\frac{e\phi}{T_i} + \frac{3}{4}\frac{\omega_{pi}^2}{\Omega_i^2}\frac{\rho_i^2}{n_i T_i}\nabla_\perp^2\frac{e\phi}{T_i}\nabla_\perp^2 p_{\perp i}$$
$$+ o(k_\perp^4) = -\frac{4\pi e^2}{T_i}(\bar{n_i} - n_e), \tag{15}$$

where the leading terms in $\nabla_\perp^2$ is the same as that from the Γ_0 expansion in Eq. (13). On the other hand, the $k_\perp^4$ terms in the equation, which are different from the Γ_0 expansion terms in Eq. (13), are valid for arbitrary distribution in the long wavelength limit, but are still negligible.

The conclusions drawn from Eqs. (10) and (15) regarding the irrelevance of the higher order terms are different from those of Parra and Catto [9], where the lack of consistency of the equation of the form,

$$\nabla\cdot\frac{\omega_{pi}^2}{\Omega_i^2}\nabla_\perp\frac{e\phi}{T_i} - \frac{1}{2}\frac{\omega_{pi}^2}{\Omega_i^2}\left|\nabla_\perp\frac{e\phi}{T_i}\right|^2 + \frac{1}{2}\frac{\omega_{pi}^2}{\Omega_i^2}\frac{1}{n_i T_i}\nabla_\perp^2 p_i,$$
$$= -\frac{4\pi e^2}{T_i}(n_i - n_e), \tag{16}$$

was discussed. The appearance of the nonlinear term (the second term) on the LHS of Eq. (16) is believed to be related to their phase-averaged gyroradius correction given by $\langle\Delta\boldsymbol{\rho}\rangle = \mathbf{v}_\perp/\Omega = \rho_i^2\nabla_\perp(e\phi/T_i)\times\mathbf{B}/B$, where $\mathbf{v}_\perp$ is the guiding center drift part of Eq. (3). To take this into account in our formulation, we have to replace the $exp(i\mathbf{k}\cdot\mathbf{x})$ term related to the perturbations in Eq. (12) by $exp(i\mathbf{k}\cdot\mathbf{x} + i\mathbf{k}\cdot\langle\Delta\boldsymbol{\rho}\rangle)$. It can be shown that the resulting nonlinear term in ϕ is of $o(k_\perp^4)$ and, like the similar terms in Eqs. (14) and (15), is, therefore, negligible. In fact, this correction is actually zero in slab geometry. This conclusion of ours seems to be consistent with the notion that higher order corrections to the particle gyroradii should not have much impact on the gyrokinetic Poisson's equation in the long wavelength limit. We should also remark here that our set of equations conserves energy as given by Eq. (8), when neglecting the higher order terms in the gyrokinetic Poisson's equation. This is very important for transport time scale simulations. We will re-visit Eq. (16) regarding the third term on the LHS later in the paper.

Presently, in the simulation community, the gyrocenter density $\bar{n}$, as defined in Eq. (12), has been used routinely for calculating the perturbations to $k_\perp\rho_i \sim o(1)$ accuracy. However, for the calculation of the background density in δf [15] codes such as GTC [16] and GTS [11], it is commonly obtained by setting $J_0 \approx 0$, valid only for $k_\perp \rho_i \approx 0$. As pointed out earlier [1], there is a difference between the guiding center density and the gyrocenter density in the presence of background inhomogeneity for $k_\perp L \sim o(1)$, where L is the scale length. Let us now re-visit the issue and examine the consequences.

To the lowest order, the gyrocenter density, $\bar{n}$, given by Eq. (12), can be calculated by

$$\bar{n}(\mathbf{x}) = \int \left(1 + \frac{1}{4}\frac{v_\perp^2}{\Omega^2}\nabla_\perp^2\right) F_{gc}(\mathbf{R}) dv_\parallel d\mu,$$

by using $\langle \delta(\mathbf{R}-\mathbf{x}+\boldsymbol{\rho})\rangle_\varphi$, expanding the Bessel function and invoking $\sum_\mathbf{k} e^{i\mathbf{k}\cdot(\mathbf{R}-\mathbf{x})}/V = \delta(\mathbf{R}-\mathbf{x})$. Let us consider the case for a Maxwellian plasma with spatial inhomogeneity for both density and temperature, i.e.,

$$\nabla_\perp F_{gc}^M = \left[\frac{\nabla_\perp n}{n} - \frac{3}{2}\frac{\nabla_\perp T}{T} + \frac{mv^2}{2T}\frac{\nabla_\perp T}{T}\right] F_{gc},$$

and

$$\nabla_\perp^2 F_{gc}^M = \left[\frac{\nabla_\perp^2 n}{n} + (-\frac{3}{2} + \frac{mv^2}{2T})(\frac{\nabla_\perp^2 T}{T} + 2\frac{\nabla_\perp T}{T}\cdot\frac{\nabla_\perp n}{n}) + (\frac{15}{4} - \frac{5mv^2}{2T} + \frac{m^2v^4}{4T^2})\frac{\nabla_\perp T\cdot\nabla_\perp T}{T^2}\right] F_{gc}^M.$$

The particle density then takes the form of

$$\bar{n}(\mathbf{x}) = n + \frac{1}{2}\rho_t^2\frac{1}{T}\nabla_\perp^2 nT, \tag{17}$$

where only the spatial dependence of the density on the RHS of Eq. (17) was kept earlier [1]. With this $\bar{n}_i$ accounting for the FLR effects for the ion number density in Eq. (15), the extra terms will add or subtract additional charges in the simulation depending on the concave or convex nature of the profiles due to the difference in gyroradius between the electrons and the ions. Consequently, a non-vanishing potential would emerge in the linear stage of the simulation giving rise to zeroth-order zonal flows.

For example, gyrokinetic Poisson's equation, Eq. (15), in the absence of higher order FLR terms with zero electron response, can be written as

$$\rho_s^2\nabla_\perp^2\frac{e\phi}{T_e} = -\frac{\delta n_i}{n_0},$$

for $\bar{n}_i = n_0 + \delta n_i$. Using Eq. (17) for the zeroth order background temperature variation (i.e., no density gradient), we then have $\rho_s^2\nabla_\perp^2(e\phi/T_e) = -(1/2)\rho_i^2(\nabla_\perp^2 T_i/T_i)$. Consequently, the zone flow potential,

$$\frac{e\phi}{T_i} = \frac{1}{2}\frac{\kappa_{Ti}^2}{k_\perp^2},$$

can be substantial for global modes when $k_\perp$ and the temperature gradient $\kappa_{Ti}(\equiv -\partial lnT_{i0}/\partial x)$ become comparable. The corresponding zonal flow velocity is

$$\frac{V_{E\times B}}{c_s} \equiv k_\perp\rho_s\frac{e\phi}{T_i} = \frac{1}{2}\frac{\kappa_{Ti}^2\rho_s^2}{k_\perp\rho_s}.$$

Thus, it is non-vanishing in the linear stage of the simulation and the effect of this term should be kept in the ion temperature gradient (ITG) instability calculations for $(k_\perp\rho_s)(e\phi/T_i) \sim o(\epsilon)$ or higher. Similar argument can be made for trapped electron modes in the presence of density gradient. The resulting zonal flows will certainly compete with those from the Dimits shift [17]. Interestingly, these extra charges represented by the second term of the RHS of Eq. (17) is the same as the last term on the LHS of the Parra-Catto equation, Eq. (16). As mentioned earlier, their existence was first discussed, based on gyrokinetic theory, in 1983 [1].

In summary, we have formulated the gyrokinetic Vlasov-Poisson equations by including higher order terms in both $e\phi/T_i$ and/or $k_\perp\rho_i$ in the long wavelength limit through two different procedures. The results confirm that the commonly used gyrokinetic Vlasov-Poisson equations are valid for an arbitrary distribution function in the long wavelength limit. In a recent paper [12], the nonlinearly-generated long wavelength (global) zonal flows due to ITG drift turbulence has been reported. It is shown that, by using the GTC code [16] for global gyrokinetic PIC simulation, that the damping of these long wavelength modes by the velocity space nonlinearity in the form of $E_\parallel\partial\delta f/\partial v_\parallel$, from the last term in Eq. (9) by using $F = F_{gc}^M + \delta f$, gives rise to the nonlinearly saturated state for tokamak core turbulence. These results are an example of the importance of the $k_\perp L \sim o(1)$ physics, which not only agreed the earlier observations by ORB5 [10] and GTS [11], but also gave rise to an interesting piece of physics related to the nonlearly-generated parallel current. In the future, it will be interesting to closely examine the effects of the gyrocenter density given by Eq. (17) on the zonal flows for the radially-compressed ITG modes that have been observed in a recent gyrokinetic PIC simulation of the tokamak edge using the XGC code [18], where the higher order terms associated with the $\mathbf{E}\times\mathbf{B}$ drift in Eq. (9) and with the pressure term in Eq. (15) may also be relevant.

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