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by

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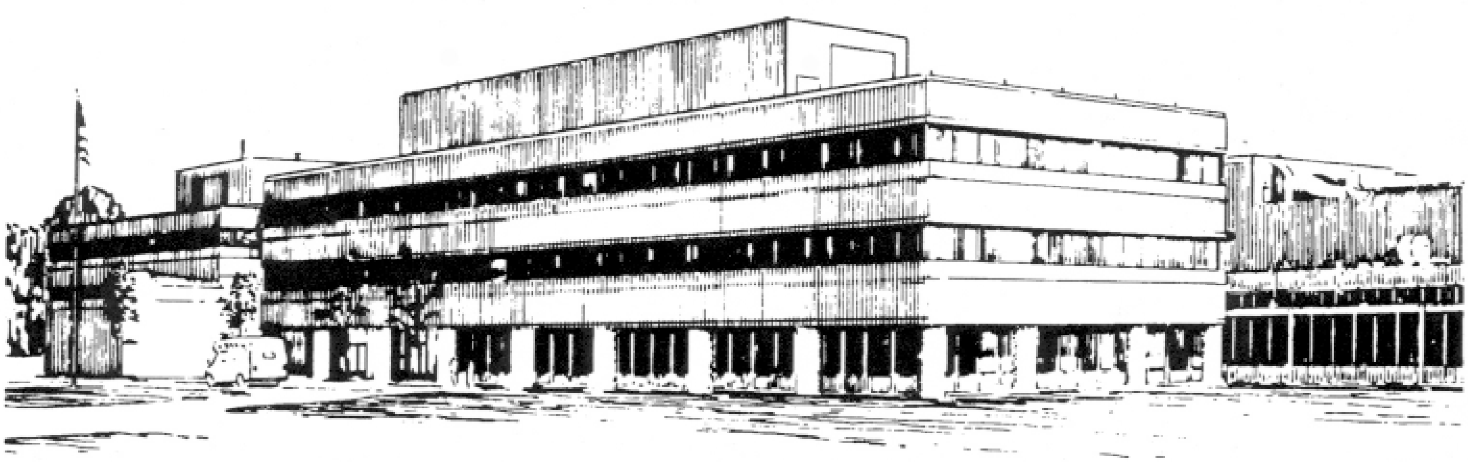

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LETTER TO THE EDITOR

Effects of 2D and finite density fluctuations on O-X correlation reflectometry

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Abstract. The correlation between O-mode and X-mode reflectometer signals is studied with a 1D and 2D reflectometer model in order to explore its feasibilities as a q-profile diagnostic. It was found that 2D effects and finite fluctuation levels both decrease the O-X correlation. At very low fluctuation levels, which are usually present in the plasma core, there is good possibility to determine the local magnetic field strength and use that as a constraint for the equilibrium reconstruction.

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1. Introduction

An essential physics parameter to determine in magnetic confinement systems is the plasma rotational transform or magnetic safety factor, q. In addition, the corrections to the toroidal magnetic field due to the finite pressure of the plasma is of interest, particularly in high-beta systems which are expected to produce a magnetic well. The standard method currently in use for q-profile measurements is the Motional Stark Effect (MSE) [1, 2] which provides a constraint on the magnetic field line pitch angle along the trajectory of a diagnostic neutral beam. Other methods make use of Faraday rotation [3] of a line integrated measurement or even the excitation of Alfvén Eigenmodes [4, 5, 6]. The MSE technique has been very effective in extracting equilibrium profiles, however, the need to develop alternative approaches is also necessary when considering the challenge of measuring the equilibrium profile in systems where a diagnostic neutral beam is technically difficult to implement or very costly.

In two recent articles [7, 8] an alternative approach to magnetic equilibrium reconstruction has been proposed based on the reflection of microwaves of different polarizations from a magnetized plasma. In this method the ubiquitous presence of turbulence in the plasma column is used by measuring the peak correlation in the reflected signal between two modes of orthogonal polarization, the ordinary or O-mode and the extraordinary or X-mode is shown schematically in figure 1. The correlation peaks near to where the two cut off layers coincide. From the frequency difference between the two orthogonal polarizations at the peak in their correlation the local magnetic field strength can be obtained. The reflection layer of the O-mode depends only on the plasma density whereas the reflection location of the X-mode depends both on the density and the magnetic field strength and weakly on the plasma temperature. By maximizing the correlation of the scattered signals between a fixed frequency O-mode channel and an X-mode channel that is scanned in frequency, a correlation as a function of the frequency separation can be obtained. The O-mode reflection layer can be obtained from the measured density profile and the wave frequency. From the frequency separation between the O- and X-mode at the peak of the cross correlation the local electron cyclotron frequency can be obtained which depends on the local magnetic field strength.

If this field strength can be resolved with sufficient precision, then the deviation of the magnetic field in the plasma from the vacuum field can be resolved. This deviation from the vacuum field can then be used as a constraint in an equilibrium solver in much the same way that MSE is used to constrain the numerical equilibrium.

The O-X correlation method was studied extensively at the LArge Plasma Device (LAPD) [9] at magnetic fields up to 0.18 T, electron densities up to $3.0\,10^{18}$ m^{-3} and density scale lengths between 0.05 and 0.20 m. Experimentally, it was found that the X-mode frequency corresponding to the maximum correlation did not coincide with the X-mode frequency at the O-mode reflection layer but was shifted toward lower frequencies than calculated from independent measurements of the magnetic field. This shift was reproduced well with a 1D full wave model taking into account the measured density correlation lengths [7]. In addition a degradation of the maximum coherence was observed with increasing magnetic field strengths. Experimentally, it was found that the decrease of the coherence depends on the magnetic field as was shown in figure 9 of reference [7]. From the decrease of the coherence with the magnetic field strength the applicability of this method for magnetic field measurements in fusion scale plasmas needs careful consideration. An important question that we address in this paper is whether the degradation in the correlation is a feature of 2D scattering or whether the decrease in the O-X correlation can be accounted for within the context of a 1D full wave analysis.

2. 1D and 2D modeling

Recently, it was found that, due to the finite plasma curvature, a 1D approach works very well for studying plasma turbulence with microwave reflectometry [10]. In the following we will study the O-X correlation measurements with both a full wave 1D and 2D model in order to investigate the influence of 2D and finite fluctuation effects to O-X correlation measurements.

The 1D simulation code is based on a tri-diagonal implicit method [11] in which the scalar wave equation

$$\left(\frac{\partial^2}{\partial x^2} + k_0^2 \varepsilon(x)\right) E(x) = 0$$

for the electrical field, $E(x)$, is solved for a wave with a vacuum wave vector $k_0 = 2\pi/\lambda_0$ (λ_0 is the vacuum wave length). This wave equation is valid for waves that propagate perpendicular to the magnetic field. The upper and lower X-mode permittivity, $\varepsilon(x)$, is given by

$$\varepsilon = 1 - \frac{(U-X)X}{(U-X)U - Y^2}$$

with $X = (\omega_{pe}/\omega)^2$, $Y = \omega_{ce}/\omega$, $U = 1 - \mathrm{i}\nu_{ei}/\omega$, $\omega_{pe} = (n_e e^2/\varepsilon_0 m_e^*)^{1/2}$ the plasma frequency, $\omega_{ce} = eB/m_e^*$ the electron cyclotron frequency and ν_{ei} the electron-ion collision frequency. The O-mode permittivity is obtained when Y is zero. The finite electron-ion collision frequency is included to account for the pole at the upper hybrid resonance. The effective electron mass, $m_e^* = m_e(1 + 5\,T_e/511)^{1/2}$, is corrected for finite electron temperature effects as outlined in [12, 13] (T_e the electron temperature in keV).

For the solution of the wave equation an incoming and outgoing wave are taken in the vacuum region outside the plasma as

$$E(x) = \exp(\mathrm{i}k_0 x) + \exp(-\mathrm{i}(k_0 x + \phi))$$

and integrated into the evanescent region beyond the cut-off to determine the (complex) phase, ϕ, which reflects the density fluctuations [14].

For the 2D simulations the time dependent wave equation

$$\left(\nabla^2 + k_0^2 \varepsilon(x,y)\right) E(x,y,t) = i\omega \frac{\partial E(x,y,t)}{\partial t}$$

is solved until a steady state solution ($\partial E(x,y,t)/\partial t = 0$) is reached [15]. This 2D code was originally developed to simulate reflectometer experiments in large tokamaks where the distance between the antenna and the reflecting layer is typically 500 or more vacuum wavelenghts of the probing beam. The full plasma geometry is used together with experimental density, magnetic, and temperature profiles. In this code a free space Green's function is used to project wavefields between the antenna and plasma boundary. Then a paraxial approximation is used from edge to a few (typical 5 to 10) wavelengths from the cut off layer. The last section to beyond the cut off layer is solved with an iterative line Jacobi method. The details of the algorithms used in this code can be found in [15]. The complex 2D phase is extracted at the antenna plane by folding the scattered wave field with the receiving antenna aperture.

3. Density fluctuation effects

For the simulation of the 1D density fluctuations we have added to the equilibrium profile a spectrum of fluctuations which gives the following density correlation function

$$\langle \tilde{n}(x_2)\,\tilde{n}(x_1)\rangle / n^2 = \left(\frac{\tilde{n}}{n}\right)^2_{x_1} \exp(-(x_2 - x_1)^2/\lambda_c^2)\cos(k_{\mathrm{fl}}(x_2 - x_1)) \qquad (1)$$

with $(\tilde{n}/n)_{x_1}$ the fluctuation amplitude, x_1 (x_2) the fixed (variable) frequency cut off layer position, λ_c the 1/e width of the correlation and $k_{\text{fl}} = 2\pi/\lambda_{\text{fl}}$ where λ_{fl} is the characteristic fluctuation wave length. In a similar manner a 2D spectrum of density fluctuations (spectral locations k_x and k_y and widths Δk_x and Δk_y) is added to the 2D density equilibrium profile for the 2D calculations.

A set of N (typically 500) random density distributions, $\tilde{n}(x)$, are generated from the correlation function (equation 1). The wave equation is then solved numerically for each of these distributions to generate an ensemble of complex phases, ϕ, for the outgoing wave. This is repeated for each frequency and each mode of propagation. Then the correlation of the complex signal of the outgoing waves, $E = \exp(\mathrm{i}\phi)$, is obtained where the normalized correlation, γ, is given by

$$\gamma = \frac{|\langle E_O E_X \rangle|}{\sqrt{\langle |E_O|^2 \rangle \langle |E_X|^2 \rangle}}$$

which is depicted in figure 2 for a 1D and 2D case at a low and a high fluctuation level. Both the 1D and 2D normalized correlations are used in the following to study the effects of finite density fluctuation levels for parameters similar to those obtained in the LAPD experiments [7]: the density scale length, $L_n = 20$ cm, the fluctuation width for the 1D simulations $\lambda_c = 1.57$ cm, and $k_{\text{fl}} = 0$ m^{-1}. For the 2D fluctuation spectra we have taken: $k_x = k_y = 0$ m^{-1} and $\Delta k_x = \Delta k_y = 2/\lambda_c = 1.274$ cm^{-1}. We have scanned fluctuation level, $\tilde{n}/n$, at two magnetic fields: $B = 0.10$ and 0.18 T. The results of the 1D and 2D calculations are shown in figure 3.

From figure 3 (a) it can be seen that the decorrelation between the X-mode and O-mode signals decreases with increasing fluctuation levels and the decorrelation is stronger at higher magnetic field strengths. At low fluctuation levels the 1D model gives normalized correlations close to unity and a gradually decrease to around 0.5 and 0.3 for $B = 0.10$ T and $B = 0.18$ T respectively. In the 2D model the peak O-X correlations are already reduced at low fluctuation levels, 0.87 and 0.80 for $B = 0.10$ T and $B = 0.10$ T respectively, and they drop to similar values as the 1D model at high fluctuation levels.

The location where the correlation peaks depends also on the fluctuation level as is shown in figure 3 (b). The position moves away from the equilibrium fixed channel location towards the antennas when $\tilde{n}/n$ increases. At low fluctuation levels (less than 3%) the 1D and 2D calculations show a similar shift away from the fixed channel location. This shift is consistent with the one reported in [7] and was explained successfully with phase matching criteria of the X- and O-mode responses. At higher fluctuation levels the peak of the correlation shifts away from the fixed channel position. This shift is much stronger in the 2D than in the 1D calculations.

There are two important mechanisms for the decorrelation between the X- and O-mode signals: finite fluctuation effects and effects due to differences in the scattered O- and X-mode wave fields.

The finite fluctuation effects are present in both the 1D and 2D calculations and can be explained as follows. The excursions of the O- and X-mode reflection points at the maximum correlation due to low fluctuation levels are very similar as can be seen in figure 4 (a) where $\tilde{n}/n = 0.01$. The O-mode reflection point is distributed symmetrically around the equilibrium reflection point, whereas the distribution of the X-mode reflection point is shifted as can be seen in figure 4 (b). At high fluctuation levels these distributions broaden and shift away from the equilibrium location toward the reflectometer antennas as is shown in figure 4 (d) for $\tilde{n}/n = 0.20$. The location of the O- and X-mode reflection points decorrelate as can be seen from the large scatter above the diagonal line in figure 4 (c), which is consistent with the decreased O-X correlation.

The decorrelation due to the difference in scattered wave fields is only present in the 2D calculations down to very low fluctuation levels. In figure 5 the permittivity for O- and X-mode are shown (top and bottom left respectively) together with the scattered O- and X-mode wave field power (right) for the same fluctuation realization. The evanecent region ($\varepsilon \leq 0$) is indicated in black and the upper hybrid resonance zone is shown in white for the X-mode case. From this figure it can be seen that there are differences in the O- and X-mode permittivity which are reflected in the scattered power fields.

For a correct determination the local magnetic field strength from O-X correlation measurements, 2D calculations are preferable because they yield a significant decrease of the maximum correlation, even at low fluctuation levels. The big disadvantage of the 2D calculations is that they are extremely time consuming (about 6 months of CPU time on a 750 MHz alpha processor was used for figure 3). At low fluctuation levels the position of the peak correlation is reproduced well with the 1D model which runs much faster than the 2D calculations. For an O-X correlation diagnostic of the local magnetic field strength it is preferable to have a low level of fluctuations (less than 3%) so that the location of the peak of the correlation can be calculated from a much faster 1D code instead of a slow 2D code.

4. Accuracy

Another important question is how accurate the magnetic field strength can be measured from O-X correlations. For this we have to investigate how accurate the top of the O-X peak correlation can be determined and how this translates into an uncertainty for the magnetic field strength.

The accuracy for the magnetic field strength measurements can be expressed as

$$\frac{|\Delta \mathrm{B}|}{|\mathrm{B}|} = \frac{\Delta\omega_X}{\omega_X}\left(\frac{\omega_X}{\omega_{ce}} + \frac{\omega_{pe}^2}{\omega_{ce}\omega_X}\right)$$

with ω_X the measured peak correlation X-mode frequency and $\Delta\omega_X$ its uncertainty. For large aspect ratio tokamaks with $\omega_{pe}^2 \ll \omega_{ce}^2$ (and $\omega_X \approx \omega_{ce}$) the relative uncertainty in the X-mode frequency is identical to the relative uncertainty in the magnetic field measurement. It is technically well possible to determine the peak of the O-X correlation with an accuracy of better than 0.1% which is needed to observe deviations from the vacuum magnetic field in tokamaks.

5. Conclusions

We have investigated the feasibility to use O-X correlations for the determination of the local magnetic field strength in plasmas. It was found that 2D effects decrease peak correlation significantly compared to 1D calculations when the density fluctuations are low. At high fluctuation levels the reduction of the peak correlation was found to be similar for the 1D and 2D calculations. The location of the peak correlation, which is important for the deduction of local magnetic field strength, in 1D was found to agree with the one from the 2D calculations at low fluctuation levels. At high fluctuation levels the peak correlation in the 2D calculations was shifted significantly towards the antenna, more than found from the 1D calculations. Hence, for a correct determination the local magnetic field strength from O-X correlation measurements, 2D calculations are preferable. The big disadvantage of the 2D calculations is, however, that they are extremely time consuming. At low fluctuation levels 1D modeling can be considered because it gives the right location of the peak correlation but the 1D model overestimate the maximum of the correlation.

The O-X correlation measurements for determining the magnetic field strength in plasmas is promising, and should be a valuable diagnostic in fusion scale plasmas provided that the peak in the correlation can be determined with sufficient accuracy. Further work should focus on the degree to which the peak correlation can be measured, which will relate directly to the accuracy of the magnetic field strength measurement and its effectiveness as a constraint to determine the MHD equilibrium.

Acknowledgments

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Figure captions

Figure 1. Schematic of O-X correlation reflectometry. Waves with O- and X-mode polarization reflect from different layers, separated by Δr, in the plasma. The O-mode reflection point depends only on the electron density, whereas the X-mode point depends on both the electron density and the magnetic field. When one of the reflectometer frequencies is varied the cross correlation, γ, between the O- and X-mode signals can be obtained as a function of Δr.

Figure 2. The 1D (full line) and 2D (squares) normalized O-X correlation function at a fluctuation level of 2% (left) and 22% (right) for a magnetic field of 0.1 T. The other plasma parameters are given in the text. The dashed curve is the 1D density correlation function from which the ensemble was drawn.

Figure 3. (a) Simulated O-X peak coherence as a function of the fluctuation level for the parameters given in the main text that are similar to the LAPD experiments at a magnetic field of 0.1 T and 0.18 T. (b) The location where the O-X correlation peaks relative to the fixed channel equilibrium O-mode location. The solid lines are from the full 1D calculation and marks are from the 2D calculation.

Figure 4. The distribution of the reflection points at the maximum of the O-X cross correlation relative to the fixed frequency location without fluctuations for an ensemble of 6000 points. (a) and (b) Fluctuation level 1%. (c) and (d) Fluctuation level 20%. (a) and (c) The location of the variable X-mode channel plotted against the fixed O-mode channel. (b) and (d) The distribution of the O-mode (white) and X-mode (shaded) reflection points relative to the O-mode location without fluctuations.

Figure 5. The O-mode (A) and X-mode (C) permittivity and the scattered O-mode (B) and X-mode (D) wave field power at 11.00 GHz and 12.32 GHz respectively, for the same fluctuation realization. The magnetic field was 0.1 T and the other plasma parameters are given in the text. The evanecent region is indicated in black and the upper hybrid resonance zone is shown in white for the X-mode case. The box size is 56 cm by 56 cm.

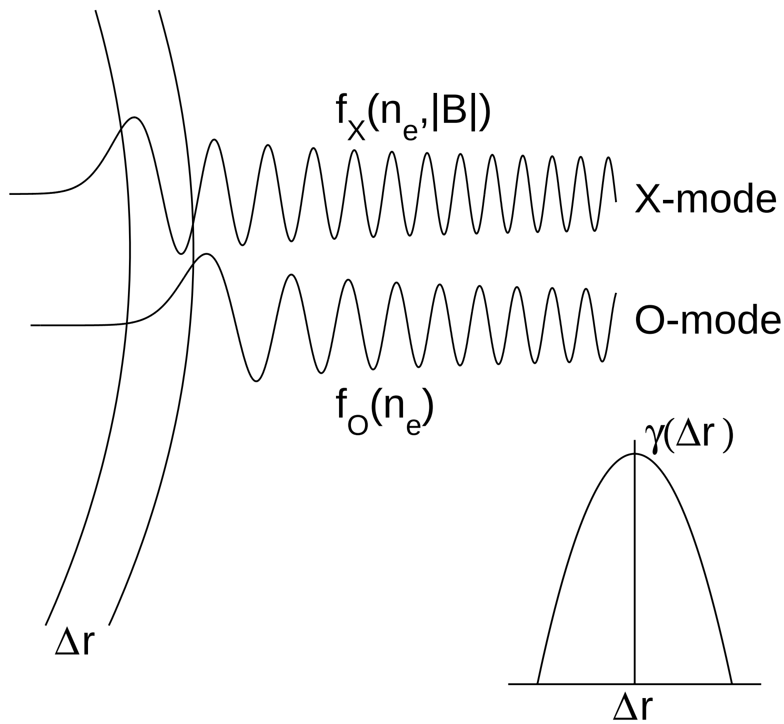

Figure 1

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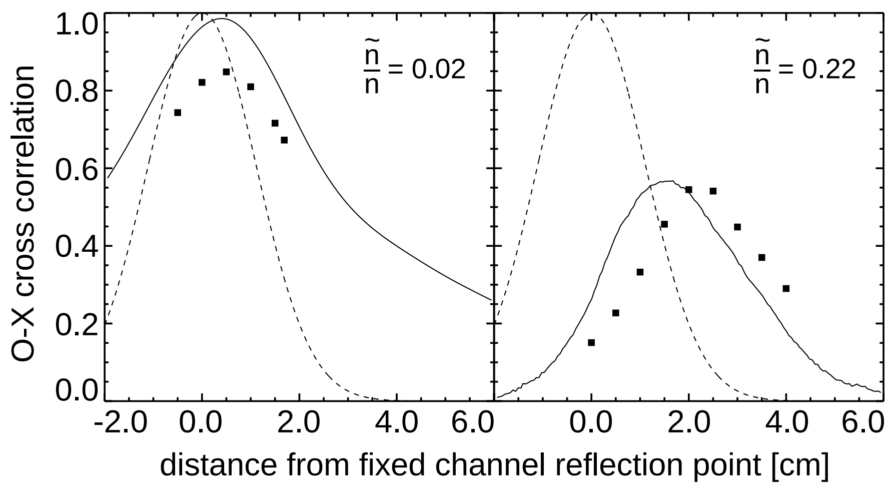

Figure 2

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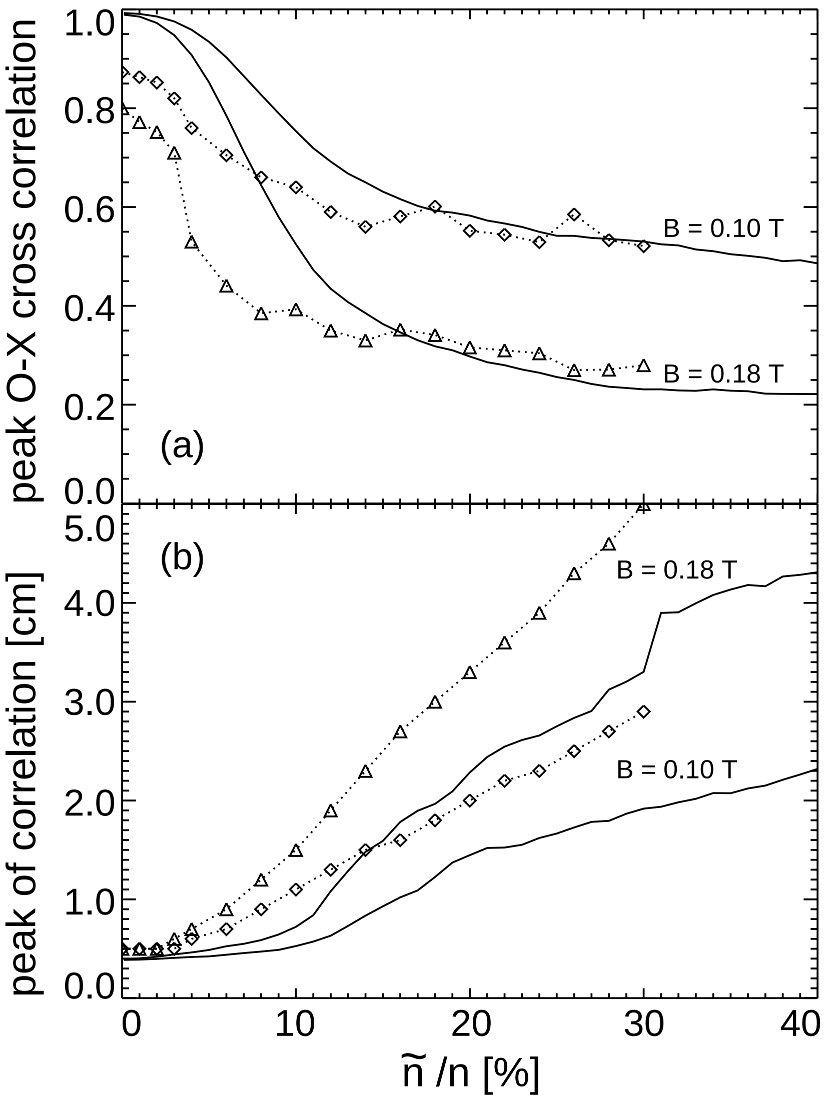

Figure 3

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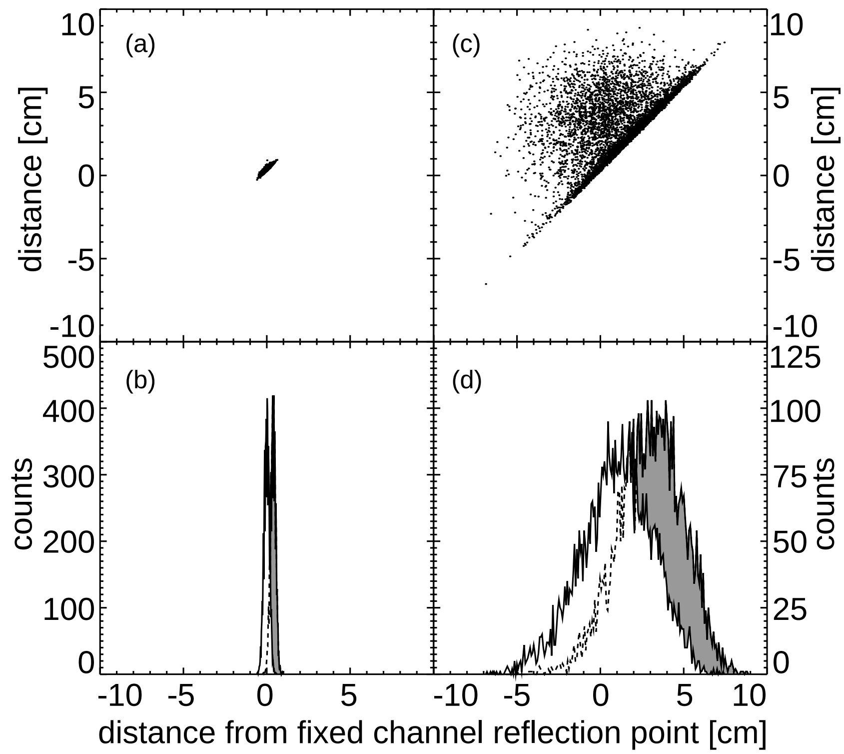

Figure 4

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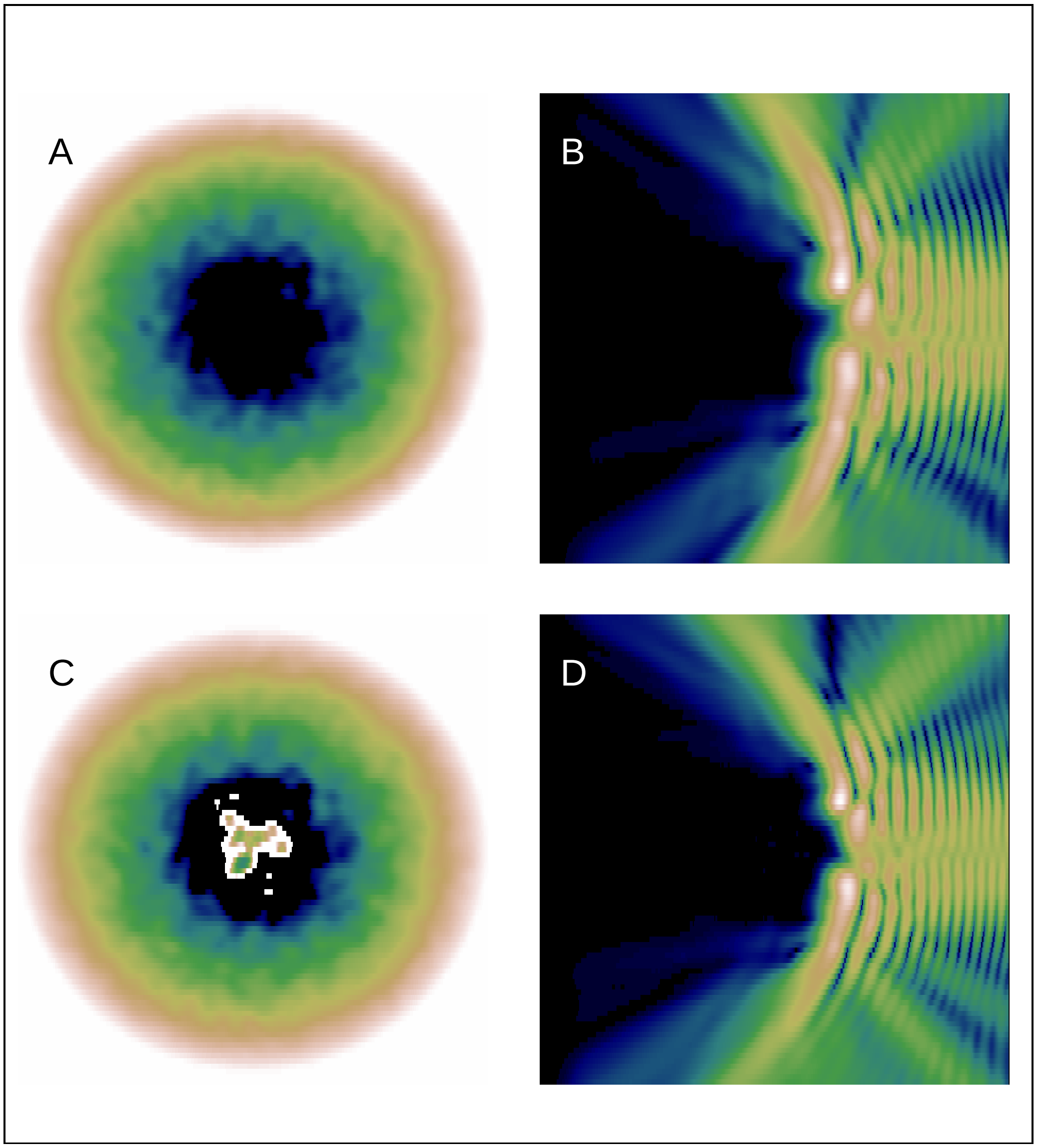

Figure 5

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