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Stabilization of the quasi-interchange mode in tokamaks by circulating energetic ions

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The influence of the circulating energetic ions on the quasi-interchange (QI) mode in tokamak plasmas with a wide shearless core and the central safety factor close to unity is considered. It is found that these ions tend to stabilize the QI mode in the case of co-injection and balanced injection, whereas the influence of counter-circulating ions is typically destabilizing because of finite-orbit-width effects. Specific examples relevant to tokamaks with large and small aspect ratio of the torus are considered.

I. INTRODUCTION

Experiments on tokamaks show that the energetic ions, either trapped or circulating, can stabilize sawtooth oscillations – a typical form of magnetohydrodynamic (MHD) activity in tokamak plasmas – when the magnetic shear ($\hat{s}$) inside the $q = 1$ radius, r_s [$q(r)$ is the tokamak safety factor] is considerable.[1,2] These facts can be explained theoretically in terms of the ideal kink mode and non-ideal MHD modes (tearing modes in various regimes of the collisionality) in the presence of the energetic ions.[3–7] The cited theoretical works imply that the shear in the plasma core is not small (the works actually are valid for $\hat{s} \sim 1 - q_0 \gg \epsilon_s$ in the region $r \lesssim r_s$, where $q_0 = q(0)$, $\epsilon_s = r_s/R$, R the large radius of the torus). Till recent time, concerning the influence of the energetic ions on MHD activity in the small-shear plasma core, $\hat{s} \lesssim \epsilon_s$, was known only that the trapped particles play a minor role because the perturbed magnetic flux encircled by the precessional drift orbits vanishes in the limit of $1 - q_0 \rightarrow 0$.[5] Thus, it is of interest to consider the effect of the circulating energetic ions in this case. First results of the study of the influence of circulating particles on the $m = n = 1$ ideal MHD modes in low-shear plasmas were reported recently in an invited ICPP (International Congress on Plasma Physics) paper.[8] However, the analysis itself was not shown in the mentioned paper. Moreover, only a particular example was considered. This motivated us to make this work where an analysis of the influence of circulating energetic ions on MHD activity in plasmas with the shearless core is carried out and tokamaks with various magnitudes of the aspect ratio of the torus and various shearless regions are considered.

Equilibria with the extended low-shear central core separated from the wall by the region with large magnetic shear are prone to the so called "infernal" mode instability when the safety factor profile $q(r)$ is flat and sufficiently close to the mode rational m/n, with n the toroidal mode number, in the central region.[9–11] The dominant poloidal harmonic of such modes is localized in the low-shear, near-resonant region, where the field-line bending is minimized. Toroidal coupling plays a crucial part in the destabilization of the infernal modes, allowing the mode to acquire quasi-ballooning character. The lowest mode number ($m = n = 1$) member of this family is known as the quasi-interchange (QI) mode.[12] The structure of the QI mode has nothing to do with the rigid kink displacement taking place for $\hat{s} \gg \epsilon_s$ in the core region. The QI instability was proposed in Ref. 12 to interpret sawtooth crashes in those JET (Joint European Torus[13]) shots where the crashes were incompatible with the Kadomtsev model described in Ref. 14. An analytical theory for the QI-mode stability in the framework of the ideal MHD was developed in Refs. 15,16. However, plasmas of modern discharges with equilibria prone to the QI mode typically contains a population of energetic ions arising, e.g., from the neutral beam injection (NBI). That is why it is of interest to extend analysis of the mentioned works to include effects of the energetic ions.

Note that flat $q(r)$ profiles in the plasma core are typical for the low-aspect ratio tokamaks (spherical tori). They may occur also in the conventional tokamaks. In particular, such profiles were observed in JET and will take place in the ITER hybrid operation scenario.

II. STABILITY ANALYSIS

We consider a plasma with strong-shear periphery and shearless core that contains well circulating energetic ions with the distribution function, F_α, given by

$$F_\alpha(\bar{r},\varepsilon,\Lambda) = \frac{m_\alpha^{3/2}}{2\sqrt{2}\pi\varepsilon_\alpha} p_\alpha(\bar{r}) H(\varepsilon_\alpha - \varepsilon)\varepsilon^{-3/2}\left[\frac{1}{2}(1-\sigma_b) + \sigma_b H(\pm v_\parallel)\right]\delta(\Lambda), \tag{1}$$

where $\bar{r}$ is the average radius of a particle during its orbital motion, $\Lambda = \mu B_0/\varepsilon$, μ is the particle magnetic moment, B_0 is the magnetic field at the magnetic axis, ε_α is the birth energy, $p_\alpha(\bar{r}) = \int d^3v m_\alpha v_\parallel^2 F_\alpha$ is the beam particle pressure, $H(x)$ is the unit step function, and $\delta(x)$ is the Dirac δ-function, $\sigma_b = 0$ for the balanced injection, and $\sigma_b = 1$ in the case of co- and counter- injections.

In order to derive equations describing the QI mode in the presence of the energetic ions we proceed from the following energy functional, $\delta\mathcal{E}$:

$$\delta\mathcal{E} = \frac{R}{\pi^2 B_0^2}(\delta W_{MHD} + \delta W_{hot}) + \frac{\gamma^2}{\omega_A^2}N, \tag{2}$$

where δW_{MHD} is the ideal MHD potential energy,[15,16] δW_{hot} is the energetic ion contribution to the potential energy, and $\omega_A = v_A/R$ with v_A the Alfvén velocity. The last term in Eq. (2) represents the kinetic energy. It is given by

$$N = \frac{1}{2\pi^2 R}\int d^3r|\vec{\xi}|^2, \tag{3}$$

with $\vec{\xi}$ the plasma displacement. The magnitude δW_{hot} includes both the fluid part and kinetic part (we used Refs. 17,18):

$$\delta W_{hot} = \delta W_f + \delta W_k(\omega = 0), \tag{4}$$

$$\delta W_f \equiv \frac{1}{2}\int \vec{\xi}^*_\perp \cdot \nabla\delta\Pi^f_\alpha d^3r = \frac{\pi^2 m_\alpha}{qR}\sum_\sigma \int dr \int v^5 dv \int \frac{\partial F_\alpha}{\partial r}\tau_b \langle |\xi_r|^2 r\cos\theta\rangle d\Lambda, \tag{5}$$

$$\delta W_k \equiv \frac{1}{2}\int \vec{\xi}^*_\perp \cdot \nabla\delta\Pi^k_\alpha d^3r = -\frac{\pi^2 m_\alpha}{\omega_{c\alpha}}\sum_{S=0,\pm1}\sum_\sigma \int v^3 dv \int dr \int d\Lambda \tau_b \times$$

$$\frac{\partial F_\alpha}{\partial \varepsilon}\frac{\omega - \omega_{*\alpha}}{\omega - (k_\parallel + S/qR)v_\parallel - \omega_d}\left|\left\langle \left(\frac{v_\perp^2}{2} + v_\parallel^2\right)\vec{\xi}_\perp \cdot \vec{\kappa}\exp\left\{i\left[\omega - \left(k_\parallel + \frac{S}{qR}\right)v_\parallel - \omega_d\right]t\right\}\right\rangle\right|^2, \tag{6}$$

where $\delta\Pi^{f(k)}_\alpha = \delta p^{f(k)}_{\perp\alpha}\hat{I} + (\delta p^{f(k)}_{\parallel\alpha} - \delta p^{f(k)}_{\perp\alpha})\vec{b}\vec{b}$ is the pressure tensor, $\hat{I}$ is the identity tensor, $\delta p^{f(k)}_{\parallel/\perp\alpha}$ is the parallel/perpendicular pressure perturbation associated with the adiabatic (non-adiabatic) response of energetic ions, $\sigma = v_\parallel/|v_\parallel|$, $\vec{\kappa}$ is the field line curvature, τ_b is the particle transit time, $\omega_{*\alpha}$ is the diamagnetic drift frequency of the energetic ions, ω_d is their precession frequency, and $\langle \ldots \rangle$ denotes the orbit averaging.

First of all, we perform the orbit averaging and calculate the velocity integrals in Eqs. (5), (6). Omitting the term odd in θ in $\vec{\xi}_\perp \cdot \vec{\kappa}$ in the integrand of Eq. (6) (this term does not contribute to δW_k) we obtain:

$$|\xi_r|^2 = \xi_1^2 + 2\xi_1\xi_2\cos\theta + \xi_2^2, \tag{7}$$

$$\vec{\xi}_\perp \cdot \vec{\kappa} = -\frac{1}{R}\xi_1(r)\cos\theta(t)\exp\{i[\theta(t) - \phi(t) - \omega t]\}, \tag{8}$$

where ξ_1 and ξ_2 are the amplitudes of the $m = 1$ radial displacements and $m = 2$ radial displacement, respectively,

$$r(\theta) = \bar{r} + \Delta_\alpha\cos\theta, \quad \theta(t) = \frac{v_\parallel}{qR}t, \quad \phi(t) = \frac{v_\parallel}{R}t, \tag{9}$$

$\Delta_\alpha = [q(\bar{r})/v_\parallel\omega_{c\alpha}](0.5v_\perp^2 + v_\parallel^2)$, $\Delta_\alpha \ll \bar{r}$, where $\omega_{c\alpha}$ is the fast ion gyrofrequency. In Eq. (8) the only term (proportional to ξ_1) is retained because the term proportional to ξ_2 does not contribute to orbit averaged magnitudes in Eq. (6).

Note that the precession of circulating ions is taken into account in Eq. (6). This precession plays an important role for the internal kink mode, providing a nonzero kinetic response at $\omega = 0$ for $S = 0$.[7] However, in the case of the QI mode the particles deposited in the shear-free core contribute to $\delta W_k^{S=0}$.[19] Because $|1-q| \sim \epsilon_s$ and $\omega_d \sim v_\parallel^2/(\omega_{c\alpha} R^2)$, we have in the shear-free core $|\omega_d/k_\parallel v_\parallel| \sim \rho_\alpha/r \ll 1$, so that we can neglect the precession. This justifies omitting a drift term in the equation for $\theta(t)$ in Eq. (9) and gives us grounds to assume that $\omega_d = 0$ in Eq. (6). Then using the result of Ref. 19 for the $S = 0$ term, we find:

$$\frac{R}{\pi^2 B_0^2}\delta W_k^{S=0}(\omega=0) = -\frac{\rho_\alpha^3 R}{5\pi}\int_0^a d\bar{r}\left|\frac{d\xi_1}{d\bar{r}}\right|^2 \frac{d\beta_\alpha}{d\bar{r}}, \tag{10}$$

$$\frac{R}{\pi^2 B_0^2}\left[\delta W_k^{S=1}(\omega=0) + \delta W_k^{S=-1}(\omega=0)\right] = -\frac{\rho_\alpha R}{12\pi}\sum_\sigma \sigma \int_0^a d\bar{r}\frac{2q(1-q)}{2-q}|\xi_1|^2 \frac{d\beta_\alpha}{d\bar{r}}, \tag{11}$$

The fluid part of the energy functional given by Eq. (5) can be written as follows [Eqs. (7), (9) were used]:

$$\delta W_f = \delta W_{coup} + \delta W_{FOW}, \tag{12}$$

$$\frac{R}{\pi^2 B_0^2}\delta W_{coup} = \frac{R}{8\pi}\int_0^a \frac{d\beta_\alpha}{d\bar{r}}\xi_1\xi_2 \bar{r} d\bar{r}, \tag{13}$$

$$\frac{R}{\pi^2 B_0^2}\delta W_{FOW} = -\frac{\rho_\alpha R}{12\pi}\sum_\sigma \sigma \int_0^a \frac{d^2\beta_\alpha}{d\bar{r}^2}\xi_1^2 \bar{r} d\bar{r}, \tag{14}$$

where $\rho_\alpha = v_\alpha/\omega_{c\alpha}$, δW_{coup} and δW_{FOW} are the term associated with the toroidal coupling and the term associated with the finite orbit width, respectively. One can see that δW_{FOW} given by Eq. (14) has different signs for co-injection ($\sigma = 1$) and counter-injection ($\sigma = -1$); it is vanishing for the balanced injection. Both fluid terms are of the same order, $\delta W_{FOW} \sim \delta W_{coup}$ for $\rho_\alpha/L_{p\alpha} \sim \epsilon_a \equiv a/R$ (a is the plasma radius, $L_{p\alpha}$ the fast ion pressure scale length) because $\xi_2 \sim \epsilon_a \xi_1$. They well exceed the kinetic terms. This conclusion can be drawn by comparing Eqs. (10), (11) and (13), (14) and taking into account that $|1-q| \sim \epsilon_s$ in the region where ξ_1 is localized. Therefore, below we neglect the kinetic terms.

Equation (2) and Eqs. (13), (14) lead to the following Euler equations ($\bar{r}$ is replaced with r):

$$\frac{d}{dr}\left\{\left[\left(\frac{1}{q}-1\right)^2+\frac{3\gamma^2}{\omega_A^2}\right]r^3\frac{d\xi_1}{dr}\right\}-G\{\xi_1\}+\frac{1}{3}\sum_\sigma \sigma\rho_\alpha Rr\beta''_\alpha\xi_1$$
$$=\hat{C}\{\xi_2\}+\frac{1}{2}Rr\beta'_\alpha\xi_2, \tag{15}$$

$$\frac{d}{dr}\left[\left(\frac{1}{q}-\frac{1}{2}\right)^2 r^3\frac{d\xi_2}{dr}\right]-3\left(\frac{1}{q}-\frac{1}{2}\right)^2 r\xi_2=\hat{C}^+\{\xi_1\}+\frac{1}{2}Rr\beta'_\alpha\xi_1, \tag{16}$$

where prime denotes the radial derivative, G is the toroidal driving term, $\hat{C}$ and $\hat{C}^+$ are the toroidal coupling operators in the absence of energetic ions. The explicit forms for G and $\hat{C}$ are given in Ref.[16]. The operator $\hat{C}^+$ is adjoint to $\hat{C}$:

$$\int_0^a drf(r)\hat{C}\{g(r)\}=\int_0^a drg(r)\hat{C}^+\{f(r)\}. \tag{17}$$

We assume that $\gamma/\omega_A=O(\epsilon_a), |1/q-1|=O(\epsilon_a), G(\xi_1)=O(\epsilon_a^2\xi_1), \hat{C}(\xi)=O(\epsilon_a\xi), \xi_2=O(\epsilon_a)\xi_1$ in the plasma core.

Due to the mentioned ordering, we can take $q=1$ in the terms $G, \hat{C}$, and $\hat{C}^+$ for the shearless core. Then Eqs. (15), (16) can be written as follows [cf. Eq. (44a,b) of Ref. 16]:

$$\frac{d}{d\tilde{r}}\left\{\left[\left(\frac{1}{q_0}-1\right)^2+3\hat{\gamma}^2\right]\tilde{r}^3\frac{d\xi_1}{d\tilde{r}}\right\}-4\epsilon_a^2\left(\frac{\tilde{r}}{4}\beta'_p+\beta_p\right)^2\tilde{r}^3\xi_1+\frac{1}{3}\sum_\sigma\sigma\frac{\rho_\alpha}{\epsilon_a a}\tilde{r}\beta''_\alpha\xi_1=$$
$$\left(\frac{\tilde{r}}{4}\beta'_p+\beta_p\right)\epsilon_a^2\frac{d}{d\tilde{r}}(\tilde{r}^3\hat{\xi}_2)+\frac{1}{2}\tilde{r}\beta'_\alpha\hat{\xi}_2, \tag{18}$$

$$\frac{d}{d\tilde{r}}\left(\tilde{r}^3\frac{d\hat{\xi}_2}{d\tilde{r}}\right)-3\tilde{r}\hat{\xi}_2=-4\tilde{r}^3\frac{d}{d\tilde{r}}\left[\left(\frac{\tilde{r}}{4}\beta'_p+\beta_p\right)\xi_1\right]+2\tilde{r}\beta'_\alpha\xi_1, \tag{19}$$

where $\tilde{r}=r/a, \hat{\gamma}=\gamma/\omega_A, \hat{\xi}_2$ is defined by $\xi_2=\epsilon_a\hat{\xi}_2$, and

$$\beta_p(\tilde{r})=-\frac{1}{\epsilon_a^2\tilde{r}^4}\int_0^{\tilde{r}}\hat{r}^2\frac{d}{d\hat{r}}\left(\beta_c+\frac{\beta_\alpha}{2}\right)d\hat{r}. \tag{20}$$

In Eq. (20) a contribution of the anisotropic fast ion population to the Shafranov shift is taken into account. This is done using the following solution of the equilibrium equations with the anisotropic pressure:[20]

$$\Delta'(r)=\frac{r}{R}\left(\frac{l_i}{2}+\langle\beta_p\rangle_S+\beta_{ph}^A\right), \tag{21}$$

$$\beta_{ph}^{A} = \frac{1}{2}\left(\frac{qR}{r}\right)^{2} \langle(\beta_{\alpha\perp} + \beta_{\alpha\parallel})\cos 2\theta\rangle_S,$$

$$\langle\beta_p\rangle = -\frac{q^2R^2}{r^4}\int_0^r d\hat{r}\hat{r}^2\frac{d}{d\hat{r}}\left[\beta_c + \frac{1}{2}\langle(\beta_{\alpha\perp} + \beta_{\alpha\parallel})\rangle_S\right],$$

where l_i is the local internal inductance and $\langle\ldots\rangle_S$ denotes the flux surface averaging.

We assume that transition between the low-shear region and large-shear region is sufficiently abrupt and use the fact that ξ_1 is small and can be neglected in the large-shear region.[15,16] The eigenvalue $\hat{\gamma}$ can then be obtained in two steps. At the first step, Eq. (16) with the neglected right-hand side will be solved numerically in the outer region, $r_0 \leq r \leq a$, where r_0 is the boundary between the shearless region and large-shear region, with imposing the following boundary conditions:

$$\hat{\xi}_2(r_0 + 0) = 1, \tag{22}$$

$$\hat{\xi}_2'(r_2) = 0, \tag{23}$$

where r_2 is defined by the equation $q(r_2) = 2$, and it is assumed that the $q = 2$ radius lies inside the plasma, $r_2 < a$. Note that Eq. (22) represents a normalization condition. At the second step, Eqs. (18), (19) will be solved numerically in the inner region, $0 \leq r \leq r_0$, with the boundary conditions

$$\xi_1'(0) = \xi_1(r_0 - 0) = 0, \tag{24}$$

$$\hat{\xi}_2(r_0 - 0) = 1, \quad \hat{\xi}_2'(r_0 - 0) = \hat{\xi}_2'(r_0 + 0), \tag{25}$$

where $\hat{\xi}_2'(r_0 + 0)$ is the magnitude obtained at the first step.

When the energetic ions are absent, Eqs.(18), (19) can be solved analytically, which yields:

$$\xi_1(\tilde{r}) = \frac{\epsilon_a^2 C_{\xi_1}}{(1/q_0 - 1)^2 + 3\gamma^2}\int_{\tilde{r}}^{\tilde{r}_0} r\beta_p(r)dr \tag{26}$$

$$\xi_2(\tilde{r}) = r^{-3}\int_0^{\tilde{r}} r^4\beta_p(r)\frac{d\xi_1}{dr}dr + [C_{\xi_1} - \beta_p(\tilde{r})\xi_1(\tilde{r})]\tilde{r}, \tag{27}$$

where the constant of integration C_{ξ_1} and dispersion relation for γ are determined by the boundary conditions given by Eqs. (24) and (25).

Below we choose the following model profiles for $\beta(r)$, $\beta_\alpha(r)$ (the ratios of the particle pressure to the magnetic field pressure), and $q(r)$:

$$\beta(r) = \beta_0 \left(1 - \frac{r^2}{a^2}\right)^{l_1}, \tag{28}$$

$$\beta_\alpha(r) = \beta_{\alpha 0} \left(1 - \frac{r^2}{a^2}\right)^{l_2}, \quad l_2 > l_1, \tag{29}$$

$$q(r) = \begin{cases} q_0 = 1 - \Delta q & \text{for } 0 \le r \le r_0 \\ q_0 + \frac{(q_a - q_0)(r - r_0)^2}{(a - r_0)^2} & \text{for } r_0 \le r \le a, \end{cases} \tag{30}$$

where $\Delta q > 0$. For these profiles, Eqs. (26), (27) with the boundary conditions (24) and (25) yield:

$$3\gamma^2 = \frac{-\tau + \alpha \xi_2'(\tilde{r}_0 + 0)}{1 - r_0 \xi_2'(\tilde{r}_0 + 0)} - \left(\frac{1}{q_0} - 1\right)^2. \tag{31}$$

In particular, for $l_1 = 2$ the expressions for τ and α are given by

$$\tau = \left(\frac{q_0^2 \beta_0}{\epsilon}\right)^2 \left\{ -\left.\frac{dP_1}{dr}\right|_{\tilde{r}_0} + P_2(\tilde{r}_0) + \tilde{r}_0 \left.\frac{dP_2}{dr}\right|_{\tilde{r}_0} \right\}, \tag{32}$$

$$\alpha = \left(\frac{q_0^2 \beta_0}{\epsilon}\right)^2 \left\{ -P_1(\tilde{r}_0) + \tilde{r}_0 P_2(\tilde{r}_0) \right\}, \tag{33}$$

where $P_1(\tilde{r})$ and $P_2(\tilde{r})$ are the polynomials,

$$P_1(\tilde{r}) = \frac{1}{6}\tilde{r}^3 \left\{ -\frac{4}{15}\tilde{r}^4 + \tilde{r}^2 - 1 \right\}, P_2(\tilde{r}) = \frac{1}{2}\left(\frac{2}{3}\tilde{r}^2 - 1\right) \left\{ \left(\frac{\tilde{r}_0^4}{3} - \tilde{r}_0^2\right) - \frac{\tilde{r}^4}{3} + \tilde{r}^2 \right\}. \tag{34}$$

In the considered case of $l_1 = 2$, the eigenfunctions corresponding to the eigenvalue (31) are also polynomials:

$$\xi_1(\tilde{r}) = \frac{C_{\xi_2}}{2} \left\{ \left(\frac{\tilde{r}_0^4}{3} - \tilde{r}_0^2\right) - \frac{\tilde{r}^4}{3} + \tilde{r}^2 \right\}, \tag{35}$$

$$\xi_2(\tilde{r}) = -\left(\frac{q_0^2 \beta_0}{\epsilon}\right) C_{\xi_2} P_1(\tilde{r}) + \left[C_{\xi_1} + \left(\frac{q_0^2 \beta_0}{\epsilon}\right) C_{\xi_2} P_2(\tilde{r}) \right] \tilde{r}, \tag{36}$$

where

$$C_{\xi_1} = \left[\tilde{r}_0 + \left(\frac{q_0^2\beta_0}{\epsilon}\right)\frac{q_0^2\beta_0}{(1/q_0-1)^2+3\gamma^2}\{-P_1(\tilde{r}_0)+\tilde{r}_0P_2(\tilde{r}_0)\}\right]^{-1}, \qquad (37)$$

$$C_{\xi_2} = C_{\xi_1}\frac{q_0^2\beta_0}{(1/q_0-1)^2+3\gamma^2}. \qquad (38)$$

We developed a code which solves Eqs. (18), (19) numerically. When $\beta_\alpha = 0$, it reproduces the analytical results for both γ and eigenfunctions very well (including the case when the system is near threshold, i.e. when γ is very small). The code makes calculations for $\beta_\alpha \neq 0$ by means of an iteration procedure, with the initial solution given by Eqs. (31), (35), (36).

Using this code, calculations were carried out for two tokamaks, which we refer to as the conventional tokamak (CT), and spherical tokamak (ST). We assumed that $\beta(0) = 0.1$, $l_1 = 2$, $l_2 = 4$, $\rho_\alpha/a = 0.05$ and varied $\beta_\alpha(0)$, r_0/a, and $A \equiv R/a$. In CT we took $A \equiv R/a = 4$ and $q_a = 3$, whereas $A = 2$ and $q_a = 5$ in ST. The ratio r_0/a was taken 0.5 in both CT and ST. In addition, we took $r_0/a = 0.3$ in CT.

The results for ST and CT with $r_0/a = 0.5$ are shown in Figs. 1, 2a. We observe that the instability is more strong in the CT: in the absence of the energetic ions $\gamma_{max}^{CT}/\gamma_{max}^{ST} = 1.7$, the growth rate decreasing by a factor of 4 for $(\Delta q)_0 = 0.06$ (the subscript "0" means that the energetic ions are absent) in CT and $(\Delta q)_0 = 0.035$ in ST. The stabilizing effect of the energetic ions is roughly the same in CT and ST, γ_{max} decreases by a factor of 2 due to the presence of the energetic ions with $\beta_\alpha = 5\%$, and $(\Delta q)_0/(\Delta q)_\alpha \sim 2$, where $(\Delta q)_\alpha$ is the magnitude of Δq for which the growth rate decreases by a factor of 4 in the presence of the energetic ions.

The results of calculations for CT with $r_0/a = 0.3$ are shown in Fig. 2b. Comparing Figs. 2 and 3 we conclude that the influence of the unbalanced injection (either co- or counter-) is much more strong in tokamaks with smaller r_0/a. In particular, when $\Delta q \to 0$, adding co-/counter- injected ions with $\beta_\alpha(0) = 1\%$ reduces/increases the growth rate by a factor of 2. Surprisingly, adding a counter-directed beam with $\beta_\alpha(0) = 1\%$ to a co-directed beam with $\beta_\alpha(0) = 1\%$ only weakly affects the growth rate of the co-injected case: the curve corresponding to the balanced injection with $\beta_\alpha(0) = 2\%$ (not shown in Fig. 2b) almost coincides with the curve corresponding to the co-injection with $\beta_\alpha(0) = 1\%$, in spite of the fact that the curve corresponding to the counter-injection with $\beta_\alpha(0) = 1\%$ lies above the curve in the absence of the energetic ions. The explanation of these results is the following. There are two mechanisms responsible for the influence of the energetic ions on the instability. First, the toroidal coupling of the mode harmonics, which leads to terms proportional to β'_α in Eqs. (18), (19), and second, the finite orbit width of the energetic ions, which leads to terms proportional to β''_α. The decrease of r_0/a leads to the decrease of the region where the dominant harmonic ($m = 1$) is localized; therefore, the ratio $\beta''_\alpha/\beta'_\alpha$ grows. This explains why the effects of the energetic ions for unbalanced injection are more pronounced for smaller r_0/a. On the other hand, finite-orbit terms compensate each other for the balanced injection. As a result, only terms proportional to β'_α, which are relatively small for small r_0/a and do not depend on the beam direction, contribute. This explains why the balanced injection has a stabilizing influence on the instability and why this influence is relatively small, so that the balanced injection with $\beta_\alpha(0) = 2\%$ produces approximately the same effect as the co-injection with $\beta_\alpha(0) = 1\%$.

In conclusion, we note that the energetic ions affects not only the stability of the QI mode but also its radial structure. Figure 3 demonstrates this for the case of a balanced injection. When a non-balanced injection is used, the mode structure can be considerably changed, especially $\xi_1(r)$ in the region of small r for $\sigma = -1$ (counter-injection). However, the term in Eq. (18) describing the contribution of the finite orbit width was obtained in approximation of the small orbit width, which implies that our calculations are correct only when effects of the near-axis particles are small. This is the case when $r_0 \gg \Delta_\alpha$.

III. SUMMARY AND CONCLUSIONS

In summary, we have shown that the circulating energetic ions can stabilize the QI mode in tokamaks with the shearless plasma core and the central safety factor slightly smaller unity. Normally, the stabilizing effect takes place when the co-injection or balanced injection is used. It weakly depends on the aspect ratio of the torus. The effect is most strong in the case of co-injection and when the shear-free region ($r \leq r_0$) is not too large, so that finite orbit width effects may considerably contribute. On the other hand, when r_0/a is relatively small and counter injection is used, the particle finite orbit width may either enhance the instability. The reason for the stabilizing (destabilizing) effect of the co- (counter-) injection is clear: when the stability is determined by the region where $\xi_1(r)$ is a smoothly decreasing function (in contrast to the internal kink mode), the co- (counter-) injected ions with their orbits shifted outward (inward) "feel" smaller (larger) ξ_1 when passing the region with unfavourable curvature, and larger (smaller) ξ_1 when passing the region with favourable curvature.

For the balanced injection, finite orbit width effects compensate each other, so that the energetic ions influence the instability only due to the toroidal coupling of the dominant mode harmonic with $m = 1$ and the satellite harmonic with $m = 2$. This mechanism is relatively does not depend on the beam direction.

The effect of the mentioned toroidal coupling does not depend on the beam direction. Therefore, when this effect exceeds the influence of the finite orbit width on the mode, some decrease of the instability growth rate can take place also during counter-injection.

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Figures

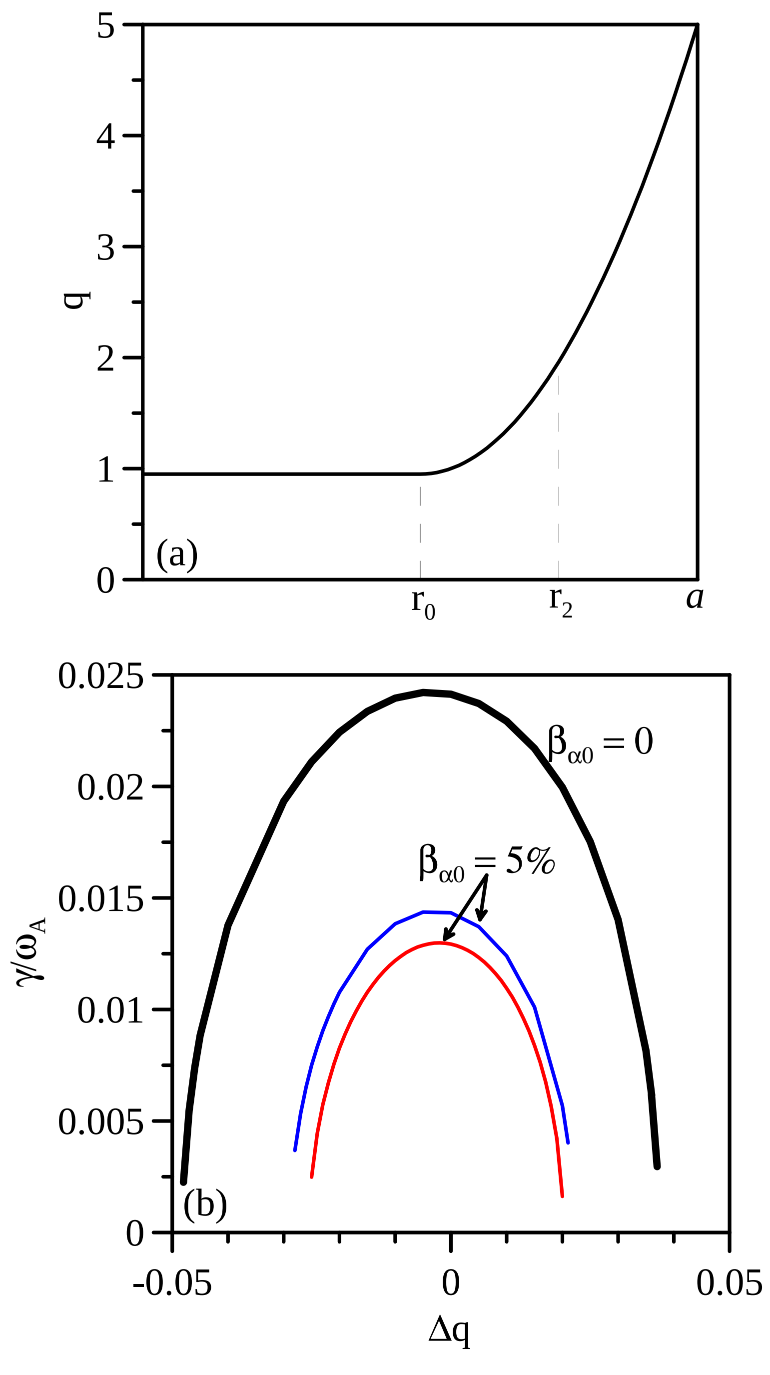

FIG. 1: Safety factor profile $q(r)$ with $q_0 < 1$ (Fig. 1a) and the QI instability growth rate for this profile but various q_0 ($\Delta q \equiv 1 - q_0$) (Fig. 1b) in a small-aspect-ratio tokamak (ST) with $A = 2$, $q_a = 5$, and $r_0/a = 0.5$. The other parameters used: $l_1 = 2$, $l_2 = 4$, $\beta_0 = 0.1$, $\rho_\alpha/a = 0.05$. Blue line corresponds to the balanced injection, red line corresponds to the co-injection.

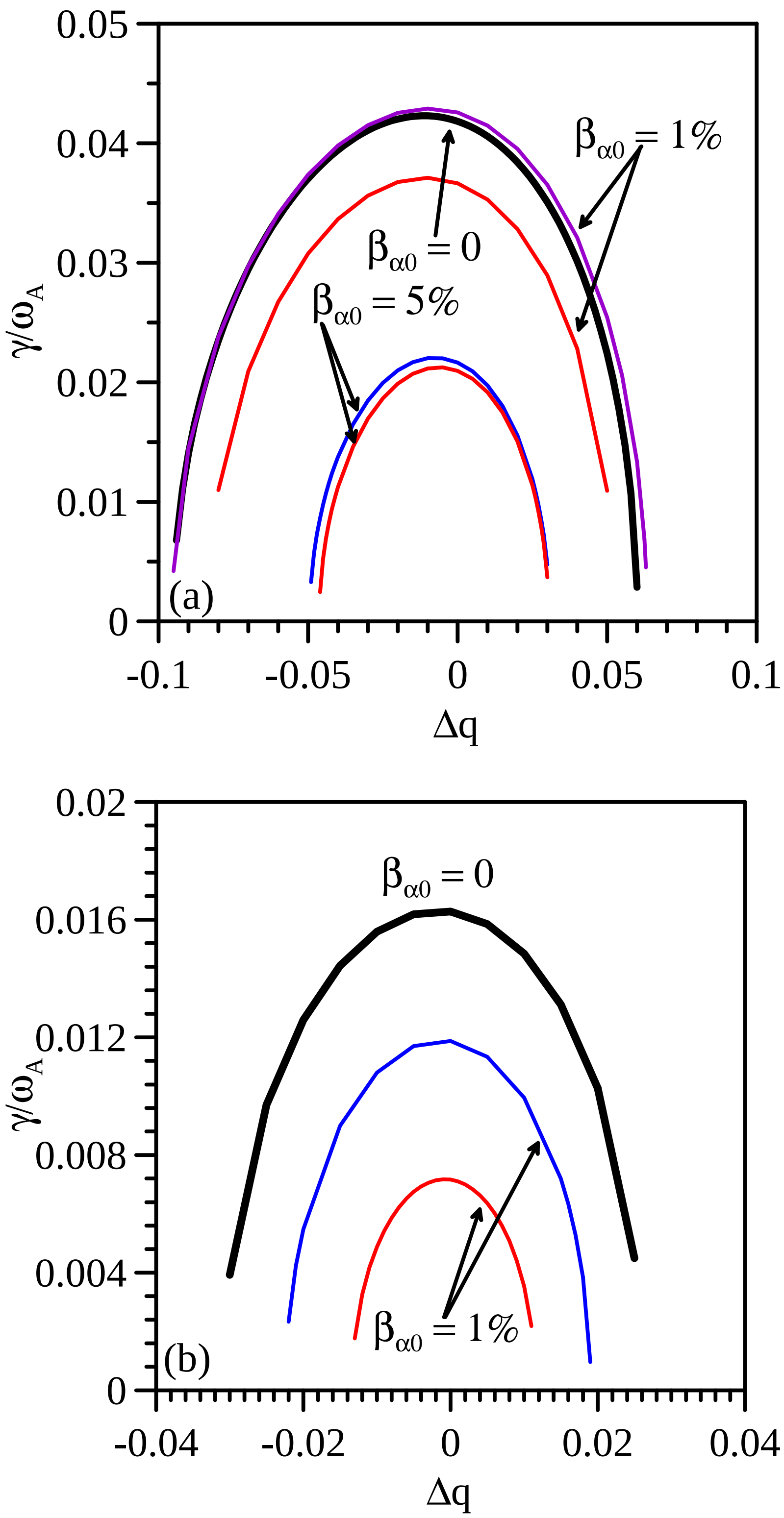

FIG. 2: The QI instability growth rate versus $\Delta q \equiv 1 - q_0$ in a tokamak (CT) with $A = 4$, $q_a = 3$: a, $r_0/a = 0.5$; b, $r_0/a = 0.3$, the other parameters are the same as in Fig. 1. Purple line, blue line, and red line correspond to the counter-injection, balanced injection, and co-injection, respectively.

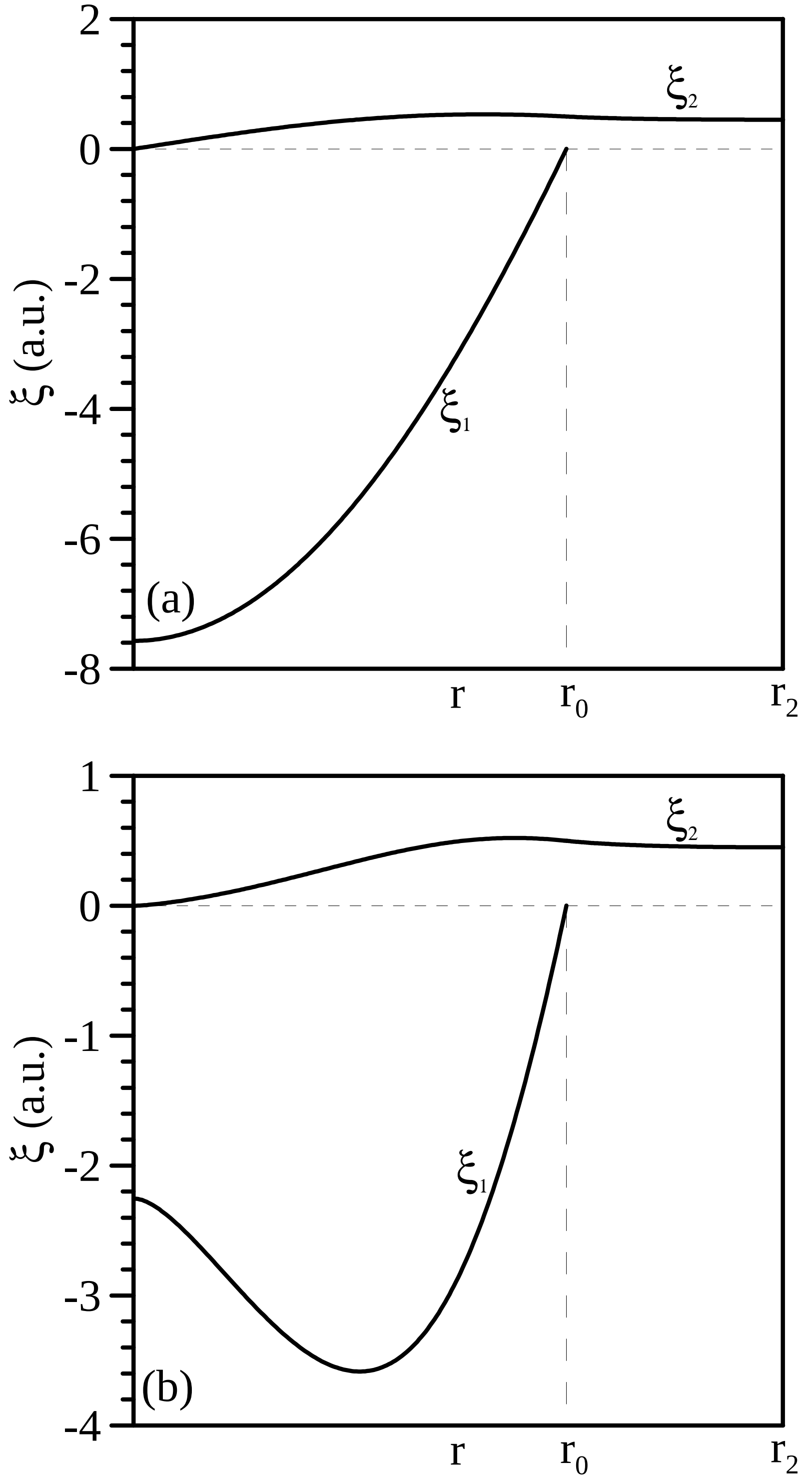

FIG. 3: Eigenfunctions $\xi_1(r)$ and $\xi_2(r)$ in a small-aspect-ratio tokamak: a, in the absence of the energetic ions; b, during a balanced injection. The used parameters are the same as in Fig. 1.

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