

# LINEAR AND NONLINEAR STUDY OF FAST PARTICLE EXCITATION OF ALFVÉN EIGENMODES<sup>3</sup>

N. N. GORELENKOV<sup>4</sup>, C. Z. CHENG, Y. CHEN, G. Y. FU, R. NAZIKIAN,  
R. B. WHITE

Princeton Plasma Physics Laboratory, Princeton University  
P. O. Box 451, Princeton, New Jersey 08543 USA.

M. V. GORELENKOVA  
TRINITY, Troitsk, Moscow Region, Russia, 142092

H. L. BERK  
IFS, The University of Texas at Austin, Austin, Texas, 78712

## Abstract

Recent new results concerning toroidicity-induced Alfvén eigenmode (TAE) linear stability and non-linear amplitude saturation and associated fast ion transport are presented for tokamaks such as National Spherical Torus Experiment (NSTX), and International Thermonuclear Experimental Reactor (ITER) using numerical codes HINST [1], NOVA [2, 3], and ORBIT [4, 5].

## 1 Introduction

Recent progress in TAE numerical simulations allows more robust analysis of both linear stability and nonlinear mode evolution. The existence of low- $n$  TAE modes and their mode structure are calculated with the ideal MHD code NOVA [2]. Previous results of TAE stability calculation for DT experiments in TFTR have demonstrated that alpha particles are responsible for the excitation of TAE modes [6] and are in good agreement with the experimental measurements for  $n = 3$  and 4 modes. The linear stability analysis of low- $n$  TAE modes due to nonideal effects is studied in this paper using a perturbative method in the NOVA-K code, which was recently improved to include finite orbit width (FOW) and larmor radius (FLR) effects [3]. Such an improvement validates use of NOVA-K for low aspect ratio tokamaks, such as NSTX.

For larger tokamaks such as ITER, the spectrum of unstable TAE's is shifted toward medium to high- $n$  modes. To study medium to high- $n$  TAE stability a high- $n$  stability code, HINST [1], was developed. The code solves the 2-D eigenmode problem using the ballooning representation in the poloidal direction and Fourier transform in the radial direction. The numerical method allows inclusion of non-ideal effects such as ion FLR, trapped electron collisional damping, etc., non-perturbatively.

Recent nonlinear analysis of TAE amplitude evolution has been done using analytical [7] as well as numerical  $\delta f$  methods [5]. Both approaches agree well [8], which enables prediction of the TAE saturation amplitude using the NOVA-K linear mode analysis, and estimation of fast particle transport due to TAEs using ORBIT. This part is applied here for tokamaks such as ITER and NSTX.

## 2 TAEs and their stability in NSTX

Plasma configuration and equilibrium in NSTX create an entirely new regime for studying the stability of TAE modes driven by beam ions, high harmonics fast wave heated ions and even the plasma tail ions. Some features of the NSTX plasma are: small aspect ratio, high energetic particle beta, large population of super-Alfvénic ions, large FLR and FOW effects, and the existence of magnetic well at the axis.

We choose a high beta NSTX plasma with the following parameters: geometrical center magnetic field and major radius are  $B = 0.3 T$  and  $R_0 = 85cm$ , minor radius of last plasma surface  $a = 68cm$ , so that the aspect ratio is  $A = 1.31$ , ellipticity is  $\kappa = 2$ , triangularity is  $\delta = 0.45$ , safety factor at the magnetic axis is  $q(0) = 0.6$ , and  $q(a) = 16$  at the edge, plasma

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<sup>4</sup>Permanent affiliation: TRINITY, Troitsk, Moscow Region, Russia, 142092

pressure is  $P(\psi) = P(0) (1 - \psi^{1.07})^{1.8}$ , density profile is  $n_e(\psi) = n_e(0) (1 - \psi^{1.64})^{0.478}$ . Plasma beta was defined as  $\beta_{av} = 8\pi \langle p \rangle / \langle B_{\varphi, vac}^2 \rangle$ , where  $\langle \rangle$  means volume average and  $B_{\varphi, vac}$  is the vacuum toroidal magnetic field. TAE analysis and stability study were done by NOVA and NOVA-K codes. NSTX equilibrium and corresponding continuum structure for  $n = 3$  are

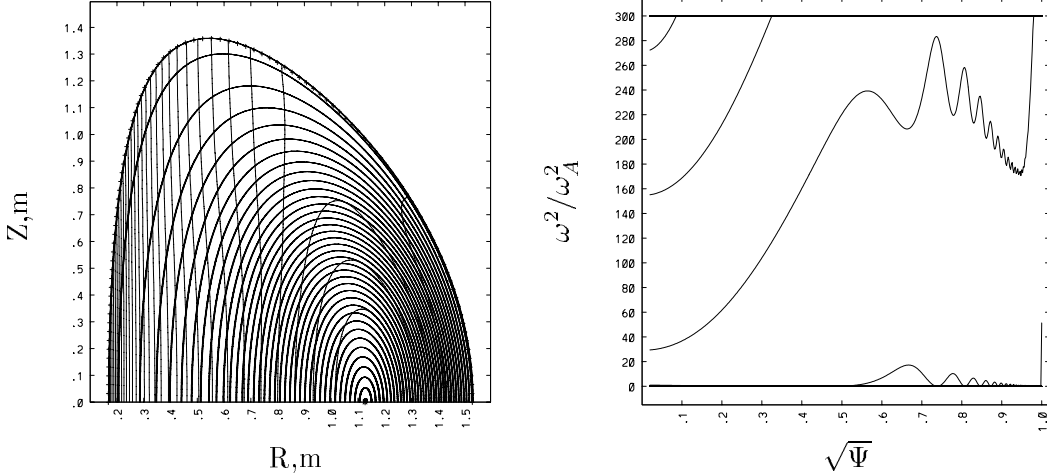


Figure 1: Equilibrium and Alfvén gap structure for NSTX at  $\beta_{av} = 34\%$

shown in Fig. 1. Results indicate that a variety of Alfvén modes exist with frequencies inside the gap, which is large because of strong toroidal coupling. We have found that for each  $n$  there are several TAE modes. TAEs usually have broad radial structure covering the whole minor radius. Global to edge localized TAEs exist in high beta ( $\beta_{av} = 34\%$ ) NSTX plasma, while core localized TAEs are not seen. We present two examples of TAE eigenmode structure in Fig.2. The first example of TAE ( $n = 3$ ) is similar to TAEs in tokamaks, which are mostly edge

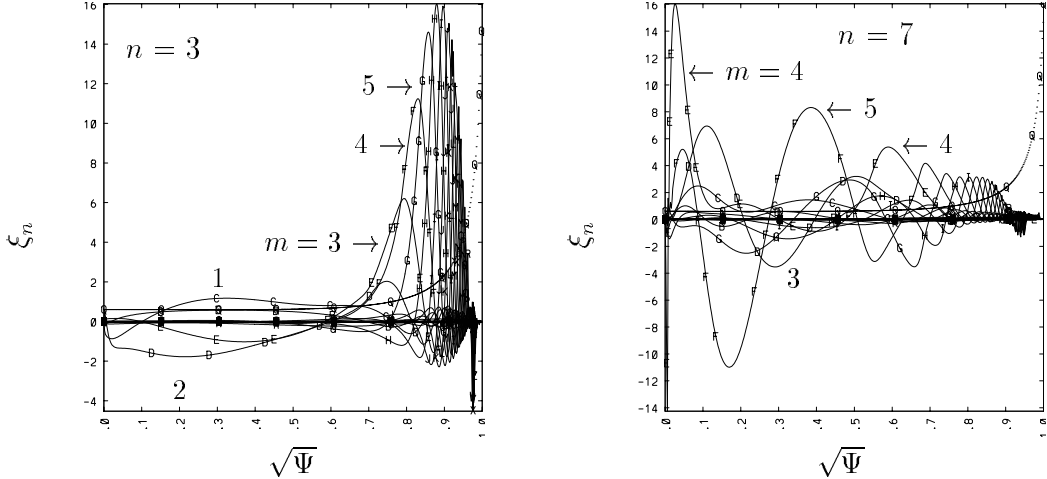


Figure 2: plasma displacement radial structures of TAEs for  $n = 3$  and  $n = 7$  in NSTX at  $\beta_{av} = 34\%$ . localized modes, while the  $n = 7$  mode shows rather “unusual” global structure.

We found several unstable modes in the specified equilibrium driven by the neutral beam injected (NBI) ions with a slowing down distribution function injected at the energy  $\mathcal{E}_{b0} = 80 keV$ . Central beam beta was  $\beta_b(0) = 2.38$ , while the total plasma beta was  $\beta(0) = 7.28$ . Table 1 is the summary of the analysis, where  $\omega^2/\omega_A^2$  is the eigenfrequency normalized to the Alfvén frequency  $\omega_A = v_A(0)/R_0 q(a)$ ,  $\gamma_b$  is beam driven growth rate, calculated in different approximations, i.e. zero orbit width (ZOW), FOW and FLR,  $\gamma_{is}$  is Landau damping of species  $s$ . Electron Landau damping is negligibly small. One stable mode is noted as “GAE” and represents the global Alfvén eigenmode solution. Note that the plasma ions can be super-Alfvénic in NSTX and can drive TAE unstable. Strictly speaking this means that ideal MHD can not be applied for this case and  $\omega_*$  effects should be included in eigenmode calculations. However for modes like the  $n = 3$  TAE as it is shown in Fig.2, the  $\omega_*$  effect may be small because the mode is localized

$n$	$\omega^2/\omega_A^2$	$\gamma_b, \% \text{ ZOW}$	$\gamma_b, \% \text{ FOW}$	$\gamma_b, \% \text{ FOW+FLR}$	$\gamma_{iD}, \%$	$\gamma_{iH}, \%$	$\gamma_{iC}, \%$	
1	20.23	-0.79	0.1	0.03	-12.5	-0.84	-2.6	
3	17.6	0.8	0.94	0.23	-0.17	-0.04	-0.36	“GAE”
	18.05	5.4	-0.07	-0.01	-7.5	-0.8	-3.9	
	20.67	0.74	1.45	0.33	0.3	-0.03	-0.13	
	23.79	0.86	0.65	0.22	0.3	0.01	-0.12	
7	21.25	0.92	1.475	0.1	4.3	0.22	0.1	
	23.63	1.8	1.9	0.134	4.9	0.18	-0.21	
	24.5	10.7	2.34	0.46	10.9	0.22	-2.3	

Table 1: Damping and growth rates for TAEs with  $n = 1, 3, 7$ .

near the edge, while the drive on the background ions comes mostly from the center. The same is true for the  $n = 7$  eigenmodes except for the mode with  $\omega^2/\omega_A^2 = 24.5$ , which is shown in Fig.2 and presents global structure, and for which the results of table 1 need to be improved by including the  $\omega_*$  effect. We conclude that two  $n = 3$  TAEs and all  $n = 7$  modes are unstable. Two effects are neglected here: the trapped electron collisional damping, which is included in NOVA-K, but can not be applied for the low aspect ratio plasma, and the radiative damping. Both effects may be significant because thermal ion FLR are large in NSTX. Further work is needed to account for these effects.

### 3 TAEs in ITER

We analyze the # 1002 ITER plasma equilibrium with following parameters from TRANSP plasma analysis code [10], which corresponds to a L-mode discharge. Magnetic field and major radius are  $B = 5.7 \text{ T}$  and  $R_0 = 8.14 \text{ m}$ , minor radius of last surface  $a = 2.8 \text{ m}$ , safety factor at the center is  $q(0) = 0.826$ , and  $q(a) = 3.61$  at the edge, central plasma beta is  $\beta_p(0) = 7.6\%$ , while alpha particles have  $\beta_\alpha(0) = 0.9\%$ . HINST, NOVA, and NOVA-K were applied to study TAE mode stability. Such an equilibrium differs from one used before in HINST ITER analysis [1], where the instability of resonant type of TAE was predicted. Also we have improved HINST, which now includes the fast particle distribution function in slowing down form and fast particle FLR effects. Results of the analysis show two regions of possible TAE instabilities. The first one is near the center with the frequency inside the continuum, the second one is near the plasma edge. TAEs in both regions were found stable. The RTAE (or EPM) may become unstable if the alphas beta is increased to  $\beta_\alpha(0) \gtrsim 2\%$ . The highest growth rate is achieved at  $n = 11$ . The main reason for the stability of RTAEs in this case is radiative collisional damping as the frequency of RTAE lies inside the continuum. The second region of TAE instability is at the edge  $r/a \simeq 0.7 - 0.9$ . Here both codes HINST and NOVA can be used. The analysis of the  $n = 10$  TAE shows good agreement for both codes and that “global” -like TAEs are stable with the small alpha driven growth rates  $\gamma_\alpha/\omega \simeq 0.5\%$ . NOVA shows that the damping is coming mostly from plasma ions and well exceeds the drive  $\gamma_{di}/\omega \simeq 1\%$ . However, in addition to that HINST predicts additional radiative damping  $\gamma_{dr}/\omega \simeq 1\%$ . The only modes, which were found unstable are KTAEs, localized closer to the center  $r/a = 0.4 - 0.55$  as shown in Fig.3 as a result of local HINST calculations. These modes have relatively small growth rate  $\gamma/\omega < 0.3\%$  for  $n = 10$ . The study of the growth rate dependence on the mode number showed that the most unstable mode have  $n = 9$  and becomes stable for  $n > 16$ .

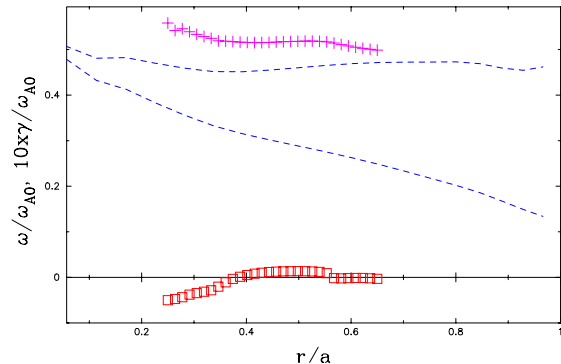


Figure 3: KTAE local eigenfrequency (+ points) and growth rates (□ points) relatively to the TAE gap for  $n = 10$  in ITER.

## 4 Nonlinear mode study

NOVA-K has been improved recently [8] to include theoretical predictions for the mode saturation amplitude [7]. The formulation was benchmarked against Monte-Carlo ORBIT simulations, which utilizes  $\delta f$  method [5]. The results of both codes applied to TFTR shot # 103101 at 2.92 sec where an  $n = 3$  TAE were observed [9] are shown in Fig.4. Both codes predict similar values of TAE amplitude  $\tilde{B}_\theta/B \simeq 10^{-5}$  at  $\gamma_L/\gamma_d = 2$  in agreement with the observed values [6]. We use the results of NOVA to calculate the saturation amplitude in NSTX and ITER and the eigenmode structure in ORBIT to calculate the effect of TAE on the fast particle losses.

For the eigenstructures of the TAEs shown in Fig.2, which are the most unstable modes in NSTX, we performed ORBIT simulations. Assuming  $\gamma_\alpha/\gamma_d = 2$ , NOVA-K produced the amplitudes of order  $\tilde{B}_\theta/B \simeq 10^{-3}$  for both  $n = 7$  and  $n = 3$  modes. Other effects included are collisional drag and scattering, and toroidal field ripple. Without TAEs the losses of beam co-injected ions were not significant  $\sim 5\%$ , though FLR effects were neglected, which allows particles to leave the plasma during gyro-motion and come back. As one can expect the  $n = 3$  TAE, being localized at the edge, does not

generate significant losses and accounts for additional  $\sim 2\%$  losses. However the  $n = 7$  mode is responsible for a higher total lost beam ion fraction ( $\sim 10\%$ ). The lost fraction due to the mode is relatively small, which is explained by better single particle confinement in spherical torus than in tokamaks. This is because of two reasons: presence of magnetic field well and strong poloidal field at the edge. Calculations presented include only single mode analysis and need to be improved for the case of multiple mode excitation. A similar approach was used to simulate the effect of TAEs in ITER. To estimate the mode amplitude we used for KTAE the value  $\tilde{B}_\theta/B = 10^{-4}$ , which was obtained in NOVA-K for edge TAE as there is no possibility to analyze KTAE in NOVA and the nonlinear theory was not applied in HINST. HINST produced the 2-D eigenmode structure, which shows coupling of several KTAE solutions, but still localized in real space within  $r/a = 0.25 - 0.65$ . The effect of KTAE on the alpha particle distribution function was found to be small with the flattening of the density profile near the mode localization similar to that reported earlier [11] for core localized TAEs. KTAEs do not increase losses, which are small in a steady state plasma (0.12%).

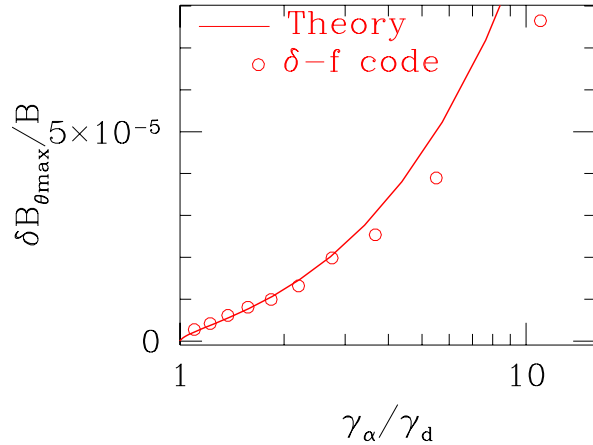


Figure 4: NOVA-K saturation amplitude vs  $\delta - f$  ORBIT simulations for TFTR shot 103101 at 2.92 sec.

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