MAGNETOSONIC EIGENMODES NEAR THE MAGNETIC FIELD WELL IN A SPHERICAL TORUS

M. V. Gorelenkova, N. N. Gorelenkov*

Troitsk Institute for Innovative and Fusion Research

Troitsk, Moscow region, Russia, 142092

(July 7, 1998)

The structure and spectrum of magnetosonic Alfvén eigenmodes in spherical torus in the presence of magnetic field well are studied. Analytical solution for eigenmodes localized in the well is obtained and compared with the numerical one. The possibility of using the eigenmode spectrum measurements for reconstructing the magnetic field well, and, thus, central magnetic safety factor profile is discussed.

*Present address: Princeton Plasma Physics Laboratory, P.0. Box 451, Princeton, NJ 08543

I. INTRODUCTION

Low aspect ratio tokamaks, such as National Spherical Torus Experiment (NSTX) [1], may have relatively low toroidal magnetic field, which can be exceeded by its poloidal component. Also, the volume averaged plasma beta is considered to be significant $10\% - 40\%$. It makes possible to create such an equilibrium, where the magnetic field has local minimum B_{min} close to the magnetic axis [2]. At such conditions the local Alfvén velocity will have local minimum at $B = B_{min}$, which may result in the existence of the magnetosonic (or fast, compressional Alfvén) eigenmodes (MSE). The eigenfrequency of such modes can provide the information about the absolute value of the magnetic field as well as the depth of the magnetic well.

MSE are believed to be responsible for the numerous observations of ion cyclotron emission (ICE) in tokamak plasmas, which detail study in experiments on Joint European Torus (JET) [3,4] and Tokamak Fusion Test Reactor(TFTR) [5,6] was reported. For the first time the existence of localized MSE has been predicted theoretically in Ref. [7]. Further development of MSE theory to explain the experimental measurements was done in Refs. [8–10]. In tokamaks MSE are localized at the minimum of the Alfvén velocity, which is located at the low magnetic field edge of the tokamak plasma rather than near the magnetic axis in spherical torus, which is to be discussed in this paper. MSE are excited via cyclotron resonances with superalfvenic charged fusion products, which are strongly anisotropic in tokamaks. Thus, as one expects strong anisotropy of beam and ICRF ions and low Alfvén velocity in spherical torus experiments, MSE instability in the frequency range of ion cyclotron frequencies seems plausible in those machines. It also may be helpful for the diagnostic of energetic ions.

Here we investigate the possibility of localized MSE solutions in the magnetic well, present MSE spectrum and eigenmode structure under the assumption that the magnetic field well is well elongated vertically, which employs the hollow cylinder approximation similar to that of Ref. [11].

As an example of plasma magnetic equilibria in spherical torus we show the numerical equilibrium for plasma with NSTX parameters. Fixed parameters used in q-solver equilibria calculation are the major radius $R_0 = 0.85m$, the minor radius $a = 0.68m$, the plasma elongation $k = 2.0$, the triangularity $\delta = 0.45$, and vacuum toroidal magnetic field strength $0.3T$ taken at the geometrical center of plasma poloidal cross section. The safety factor at the magnetic axis was chosen $q(0) = 2.8$ and $q(1) = 14.0$ at the plasma edge, and having the following profile $q(\psi) = 2.8 + \hat{\psi}\left[11.2 - 124.8\left(\hat{\psi} - 1\right) / \left(\hat{\psi} - 1.064\right)\right]$ where $\hat{\psi}$ is the normalized poloidal flux, which is zero at the magnetic axis and unity at the plasma boundary. The pressure profile was chosen in the form $p(\psi) = p(0)(1 - \hat{\psi}^{1.6})^{1.8}$. A plot of the magnetic field strength contours for equilibrium with averaged plasma beta $\beta_{av} \equiv 8\pi < p > / < B^2 >= 40\%$ is shown in Fig.1. Also shown as dashed lines are the magnetic field surfaces. The existence of the magnetic well is clearly seen on Fig.1 as well as on Fig.2, where the dependence of B vs. R in the midplane is shown and B has a minimum at $R_m = 1.34m$. One can see that the magnetic well is asymmetrical in the $R-$direction (Fig.2) and has strong elongation in Z direction (see Fig.1).

II. EIGENMODE EQUATION

To obtain the equation for magnetosonic eigenmodes localized in the magnetic well we consider the model of an inhomogeneous, magnetized plasma in a tokamak with strong elongated cross-section. An equation for the perturbed electric field is reduced from Faraday's and Ampére's laws in the assumption of vanishing parallel electric field [12]

$$\nabla \times \nabla \times \mathbf{E} = \frac{\omega^2}{c^2}\hat{\epsilon} \cdot \mathbf{E}, \tag{1}$$

where $\mathbf{E}$ has two components perpendicular to the equilibrium magnetic field and dependents on time as $\mathbf{E}(t) \sim exp(-i\omega t)$. The cold plasma permeability tensor $\hat{\epsilon}$ has elements

$$\hat{\epsilon}_{11} = \hat{\epsilon}_{22} = \sum_i \frac{\omega_{pi}^2}{\omega_{ci}^2 - \omega^2},$$
$$\hat{\epsilon}_{12} = -\hat{\epsilon}_{21} = \sum_i \frac{\omega}{\omega_{ci}} \frac{\omega_{pi}^2}{\omega_{ci}^2 - \omega^2} \tag{2}$$

in the orthogonal coordinates perpendicular to the equilibrium magnetic field $\mathbf{B}$, where ω_{pi} and ω_{ci} are the ion plasma and cyclotron frequencies, respectively. We choose the cylindrical coordinate system (R, Z, φ), which is related to the toroidal coordinate system $(r, \theta, \varphi^{'})$ as follows $R = R_m + r\cos(\theta), Z = r\sin(\theta), \varphi = \varphi^{'}$, where the axis is at $\mathbf{B}$-minimum. We also assume that perpendicular electric field has only E_R and E_Z components in order to obtain an analytical solution of Eq.(1). Assuming in addition zero parallel component of wave vector $k_{||} = -i\mathbf{e}_{||} \cdot \nabla lnE$ we can rewrite Eq.(1) as a system of two coupled equations for E_R and E_Z components of the perturbed electric field [9]

$$-\left(\frac{\partial^2}{\partial Z^2} + F\right) E_R + \left(\frac{\partial^2}{\partial R \partial Z} - H\right) E_Z = 0$$
$$\left(\frac{\partial^2}{\partial R \partial Z} + \frac{1}{R}\frac{\partial}{\partial Z} + H\right) E_R - \left(\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} + F\right) E_Z = 0, \quad (3)$$

where $F = (\omega^2/c^2)\hat{\epsilon}_{11}$ and $H = (\omega^2/c^2)\hat{\epsilon}_{12}$. The system of equations (3) can be simplified by multiplying the first equation with $(\partial^2/\partial R\partial Z + (1/R)\partial/\partial Z + H)$, the second equation with $(\partial^2/\partial Z^2 + F)$, and summing them. To simplify these equations further we make use of the assumption $\partial lnB/\partial z \ll \partial lnE_{R,Z}/\partial z$. Finally after some algebra we obtain the eigenmode equation

$$\left[\frac{\partial^2}{\partial Z^2} + \frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial}{\partial R} + \frac{F^2 + H^2}{F} + \frac{H}{F}\frac{\partial}{\partial Z}\right] E_Z = 0. \quad (4)$$

We represent $B(R, Z)$ near its minimum, i.e. at $Z = 0, R = R_m$ as

$$B(R, Z) = B_{min}\left(1 + \frac{(R - R_m)^2}{\Delta_R^2} + \frac{Z^2}{\Delta_Z^2}\right), \quad (5)$$

where Δ_R and Δ_Z are the magnetic field well characteristic widths in the R and Z directions and we assume $\Delta_R \ll \Delta_Z$. We also present Z-component of the perturbed electric field in the form $E_Z(R, Z) = \hat{E}_Z(Z, R)exp(ilZ/\Delta_Z)$, where $l \gg 1$, $\hat{E}_Z(Z, R)$ is a mode envelope being slowly varying function of Z, and consider the limit $\omega^2 \gg \omega_{ci}^2$. Then neglecting slow variation of the envelope (i.e. considering $\hat{E}_Z(Z, R) \simeq \hat{E}_Z(R)$) we result in the equation which determines the radial structure of MSE and which is zero order equation in a small parameter $\Delta_R/l\Delta_Z \ll 1$:

$$\left[\frac{\partial^2}{\partial R^2} - V(R)\right]\Psi(R) = 0, \tag{6}$$

where $\Psi(r) = \hat{E}_Z(R)R^{1/2}$. Note that the next order equation in Δ_R/Δ_Z will give the MSE structure in Z as was shown in Ref. [9] for the poloidal eigenstructure in tokamaks. The potential in Eq.(6) is given by

$$V(R) = \frac{l^2}{\Delta_Z^2} + \frac{l\omega}{\Delta_Z \omega_{ci} R} - \frac{1}{4R^2} - \frac{\omega^2}{v_A^2}, \tag{7}$$

where $v_A^2 = (B^2/4\pi n_e)\sum_{i=1,2}(z_i^2 n_i/m_i n_e)$ is the Alfvén velocity for a two-component plasma, z_i and n_i are the electric charge and density of ion species i, respectively.

III. EIGENVALUES AND EIGENFUNCTIONS

To obtain a spectrum of MSE localized in the magnetic field well we define

$$x^2 = \Delta_R^{-1}(R - R_m)^2 \sqrt{\frac{2\omega^2}{v_{Am}^2} - \frac{l}{\Delta_Z}\frac{\omega}{\omega_{cim} R_m}}, \tag{8}$$

where subscript in variables v_{Am} and ω_{cim} denote that they are evaluated at the magnetic field minimum. Using Eq.(5) we further simplify equation (6) for localized in the well solution into the equation of a harmonic oscillator

$$\left[\frac{\partial^2}{\partial x^2} + f(\omega) - x^2\right]\Psi(x) = 0, \tag{9}$$

where

$$f(\omega) = \Delta_R \left(\frac{\omega^2}{v_{Am}^2} + \frac{1}{4R_m^2} - \frac{l^2}{\Delta_Z^2} - \frac{l}{\Delta_Z}\frac{\omega}{\omega_{cm} R_m}\right) / \sqrt{\frac{2\omega^2}{v_{Am}^2} - \frac{l}{\Delta_Z}\frac{\omega}{\omega_{cm} R_m}}.$$

Equation (9) allows the localized eigenfunctions in the form

$$\hat{E}_Z(R) = \frac{1}{\sqrt{R}} e^{-x^2/2} H_n(x), \tag{10}$$

where $H_n(x)$ is n-th order Hermite polynomials, and the following spectrum of the eigenfrequencies $\omega = \omega_{n,l}$

$$\omega_{n,l} \simeq \frac{v_{Am}}{2\Delta_R}\left(2n+1+\frac{v_{Am}}{\omega_{cm}}\frac{l}{AR_m}+\sqrt{\left(2n+1+\frac{v_{Am}}{\omega_{cm}}\frac{l}{AR_m}\right)^2+2\left(2n+1\right)^2+\frac{4l^2}{A^2}}\right), \quad (11)$$

where $A = \Delta_Z/\Delta_R$.

From Eqs.(8,10) one notes that the higher l number the more localized (near B_{min}) MSE solution is. An example of the localized MSE eigenfunctions for $l = 10$ and $n = 0, 1, 2$ is presented in Fig.3 for the parameters corresponding to the deuterium plasma at average pressure $\beta_{av} = 40\%$: $\Delta_R = 0.3m$, $\Delta_Z = 2\Delta_R$, $R_m = 1.34m$, $n_{e,i} = 0.6 \times 10^{20} m^{-3}$, $B_m = 0.165T$. In Fig.3 we also show the results of numerical solution of Eq.(6) with the equilibrium magnetic field shown in Fig.2, which is asymmetrical in R around R_m. It is compared with the analytical solution Eq.(10) and shows good agreement validating our approach of using parabolic symmetrical approximation for the magnetic field near the well Eq.(5).

As one might expect the spectrum of eigenvalues is discreet and depends on the chosen l mode number in Z-direction. For example, at $l = 10$ the lowest MSE eigenfrequencies are: $\omega_{0,10} = 2.3\,\omega_{cDm}$, $\omega_{1,10} = 2.6\,\omega_{cDm}$, $\omega_{2,10} = 2.9\,\omega_{cDm}$ ..., where cyclotron frequency for deuterium is estimated as $\omega_{cDm} \simeq 0.7 \times 10^7 rad/s$.

IV. SUMMARY AND DISCUSSION

We have demonstrated the existence of localized magnetosonic eigenmodes in the magnetic well in a spherical torus with NSTX equilibria at averaged plasma beta $\beta_{av} = 40\%$. We have also determined the spectrum and radial structure of eigenmodes. If these modes are excited one might suggest the measurements of MSE signal at fixed l (mode number in Z-direction) using, for example, the coherent signal from several magnetic probes on the low field side of the torus, which needs to be capable of resolving Δ_Z/l wavelength. If the emission signal is measured and the spectrum Eq.(11) is reconstructed, the relation between the depth and the magnetic well radial width (B_{min}/Δ_R) can be obtained. Together with other diagnostics it may provide the information for plasma equilibrium reconstruction in the spherical torus. Other possibility to search for MSE spectrum might be the excitation of the modes using the set of magnetic probes working as antennas phased in such a way that l number would be fixed and $k_{||} = 0$.

We note, that MSE solutions presented in this paper have zero $k_{||}$, which makes them low damped modes. We expect such modes to be driven unstable by superalfvenic NBI or ICRF heated ions, though the study of such instability needs to be done and is beyond the scope of this paper.

Finally, several effects neglected here need to be included in the theory of MSE to be applied in the experiments, such as nonzero $k_{||}$, finite toroidicity within the mode location, and more realistic equilibria.

ACKNOWLEDGMENT

The authors would like to thank Drs. R. Majeski for discussions stimulating this work. Partly (NNG) this work was supported by US DoE contract DE-AC02-76-CHO-3073.

[1] J. Spitzer, M. Ono, M. Peng, D. Bashore, T. Begilow, A. Brooks, J. Chrzanowski, H. M. Fan, P. Heitzenroeder, T. Jarboe, R. Kaita, S. Kaye, H. Kugel, R. Majeski, C. Nuemeyer, R. Parsells, E. Perry, N. Pomphrey, J. Robinson, D. Stickler, and R. Wilson, Fusion Technol. **30**, 1337 (1996).

[2] J. E. Menard, S. C. Jardin, S. M. Kaye, C. E. Kessel, J. Manickam, Nucl. Fusion **37**, 595, (1997).

[3] G. A. Cottrell., et al.,Nucl.Fusion **33**, 1365 (1993)

[4] G. A. Cottrell., R. O. Dendy, Phys.Rev.Lett. **60**, 33 (1988)

[5] G. T. Green, TFTR GROUP, in Controlled Fusion and Plasma Heating (Proc. 17th Eur.Conf. Amsterdam, 1990), Vol. 14B, Part IV, European Physical Society, Geneva (1990).

[6] S. Cauffman, R. Majeski, Rev.Sci.Instrum. **66**, 817 (1995).

[7] S. M. Mahajan, D. W. Ross, Phys. Fluids **26**, 2561 (1983).

[8] B. Coppi, Phys. Fluids **29**, 4060 (1986).

[9] N. N. Gorelenkov, C. Z. Cheng, Nucl.Fusion **35**, 1743, (1995).

[10] G. Kamelander, Ya. I. Kolesnichenko, Phys. Plasmas (1996)

[11] V. D. Yegorenkov, A. R. Polevoy, K. N. Stepanov, S. E. Sharapov, in Controlled Nuclear Fusion and Plasma Physics (Proc. 17th Eur.Conf. Berlin, 1991), Vol. iV, European Physical Society, Geneva (1991).

[12] A. I. Akhiezer, Plasma Electrodynamics, Vol.1, Pergamon Press, Oxford (1975).

FIGURE CAPTIONS:

Fig.1 Contours of absolute value of magnetic field for NSTX equilibrium for $\beta_{av} = 40\%$. The magnetic field changes from $0.152\,T$ at the B minimum at $R = R_m = 1.34$ to $1.515\,T$ at the edge at the high field side. Contours $|B| = const$ are plotted for $|B|$ values separated by $\Delta B = 0.0454\,T$.

Fig.2 The absolute value of the magnetic field for $\beta_{av} = 40\%$ in the midplane.

Fig.3 First three localized eigenmode eigenfunctions for $l = 10$ and $n = 0, 1, 2$. Solid lines represent analytical results, dash lines correspond to numerical solutions.

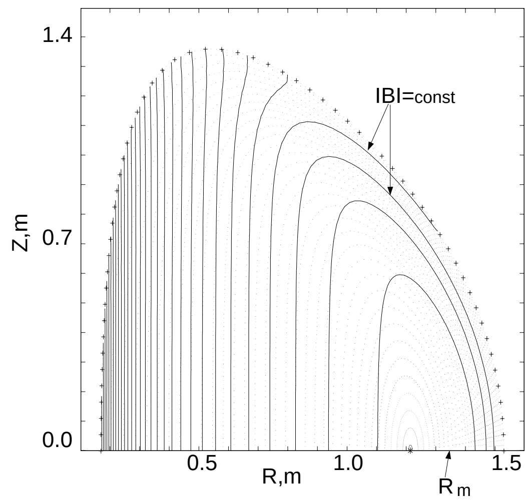

FIG. 1.

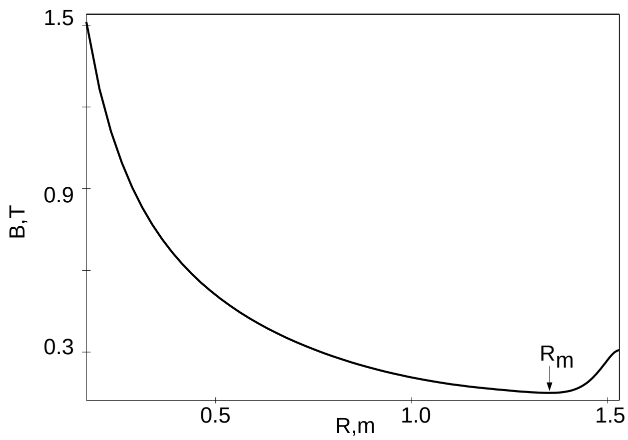

FIG. 2.

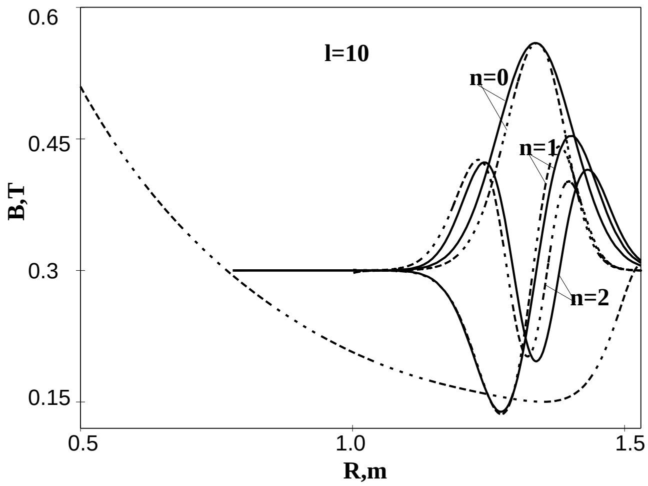

FIG. 3.