PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76CH03073

PPPL-4002
UC-70

PPPL-4002

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September 2004

PPPL
PRINCETON PLASMA
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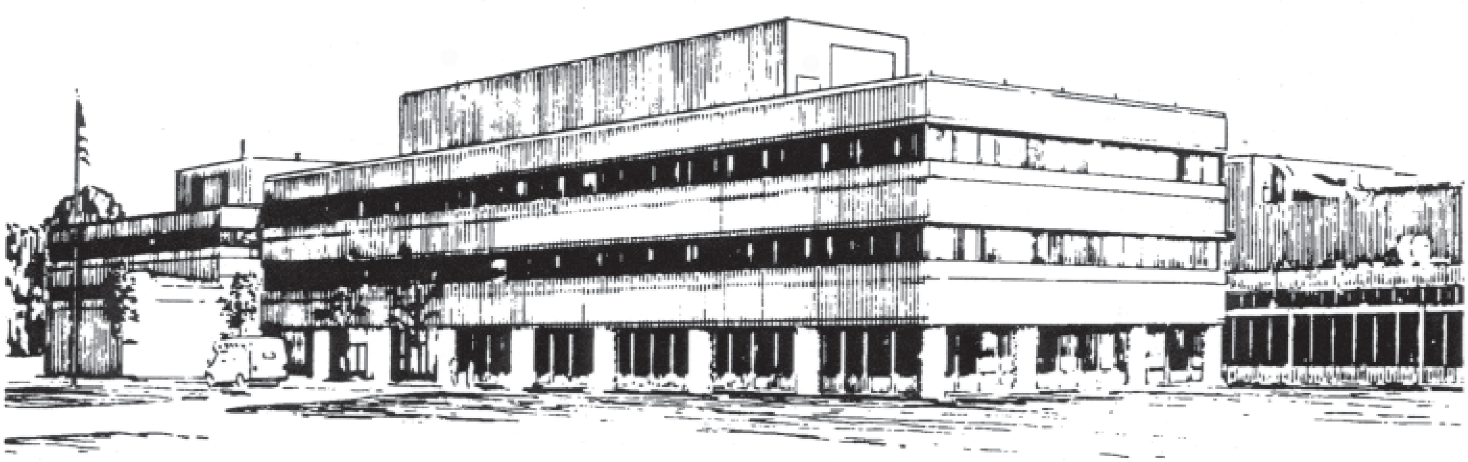

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LETTER TO THE EDITOR

Finite pressure effects on reversed shear Alfvén eigenmodes

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Abstract. The inclusion of finite pressure in ideal magnetohydrodynamic (MHD) theory can explain the Reversed magnetic Shear Alfvén Eigenmodes (RSAE) (or Alfvén cascades) that have been observed in several large tokamaks without the need to invoke energetic particle mechanism for the existence of these modes. The chirping of the RSAEs is cased by changes in in the minimum of the magnetic safety factor, $q_{\rm min}$, while finite pressure effects explains the observed non-zero minimum frequency of the RSAE when $q_{\rm min}$ has a rational value. Finite pressure effects also play a dominant role in the existence of the downward chirping RSAE branch.

Submitted to: *Plasma Phys. Control. Fusion*

1. Introduction

In advanced Tokamak scenarios with very weak or inverted magnetic shear profiles a class of Alfvén eigenmodes can be excited in the weak shear region near the minimum of the magnetic safety factor or q-profile. These modes are called reversed shear Alfvén eigenmodes (RSAE) [1, 2] and have been reported in various tokamaks: JT-60U [3, 4], Alcator C-mod [5], JET [6] (where they are called Alfvén cascades), and TFTR [7]. These modes often appear in discharges that are designed to study advanced plasma scenarios for future burning plasma experiments such as the International Thermonuclear Experimental Reactor (ITER) and the Fusion Ignition Research Experiment (FIRE).

One of the hallmarks of these modes is that their frequencies change on a fast time scale: usually they chirp up in frequency from below one-third of the Toroidicity induced Alfvén Eigenmode (TAE) [8] frequency to the TAE frequency on a ten to a hundred millisecond time scale. This behavior is due to the decrease of the minimum value of the magnetic safety factor, $q_{\rm min}$, as reported in [9]. In fact, the RSAEs have been used to monitor the evolution of $q_{\rm min}$ during the reversed shear phase of plasma discharges [10].

The mode structure of the RSAEs is close to a cylindrical mode with toroidal mode number n and poloidal mode number m. For such a cylindrical mode the frequency at a given $q_{\min}$ is given by

$$\omega \approx k_{\|} V_A = (m - nq_{\min})V_A/q_{\min}R \tag{1}$$

with V_A the Alfvén velocity and R the major radius of the tokamak. When $q_{\min}$ is rational, i.e. $q_{\min} = q_{nm} = m/n$, then the mode frequency is zero. The TAE gaps for toroidal mode number n near $q_{\min}$ are located at $q_{\mathrm{TAE}} = q_{nm} \pm 1/2n$ and the TAE frequency is given by $\omega_{\mathrm{TAE}} \approx V_A/2q_{\mathrm{TAE}}R$. When $q_{\min}$ decreases from q_{nm}, the mode frequency chirps up almost linearly with $q_{\min}$ to the TAE frequency. Experimentally however, the minimum frequency that is observed for the RSAEs at rational $q_{\min}$-surfaces is not zero but usually of the order of 20 to 40% of the TAE frequency (see experimental results in [3, 4, 6]). We show that ideal magnetohydrodynamic (MHD) theory with finite plasma pressure accounts for the frequency of the RSAEs at the rational $q_{\min}$ surfaces, as well as the existence of down chirping modes.

These modes have been studied theoretically in the past. In one of the first papers to explain these RSAEs it was concluded that i) the mode frequency follows the frequency of the lower Alfvén continuum at the shear reversal point, and that ii) the only viable option to explain the observed RSAEs is the energetic particle mechanism as presented in [9]. More recently, it was shown that there are two complementary mechanisms for establishing RSAEs: i) an energetic particle mechanism as described previously [9] and a geometrical mechanism due to second order effects in the inverse aspect ratio [11]. In [11], however, only the limiting case of zero plasma pressure and high toroidal mode numbers was considered and only the coupling between three adjacent poloidal harmonics was taken into account. This did not explain the non-zero RSAEs frequencies that are observed at rational q-surfaces and the disappearance of the RSAEs when it reaches the TAE gap as is sometimes observed. Moreover, the theory predicts that downward chirping RSAEs only exist in the presence of an energetic particle population.

In this paper we will show that the RSAE can exist without energetic particle effects, that the non-zero minimum RSAE frequency is caused by finite plasma pressure effects and that the downward chirping RSAE can exist in ideal magnetohydrodynamic (MHD) theory when the plasma-β is non-zero.

2. Simulation of RSAEs

For studying the behavior of the minimum RSAE frequency at rational $q_{\min}$-surfaces we have used the the nonvariational ideal MHD code NOVA [12] with the following plasma geometry: major radius, $R = 3.0$ m, minor radius, $a = 1.0$ m, with a circular cross section, and toroidal magnetic field, $B_T = 3.0$ T. The used density, pressure, and weakly reversed q-profiles are shown in fig. 1. The pressure profile was scaled to obtain the requested volume-averaged β ($\langle\beta_0\rangle$) values.

In fig. 2 the normalized frequency of the RSAE (normalized to $\omega_0 = V_A(r/a = 0)/2Rq(r/a = 1)$ with $V_A(r/a = 0)$ the Alfvén velocity at the plasma center and

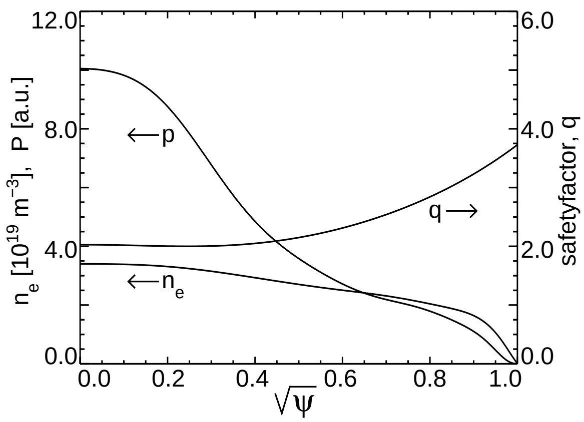

Figure 1. Density, pressure, and q-profile that were used in the model plasma simulations.

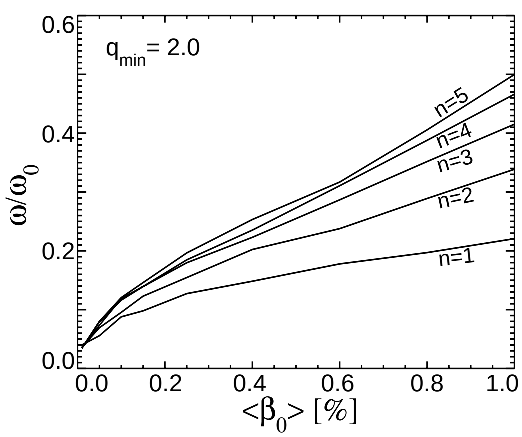

Figure 2. The minimum RSAE frequency as function of the volume-averaged β for n=1 to 5 at $q_{\rm min} = 2.0$.

$q(r/a = 1)$ the magnetic safety factor at the edge) is shown for toroidal mode numbers between one and five when $q_{\rm min} = 2.0$. From this figure it can be seen that the mode frequency at integer-q values scales with the plasma pressure. Note also that there is a difference in the minimum frequency for different toroidal mode numbers.

The variation of the RSAE frequency with $q_{\rm min}$ is shown in fig. 3 for an $n = 3$ mode at $\langle\beta_0\rangle$ of 0.01, 0.25 and 1.0%. The variation in $q_{\rm min}$ was obtained by adding a constant offset to the q-profile with $q_{\rm min} = 2.0$.

From fig. 3 it can be seen that the minimum RSAE frequency at $q_{\rm min} = 2$ is obtained for the lowest $\langle\beta_0\rangle$. Not only the β-dependence of the minimum RSAE frequency is visible but also the β-dependence of the TAEs to which the RSAE evolves when $q_{\rm min}$ decreases. Note also that the minimum frequency of the RSAE is the lowest at the lowest β while the TAE frequency is the highest for the lowest β [13].

The up chirping RSAE exists for all the different β values and at low β the frequency is very close to the frequency for the cylindrical mode as given in eq. 1. The down-

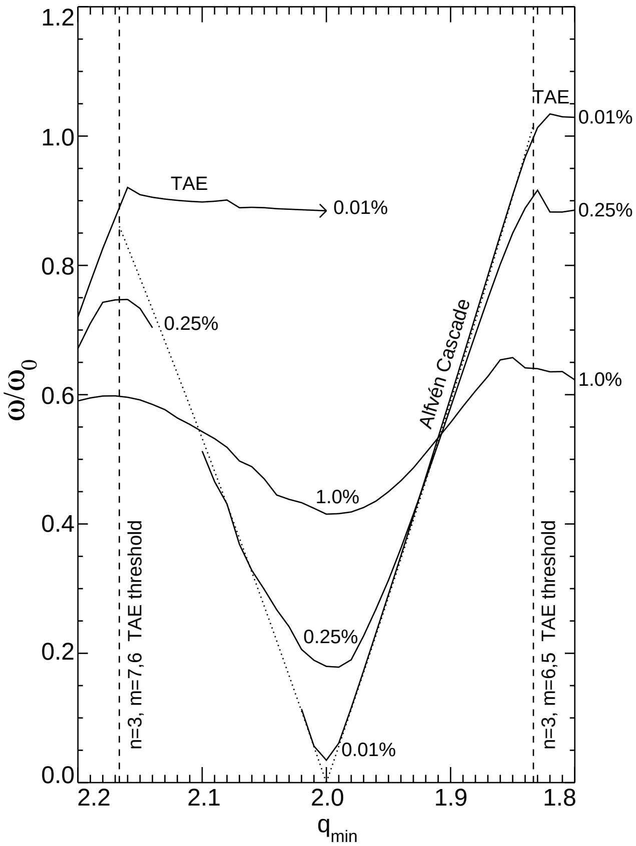

Figure 3. Normalized RSAE frequency as a function of $q_{\min}$ for an $n = 3$ mode for volume-averaged β of 0.01, 0.25 and 1.0%. The dotted lines are the predicted frequencies for a cylindrical mode at zero β. No eigenmode solutions were found in the gaps of the curves for $\langle \beta_0 \rangle = 0.01$ and 0.25%.

chirping RSAE does not exist when β is very low (0.01% in fig. 3) which is in agreement with the results of ref. [11] where only plasmas with zero pressure were considered. When β is increased (to 0.25%) the down chirping branch of the RSAE starts to appear in ideal MHD theory with $q_{\min}$ slightly above a rational value (m/n). When β is increased further, the range of $q_{\min}$ where the mode is chirping down becomes larger and at high enough β the mode is observed over the entire range of $q_{\min}$ values (1.0% in fig. 3). Thus, the downward chirping RSAE exist in ideal MHD theory when the plasma-β is sufficiently high.

3. Analytical model

The theory as presented in [11] does not provide a solution for an eigenmode in the case of a rational $q_{\min}$ because at low frequencies no potential well is formed. From our simulations we have found that the non-zero minimum RSAE frequency depends on the plasma pressure. Compressibility effects as discussed in [14] for beta-induced Alfvén eigenmodes don't play a role as we have found the RSAE at rational $q_{\min}$ values solutions when $\gamma = 0$. Thus the only posability for the RSAE to exist is allowing neighboring harmonics to create an effective potential well trough toroidicity and pressure effects. To demonstrate this we evaluate the eigenmode equation with pressure and toroidicity effects and the following q-profile:

$$q(r) = q_{\min}/(1 - (r - r_{\min})^2/w_q^2) \tag{2}$$

where $q_{\min}$ is located at $r = r_{\min}$ and $q_{\min}/w_q^2$ is the second derivative of $q(r)$ at $q_{\min}$. Under the assumption of radial localization of the mode structure near $r_{\min}$ (in the limit of high m) the set of three coupled eigenmode equations becomes [15, 16]:

$$\frac{\partial}{\partial x} D_m \frac{\partial}{\partial x} \Phi_m - D_m \Phi_m + L_+ \Phi_{m+1} + L_- \Phi_{m-1} = 0 \tag{3}$$

$$\frac{\partial}{\partial x} D_{m+1} \frac{\partial}{\partial x} \Phi_{m+1} - D_{m+1} \Phi_{m+1} + L_- \Phi_m = 0 \tag{4}$$

$$\frac{\partial}{\partial x} D_{m-1} \frac{\partial}{\partial x} \Phi_{m-1} - D_{m-1} \Phi_{m-1} + L_+ \Phi_m = 0 \tag{5}$$

with

$$D_m = \omega^2 - \omega_{A0}^2 (n - m/q)^2, \tag{6}$$

$$L_\pm = 2\omega^2 (\epsilon + \Delta') \frac{\partial^2}{\partial x^2} - \omega_{A0}^2 \frac{r\beta'}{2\epsilon} (-1 \pm \frac{\partial}{\partial x}), \tag{7}$$

$x = (r - r_{\min}) m / r_{\min}$, $\omega_{A0} = V_{A0}/R$ the central Alfvén frequency, ϵ the inverse aspect ratio, Δ' the Shafranov shift parameter, and β' the radial derivative of β. From the eigenmode equations (eqs. 3-5) and the expression for D_m (eq. 6) it can already be seen that at $q = q_{\min} = m/n$ and $r = r_{\min}$ the mode with poloidal harmonic m dominates because $D_m = \omega^2$ is much smaller than the absolute value of $D_{m\pm1} = \omega^2 - \omega_{A0}^2/q_{\min}^2$. It is instructive to use a WKB analysis to demonstrate the exisitence of the eigenmode solution:

$$\Phi_m = \exp(-i \int k(x)\, dx), \tag{8}$$

with the radial wave number, k, much larger than the spatial variation of D_m. Using eq. 4 we can express Φ_{m+1} in terms of Φ_m at the mode location:

$$(k^2 + 1) \Phi_{m+1} = L_- \frac{q^2}{\omega_{A0}^2} \Phi_m \tag{9}$$

and similarly, from eq. 5, Φ_{m-1} can be expressed in terms of Φ_m at the mode location:

$$(k^2 + 1) \Phi_{m-1} = L_+ \frac{q^2}{\omega_{A0}^2} \Phi_m. \tag{10}$$

Substituting eqs. 9 and 10 into eq. 3 we obtain the (WKB) dispersion relation:

$$D_m (k^2 + 1)^2 - 2 L_+ L_- \frac{q^2}{\omega_{A0}^2} = 0. \tag{11}$$

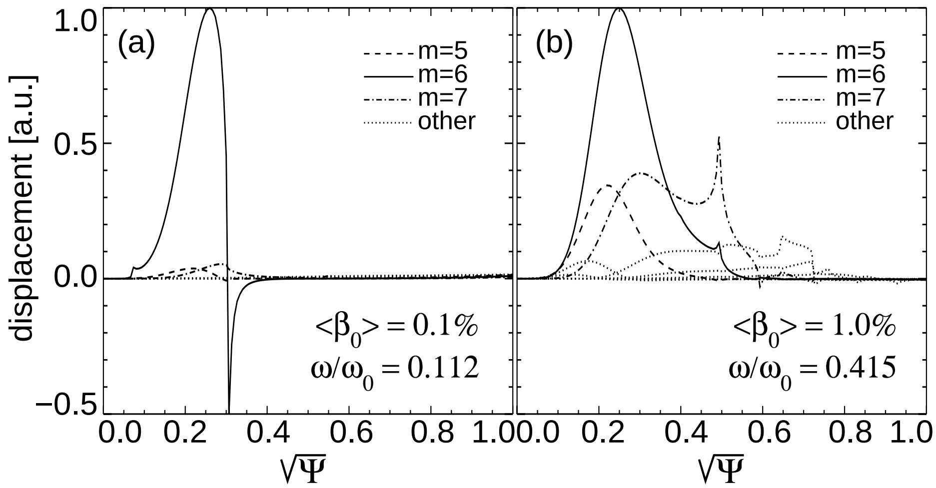

Figure 4. NOVA eigenmode solutions for the $n = 3$ RSAE at $q_{\min} = 2.0$ for (a) $\langle\beta_0\rangle = 0.1\%$ and (b) $\langle\beta_0\rangle = 1.0\%$. Note the relative increase in the sideband harmonics when β increases.

If we neglect the toroidal coupling term in the expression for $L_\pm$ (the first term in eq. 7 which is typically small), the dispersion relation becomes:

$$k^2 = 2\frac{q^2\omega_{A0}^2}{D_m}(\frac{r\beta'}{2\epsilon})^2 - 1. \tag{12}$$

One can see from this expression that wavelike solutions exist since at the bottom of the potential well, D_m is proportional to ω^2 and at sufficiently small frequencies k^2 is positive. We can see also from eq. 12 that the eigenmode frequency is proportional to the plasma pressure as was found in the simulations. The above solution implies the appropriate quantization condition:

$$\int_{x_1}^{x_2} k(x)\, dx = \pi \tag{13}$$

where the integration boundaries (x_1, x_2) limits the domain where k^2 is positive.

In practical cases, however, the WKB approximation is not satisfied near the integration boundaries because k becomes the dominant scale length there. When we search for WKB-eigenfrequencies, however, by integrating eq. 13, which can be done despite the the singularities at the integration boundaries, we find that the WKB-eigenfrequencies are proportional to β', which is similar to the simulation results (fig. 2). We also obtain the same behavior of the WKB-eigenfrequencies with n, the lowest n has the lowest frequency at constant β' and the frequency between two successive n's decreases with increasing mode number (fig. 2). Finally, the values we obtain for the WKB-eigenfrequencies are of the same order of magnitude as found in the simulations.

To illustrate the importance of coupling between the dominant poloidal harmonic and the sideband harmonics we show in figure 4 the NOVA solutions for an $n = 3$ RSAE with $q_{\min} = 2.0$ at $\langle\beta_0\rangle$ is 0.1% and 1.0%. It can be seen that the sideband harmonics increase relative to the main harmonic with increasing $\langle\beta_0\rangle$ as was found above.

4. Conclusion

In this paper we have shown that the up and down chirping RSAEs exist in ideal MHD without having to invoke energetic particle mechanisms for the existence of these modes when finite β-effects of the background plasma are taken into account. A finite plasma-β also resolves the discrepancy between the expected zero frequency and the observed non-zero frequency when $q_{\rm min}$ is at a rational value. This is accordance with experiments where RSAE frequencies have been measured at rational q-surfaces between 20 and 40% of the TAE frequency [3, 4, 6]. The minimum RSAEs frequency at rational q-surfaces might be used as a measure of the plasma-β because it scales with the plasma pressure. The down-chirping branch of the RSAEs is also found to exist in ideal MHD when β is sufficiently large.

Acknowledgments

This work was supported by DOE Contract No. DE-AC02-76-CH03073

References

[1] A. Fukuyama and E. Yokota, Proceedings of the 6th IAEA Technical Committee Meeting on Energetic Particles in Magnetic Confinement Systems, 1999, Naka (Japan Atomic Energy Research Institute, 2000) 61

[2] A. Fukuyama and T. Akutsu, Proceedings of the 19th IAEA Fusion Energy Conference, 2002, Lyon (IAEA, 2003), TH/P3-14.

[3] Y. Kusama, H. Kimura, T. Ozeki, M. Saigusa, G.J. Kramer, T. Oikawa, S. Moriyama, M. Nemoto, T. Fujita, K. Tobita, G.Y. Fu, R. Nazikian, and C.Z. Cheng 1998 Nucl. Fusion **38** 1215

[4] H. Kimura, Y. Kusama, M. Saigusa, G.J. Kramer, K. Tobita, M. Nemoto, T. Kondoh, T. Nishitani, O. da Costa, T. Ozeki, T. Oikawa, S. Moriyama, A. Morioka, G.Y. Fu, C.Z. Cheng, and V.I. Afanas'ev 1998 Nucl. Fusion **38** 1303

[5] J.A. Snipes, A. Fasoli, P. Bonoli, S. Migliuolo, M. Porkolab, J.E. Rice, Y. Takase, and S. Wolfe 2000 Plasma Phys. Contr. Fusion **42** 381

[6] S.E. Sharapov, D. Testa, B. Alper, D.N. Borba, A. Fasoli, N.C. Hawkes, R.F. Heeter, M. Mantsinen, M.G. Von Hellermann, and contributors to the EFDA-JET workprogramme 2001 Phys. Lett. A **289** 127

[7] R. Nazikian, G.J. Kramer, C.Z. Cheng, N.N. Gorelenkov, H.L. Berk, and S.E. Sharapov 2003 Phys. Rev. Lett. **91** 125003

[8] C.Z. Cheng and M.S. Chance, 1986 Phys. Fluids **29** 3695

[9] H.L. Berk, D.N. Borba, B.N. Breizman, S.D. Pinches, and S.E. Sharapov 2001 Phys. Rev. Lett. **87** 185002

[10] E. Joffrin, G. Gorini, C.D. Challis, N.C. Hawkes, T.C. Hender, D.F. Howell, P. Maget, P. Mantica, D. Mazon, S.E. Sharapov, G. Tresset, and contributors to the EFDA-JET Workprogramme 2002 Plasma Phys. Contr. Fusion **44** 1739

[11] B.N. Breizman, H.L. Berk, M.S. Pecker, S.D. Pinches, and S.E. Sharapov 2003 Phys. Plasmas **10** 3649

[12] C.Z. Cheng and M.S. Chance, 1987 Journ. Comp. Phys. **71**, 124

[13] G.Y. Fu and C.Z. Cheng, 1990 Phys. Fluids B **2** 985

[14] M.S. Chu, J.M. Greene, L.L. Lao, A.D. Turnbull, and M.S. Chance, 1992 Phys. Fluids B **4** 3713

[15] H.L. Berk, J.W. Van Dam, Z. Guo, and D.M. Lindberg, M.S. Chance, 1992 Phys. Fluids B **4** 1806

[16] C.Z. Cheng, N.N. Gorelenkov and C.T. Hsu, 1995 Nucl. Fusion **35** 1639

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