

**INVESTIGATION OF LOWER HYBRID WAVE DAMPING**

S. Bernabei, A. Cardinali\*, G. Giruzzi#, G. T. Hoang#, D. Ignat, R. Kaita, M. Okabayashi, F. Paoletti§ and S. von Goeler.

Plasma Physics Laboratory, Princeton University, Princeton NJ 08543 USA.

\* Associazione Euratom-ENEA sulla Fusione, Frascati, Italy.

# Association Euratom-CEA sur la Fusion, Centre d'Etudes de Cadarache, France.

§ Via dei Provoletti, Tor Bella Passera, Roma, Italy.

**Abstract**

Lower Hybrid (LH) current drive experiments on PBX-M have shown that the current profile can be changed by varying the phase velocity of the waves. The radial profile of the current carrying electrons was deduced from 2-D hard X-ray tomography. For a certain range of phase velocities, there is a correlation between the peak of the fast electron profile and the launched wave spectrum, despite the presence of a wide spectral gap. A model is proposed to explain how first-pass damping is possible in such plasmas. The radio frequency (RF) power can form a tail of energetic electrons, and waves with moderate phase velocity can damp on them. For waves with very fast phase velocity, there must be an upshift of the  $n$ -spectrum for any damping to occur. These hypotheses are supported by ray tracing results which were coupled to relativistic Fokker-Planck calculations of the electron distribution function.

## **1. Introduction.**

Lower Hybrid (LH) waves are routinely used to drive currents in tokamak plasmas non-inductively, thus significantly extending the tokamak operation space to long-pulse, steady-state discharges. A more ambitious goal is the use of LH current drive to control the current density profile, in order to improve both the plasma stability and confinement properties. This generally requires less power, but a more flexible control of the launched wave spectrum during the time of the discharge, eventually coupled to an appropriate feedback system. In addition, there are more intrinsic difficulties. The propagation and damping of LH waves (usually described as a bundle of rays) is a complex phenomenon, which is a sensitive function of quantities that are difficult to measure and control, such as the density, temperature and safety factor profiles, magnetic ripple, density and magnetic field fluctuations etc. Two regimes, with opposite properties, have been identified: the single-pass regime (high temperature, density, and parallel refractive index  $n_{\parallel}$ ), in which the wave propagation properties are simple, since full Landau damping takes place before the wave completely crosses the plasma, and the multi-pass regime (low temperature, density, and  $n_{\parallel}$ ), in which the wave damping is weak and the behaviour of the rays becomes stochastic. Unfortunately, most of the experiments in present-day tokamaks are realized in intermediate regimes, in which the rays are absorbed after crossing the plasma a few times and the theoretical prediction of the driven current profile becomes difficult.

Therefore, the experimental investigation of the LH wave damping in such regimes is an important task, which calls for appropriate diagnostic tools, in order to be able to compare measured and computed power deposition profiles. The PBX-M experiment is equipped with both a flexible LH wave launching system and a unique diagnostic set for the measurement of the spatial distribution of the superthermal electrons, which can be used to monitor the wave power deposition profile. This has motivated the extensive experimental and theoretical investigation of the LH wave damping in various physical regimes, which is presented in this paper.

The Lower Hybrid experiments have been performed on the PBX-M tokamak [1] with either indented or circular plasma configurations: in this

paper we refer mostly to the indented configuration. Typical parameters were: plasma current  $I_p \sim 180$  kA, density  $n_e = 1 \sim 3 \times 10^{19} \text{ m}^{-3}$ , and RF power up to 800 kW at 4.6 GHz delivered to the plasma through two fully-phased arrays of 32 waveguides each [2].

Partly because of the limited power available, steady state current drive was never achieved. Therefore we used the loop voltage drop to determine the current drive (CD) efficiency, corrected for changes in current and internal inductance, with a fitting method explained in section 2.

The current drive efficiency can also be obtained by fitting the experimental results using Fisch's theory with the "hot" conductivity [3]. The values obtained are in good agreement with the method of section 2. It is noteworthy that from this fit, it is possible to obtain the value of the conductivity of a plasma with suprathermal electrons. This is presented in section 3.

The phase dependence of the damping is presented in section 4. Some apparent anomalies can be explained by considering the two damping regimes which are apparent from the analysis of the CD efficiency.

In section 5, computer modeling is performed which couples a ray tracing code with a relativistic Fokker-Planck code. The agreement with the experimental results confirms the damping scenarios of section 4.

Discussion and conclusions are in section 6 and 7.

## **2. Current drive efficiency**

Figure 1 shows a typical experimental condition in which 370kW of RF power are injected in the plasma, and it causes the loop voltage to drop approximately 60%.

Figure 2a plots the relative loop voltage drop  $[-(V_{rf} - V_{oh})/V_{oh}]$  versus RF power normalized to line averaged density, current and major radius of the plasma.  $V_{oh}$  is the loop voltage in the purely ohmic plasma and  $V_{rf}$  is the residual loop voltage after application of RF power. The quantity  $(P/nI_pR)$  is the inverse of the wellknown "current drive figure of merit"  $\eta$ . The curve "rolls over" rather than increasing linearly with  $(P/nI_pR)$ , because of the diminishing effect of the electric field as more current is driven by the RF and less by the OH system. This has been verified by using the quasilinear Fokker/Planck code, described in section 5. The theoretical

fit in fig. 2b shows the good agreement obtained with the experimental, which illustrates the efficiency of the quasilinear interaction between the wave electric field and the static electric field of the transformer. The electrons accelerated by the static field are described by a shifted Maxwellian. The static field, therefore, moves electrons from low to higher velocities, making their interaction with the wave field more efficient.

The value of  $(P/nI_pR)^{-1} = \eta_0$  at which the curve crosses the line  $-\Delta V/V_{oh}=1$  defines the efficiency of current drive at  $V_{loop}=0$ . For any other value of  $-\Delta V/V_{oh}$ , the efficiency  $\eta(V_{rf})$  is given by the intercept of the line from the origin passing through this value and  $-\Delta V/V_{oh}=1$ . It is evident that  $\eta(V_{rf})$  is a monotonically decreasing function with increasing RF power.

In order to calculate the current drive efficiency  $\eta_0$  at  $V_{loop}=0$ , we follow the approach of Karney-Fisch [4] by plotting

$$\frac{P_{el}}{P_{abs}} \quad \text{vs.} \quad u \dots \frac{V_\phi}{V_{Dreicer}} = \frac{c}{n_{||abs} V_{Dreicer}}$$

where

$$P_{el} = V_{loop} I_{RF} \equiv V_{loop} \left( -\frac{\Delta V_{loop}}{V_{oh}} \right) I_p \quad (1)$$

is the RF power converted into electromagnetic energy,

$$P_{abs} = \alpha P_{injected}$$

is the fraction of RF power absorbed by the electrons, and

$$n_{||abs} = \beta n_{||o}$$

is the value of  $n_{||}$  absorbed by the electrons, which is usually upshifted (due to toroidal effects [5]) relative to the injected value. All the other quantities have the standard definitions.

In this set of equations, the parameters  $\alpha$  and  $\beta$  are varied in order to have the experimental points fall between the curves for  $\frac{P_{el}}{P_{abs}}$  vs.  $u$  corresponding to  $Z_{eff} = 2$  and  $Z_{eff} = 5$ , obtained from the polynomial approximation given by Karney-Fisch to match the experimental value  $Z_{eff} \approx 3$ .

Figure 3 plots the values of  $\frac{P_{el}}{P_{abs}}$  vs.  $u$  for all phases. In this plot  $\alpha=0.65$ , while  $\beta$  varies for each phase.

The efficiency at  $V_{loop} = 0$  is then calculated from:

$$\eta_0 = \frac{31}{\log \Lambda} \frac{4}{(Z_{eff} + 5)} \frac{\alpha}{(\beta^2 n_{\parallel 0}^2)} \quad (2)$$

Clearly, various combinations of  $\alpha$  and  $\beta$  will fit the data between the Fisch-Karney curves, but the ratio  $\alpha/\beta^2$  is rather constant and  $\eta_0$  is well determined. For very low values of  $\alpha$ , we obtain an  $n_{abs}$  which is too low to be explained. On the other hand, values of  $\alpha$  close to 1 would imply an  $n_{abs}$  which is far too high. Instead the choice of  $\alpha=.65$  produces a value for  $\beta$  in the case of total upshift of the spectrum to nearly  $n_{abs} = 7$ , which is what is calculated with the ray tracing code.

The results are shown in figure 4 where  $\eta_0$  is plotted versus the injected value of  $n_0$  for circular and indented configurations. Systematically, the efficiency in circular plasmas is lower than in indented plasmas, but in both cases it shows a maximum near  $n_0=2$ .

For  $n_0 \geq 2.1$  the efficiency decreases as  $n_0$  is increased as expected [6]. For values  $n_0 \leq 2.1$  efficiency decreases with decreasing  $n_0$ . This is a surprising result since it is well known that faster phase velocity waves are more efficient in driving current.

The explanation can be seen in figure 5, where the value of  $n_{abs}$  is plotted against  $n_0$ . For  $n_0 \geq 2.1$ , the upshift is modest and fairly constant, while for  $n_0 \leq 2.1$ , the waves need a large upshift before damping. This points to two different damping regimes.

### 3. Determination of the hot conductivity of the plasma.

The data of figure 2a can be fitted by using Fisch's theory, which includes the "hot" conductivity due to the suprathermal electrons. From this fit, it is possible to derive the value of the hot conductivity itself. Following reference [3], we call  $J_{\text{ohm}} = \sigma_{\text{Spitzer}} E$  the purely inductive part of the current density,  $J_{\text{rf}}$  the part generated by the LH waves, and  $J_{\text{hot}} = \sigma_{\text{hot}} E$  the cross term proportional to RF power and loop voltage. We can write then the total current density as:

$$J = J_{\text{ohm}} + J_{\text{rf}} + J_{\text{hot}} \quad (3)$$

The use of Eq. (3) implies that we neglect higher-order cross terms, e.g. those proportional to the square of RF power, the square of the electric field, etc. This is appropriate if we are far from the runaway regime, and it is consistent with the fact that in the experimental parameter range explored, the ratio of the phase velocity and the Dreicer velocity is always less than one (see Fig. 3). Note also [3] that  $\sigma_{\text{hot}}$  is generally proportional to  $P/n^2$ , weakly dependent on  $Z_{\text{eff}}$  and generally expected to increase with temperature. Its temperature dependence however, will be related to the filling of the spectral gap in a complicated way, similar to the current drive efficiency.

Combining equations (1) and (3), we obtain

$$-\frac{\Delta V}{V_{\text{oh}}} = \frac{(A + B)x}{1 + Bx} \quad (4)$$

where

$$A = \alpha \eta_0$$

$$B = \frac{n I_p R}{P} \frac{\langle \sigma_{\text{hot}} \rangle}{\langle \sigma_{\text{Spitzer}} \rangle}$$

$$x \equiv \frac{P}{nI_p R}$$

and the symbol  $\langle \dots \rangle$  denotes averaging over the plasma cross-section. Since  $\sigma_{\text{hot}}$  is proportional to  $P/n^2$ , the parameter  $B$  is actually independent of power, but is proportional to  $I_p R/n$  and depends on  $T_e$  and  $Z_{\text{eff}}$  through  $\sigma_{\text{hot}}$  and  $\sigma_{\text{Spitzer}}$ . The parameter  $A$  depends on  $T_e$  and  $Z_{\text{eff}}$  as well. Assuming that  $I_p$ ,  $n$ ,  $T_e$  and  $Z_{\text{eff}}$  are not strongly varying over the data base considered, Eq. (4) can be used to fit  $\Delta V/V_{\text{oh}}$  as a function of  $x$ . The fit for the data of figure 2a is shown in figure 6; the value for the efficiency at  $V_{\text{loop}}=0$  agrees very well with the one obtained in section 2. From the value of  $B$ , it is possible to obtain the "hot" conductivity.

It is also possible to characterize the CD efficiency as a function of the residual loop voltage,  $V_{\text{res}}$ , with

$$\eta(V_{\text{res}}) = \frac{\alpha\eta_0 + \left(\frac{1}{x}\right) \frac{\langle \sigma_{\text{hot}} \rangle}{\langle \sigma_{\text{Spitzer}} \rangle}}{1 + \frac{\langle \sigma_{\text{hot}} \rangle}{\langle \sigma_{\text{Spitzer}} \rangle}} \quad (5)$$

#### **4. Phase dependence of the damping.**

In PBX-M, we made extensive use of a 2-D Hard X-ray pinhole camera [7] to study the effect of the application of Lower Hybrid power to the plasmas. The diagnostic yielded information on the location of the damping of the waves, allowed estimates of the fast electron diffusion constant, and demonstrated the effect of MHD instabilities on the fast electron tail. [8,9]

As is well known, changing  $n_{\text{r0}}$  varies the phase velocity and damping of the waves. In this section, we discuss the relationship of  $n_{\text{r0}}$  to the damping and the RF-induced current. When the electron temperature is high enough for the waves to be damped on first pass, i.e.  $T_e(\text{keV}) \approx (7/n_{\text{r0}})^2$  (with values of  $n_{\text{r0}}$  typically up to 3), the value of  $n_{\text{r0}}$  controls the damping

location and therefore the current profile. In experiments with relatively low electron temperature, such as in PBX-M, multiple passes of the LH waves through the plasma are expected, until the value of  $n_r$  is upshifted enough for the damping to occur. This situation could lead to the loss of control of the damping location. In other words, the absorbed LH waves could lose "memory" of the original  $n_{r_0}$ . In PBX-M, we found that this last statement is only true for waves with very high phase velocity ( $n_{r_0} < 2$ ). For slower waves, first pass damping is still dominant and allowed us to achieve a degree of current profile control [10].

In PBX-M, there are two perplexing results which counter the notion that a large spectral gap makes the damping of the LH waves independent of the original  $n_{r_0}$ . The location of the damping and the velocity of the fast electrons generated were observed to depend on the value of  $n_{r_0}$ .

Figure 7 shows the location where the maximum of the fast electron tail is formed as obtained from inverting the hard X-ray emission [11] as a function of  $n_{r_0}$ . The continuous line, obtained from the ray tracing code LSC [12], is the locus of the maximum penetration of the wave in its first pass across the plasma. That the hard X-ray emission follows the first pass of the wave is a surprise, since at this location the value of  $n_r$  has changed very little from the launched value. Given that the typical temperature of the plasma is  $\approx 1$ keV, the wave is far too fast for damping; this is usually referred to as the 'spectral gap'. Instead, the damping should be expected close to  $r/a=0.5$ , where the value of  $n_r$  is allowed to reach a maximum [13]. At this location, which is rather insensitive to the original value  $n_{r_0}$ , because of the large upshift,  $n_r \approx 7$ .

The other anomaly concerns the effective temperature of the photon spectrum, as defined by von Goeler in [14] and deduced from the 2D hard X-ray camera measurements. As shown in fig. 8, the photon temperature is approximately inversely proportional to  $n_{r_0}$ , for  $n_{r_0} \geq 2.2$ : this is what is normally expected since faster waves interact with faster electrons. Below  $n_{r_0} = 2.2$  the situation is inverted and the photon temperature decreases for higher phase velocity.

Considering the results on efficiency obtained in the preceding sections, a mechanism is proposed which can explain how first pass damping is possible in situations with a large spectral gap.

As the waves propagate in the plasma, they must upshift in order to damp. As an electron tail is formed, enhanced by the electric field of the



transformer, less upshift is needed for damping. Ultimately, even the incoming wave (with a practically unchanged  $n_r$ ) can deposit some of its power into the tail. At equilibrium, the tail is maintained by a spectrum broader than the launched one: the value of the upshift is then intended as a 'weighted average' upshift of the various absorbed  $n_r$ . This scenario applies to the spectra centered at  $n_{r0} \geq 2$ .

For values of  $n_{r0} \leq 2$ , the incoming wave with  $n_r \approx n_{r0}$  is confined by accessibility to the outer portion of the plasma. For these faster waves, the spectral gap is too wide to be bridged, and only upshifted values of  $n_r$  are damped.

The waves with lower  $n_r$  have the following characteristics:

1. efficiency is lower than for  $n_{r0} \geq 2.1$
2. damping location is not correlated to accessibility, but to upshift, which is largely independent of the original  $n_{r0}$ . In fact, as seen in figure 7, the hard X-rays are observed near mid-radius where the upshift reaches a maximum [13].
3. the photon temperature decreases as the phase velocity increases. If the photon temperature is plotted versus the absorbed value of  $n_r$ , an almost monotonic function is obtained (see figure 9).

## 5. **Modeling.**

To simulate the propagation and absorption of the LH waves in PBX-M, we have constructed a numerical code which couples the solution of the system of differential equations for the LH ray trajectories (describing the wave propagation) with the solution of the quasilinear Fokker-Planck equation for the electron distribution function. This is essential in the determination of the power deposition profiles.

Given the LH power spectrum launched at the plasma edge, the objective is first to calculate the ray trajectories for each component of the wave spectrum (each  $n_{r0}$ ). This is so that, if we consider a fixed number of ray reflections between the plasma center ("whispering gallery reflection point") and the edge ("cut-off reflection point") [15], we know the evolution of the parallel wavenumber along the trajectory, and as a function of the flux surface coordinate  $\psi$ .

Since the time of the LHW propagation  $t_{lh}$  is much shorter than  $t_{ql}$  (the typical time of the quasilinear evolution of the distribution function), we can assume that, if there is no first-pass-absorption, the effect of the wave electric field on the plasma for each magnetic surface can be described by summing the quasilinear diffusion coefficient associated with the wave, at each passage on that magnetic surface. Specifically,

$$D_{ql}(\psi) = D_{ql1}(\psi) + D_{ql2}(\psi) + D_{ql3}(\psi) + \dots + D_{qln}(\psi) \quad (6)$$

where  $1, 2, \dots, n$  is the number of passages of the wave on the particular magnetic surface. Clearly, a value  $D_{qln}(\psi)$  is associated with each interval in the parallel velocity space  $v_1 - v_2$ , determined by the resonance condition  $v = \omega/k_{\parallel}$ , in which the wave interacts.

In this way, we are able to obtain the electron distribution function from the 2-D relativistic Fokker-Planck code [16], and determine the quasilinear damping of the wave, i.e.,

$$\gamma \approx \left. \frac{\partial f_e}{\partial v_{\parallel}} \right|_{v_{\parallel} = \omega/k_{\parallel}} \quad (7)$$

for all the spectra we are considering, the current generated, the absorbed power, and the efficiency of the LHCD.

In the following, we have summarized the results obtained for the simulation of the LHCD on PBX-M tokamak in an indented magnetic surface configuration.

Given a power spectrum coupled to the plasma edge, between  $n_{\parallel 1}$  and  $n_{\parallel 2}$  and centered on  $n_{\parallel 0}$ , with a Maxwellian shape, we run the Fokker-Planck code with three power spectra on each magnetic surface: all these spectra evolve according to the ray-tracing calculation. The first spectrum represents the upshifted  $n$ -spectrum at the third passage through the plasma, the second is the spectrum at the second passage, and the third is the spectrum at the first passage, which is essentially equal to the launched one. Their amplitudes represent the power that is transmitted in each one. The quasilinear interactions of these three spectra enables us to calculate the wave power dissipated in the plasma, the generated rf-current, and the quasilinear damping. In our calculation, we have divided the LH power spectrum into 100 rays, each carrying a fraction of the total power, and the

plasma in 100 magnetic surfaces from the edge ( $\psi=0$ ) and the center ( $\psi=1$ ). On each magnetic surface, we calculate self-consistently the quasilinear diffusion coefficient (by integrating the Poynting equation, which gives the square amplitude of the electric field), the velocity interval, and the quasilinear damping of the rf-power (square amplitude of the electric field), for each  $n$ .

The quasilinear absorption of the rf-power (integrated over the whole  $n$ -spectrum) for the three waves is calculated versus  $\psi$  for plasma parameters typical of the PBX-M experiment, i.e., average density  $n_e=1 \times 10^{13} \text{ cm}^{-3}$ , with a parabolic profile, ion and electron temperatures of about 1 keV with parabolic profiles, magnetic field on the axis  $B=1.95 \text{ T}$ , and indentation. The injected spectrum is centred on  $n_{\parallel 0} = 2.5 \pm 1.0$  with an extent of  $\Delta n = 2$ , and a Maxwellian shape

$$P(n_{\parallel}) = \frac{\alpha}{\sqrt{\pi}} \exp\left\{-\left[\alpha(n_{\parallel} - n_{\parallel c})\right]^2\right\}. \quad (8)$$

The frequency is 4.6 GHz, and the coupled power is about 140 kWatt.

Only a fraction (45%) of the power carried by the wave (with an upshifted spectrum centered on  $n_{\parallel c} \approx 5.5$ ) is damped into the plasma to generate the hot electron population. Altogether, almost 85% of the total power carried by the wave is quasilinearly absorbed by the plasma. Without mutual interaction among these spectra, no power is absorbed by the plasma from the nonupshifted and slightly upshifted spectra.

Figure 10 shows the quasilinear absorbed power for the three spectra we have considered versus  $n$ . The solid line represents the launched spectrum, while the dotted one is the absorbed power for the nonupshifted, slightly upshifted, and upshifted spectra.

As is shown by this figure, a large fraction of the power of the nonupshifted spectrum is absorbed because of the efficient quasilinear interaction among the waves. In particular, the fraction of power dissipated due to the upshifted spectrum results in a sink of electrons (tail formation) at higher velocities. These fast electrons are further accelerated by dissipation of the nonupshifted wave spectrum on this tail. The power deposition is concentrated at the half radius, where the  $n$ -upshift is maximum. Integrating the power density over the volume and the current density over the surface of the cylindrical plasma, we obtain the absorbed

power in Watts ( $\approx 140$  kWatt) and the generated current in Amperes ( $\approx 70$  kAmps), with an efficiency of 0.47.

Finally, in fig. 11 the calculated efficiency of current drive is plotted versus  $n_{r0}$  and compared to experiment. Again, the plot shows the expected behaviour of the efficiency for  $n_{r0} > 2$ ; the higher the launched  $n_{r0}$ , the lower the efficiency. This is in agreement with the analytical expression for the efficiency :

$$\eta = \frac{\langle v_{\parallel}^2 \rangle}{\sigma} = \frac{1}{\sigma} \frac{v_2^2 - v_1^2}{\ln(v_2 / v_1)} \quad (9)$$

where  $\sigma$  is a numerical coefficient which takes into account the 2-D Fokker/Planck correction to the efficiency and is proportional to the temperature, and  $v_i = \frac{v_{\parallel i}}{v_{th}} = \frac{c}{n_{\parallel i} v_{th}}$ , which decreases by increasing the minimum allowed value of  $n_r$  in the plasma. For values of  $n_{r0} \leq 2.1$ , the situation is reversed. In this case, the nonupshifted spectrum does not interact quasilinearly with the other two spectra (the slightly upshifted and the upshifted), and therefore, it does not damp on the tail formed by the upshifted one. The efficiency drops down abruptly. The ray tracing for  $n_{r0} < 2$  shows that in this case, the wave is not immediately accessible to the interior of plasma. Instead, the low values of  $n_r$  remain confined in an external layer where the wave suffers reflections between the slow and the fast branch of the dispersion relation. The wave, after a series of fast/slow reflections, penetrates to the plasma core with a totally upshifted value of  $n_r$ . Therefore, in the central zones of the plasma, only rays with upshifted values of  $n_r$  are present.

Figure 12 illustrates the scenario obtained from the computer modeling. Fig. 12a shows the maxwellian with the tail produced by the upshifted waves originated from  $n_{r0} < 2$ , and fig 12b shows the situation for  $n_{r0} > 2$ , with the tail extending to  $n_r = n_{r0} \approx v_r / 11 v_{th}$ .

## 6. Discussion.

The fact that the RF current for  $n_{\gamma_0} \geq 2.1$  is observed near the maximum penetration of the wave, while the tail itself is initially formed near the mid-radius, can be explained by the following model. Radial fast electron diffusion can allow a good fraction of the tail to be present on the magnetic surface where the radial wavenumber  $n_r$  becomes zero and the electric field of the wave grows, favouring damping.

Radial diffusion of fast electrons generated by LHCD in PBX-M has been extensively investigated in ref. 11. The main result of that study, which applies to the same type of discharges discussed here, is a fairly precise estimate of the fast electron radial diffusion coefficient  $D_r$ , namely,  $D_r \approx 1 - 2 \text{ m}^2/\text{s}$ .

This range of diffusion coefficient values is also needed to explain the change in the current profiles measured in these plasmas during LHCD [17,18].

We now show that this level of radial diffusion is sufficient to provide a substantial fast electron population in the central part of the discharge, by inward diffusion of electrons originally driven at  $r \approx a/2$ . The radial distribution of electrons generated at a given position  $r'$  is given by the Green's function  $g(r, r')$  of the steady-state kinetic equation, including radial diffusion and Coulomb collisions. An approximate Green's function, can be evaluated assuming that both  $D_r$  and the collision frequency  $\nu$  are radially constant. This is justified in the central part of the plasma, where the density profile is rather flat. The derivation of this Green's function is given in the appendix. For  $r < r'$ ,  $g(r, r')$  is proportional to  $I_0(hr)$ , where  $h = (\nu/D_r)^{1/2}$  and  $I_0$  is a modified Bessel function. Thus, the ratio of the electron populations at the locations  $r$  and  $r'$  for inward diffusion (i.e.,  $r < r'$ ) can be simply quantified by the ratio

$$\frac{g(r, r')}{g(r', r')} = \frac{I_0(hr)}{I_0(hr')} \quad (10)$$

This quantity is plotted in Fig. 13 for  $r' = a/2$ ,  $n_e = 2 \cdot 10^{13} \text{ cm}^{-3}$ ,  $T_e = 1 \text{ keV}$ ,  $D_r = 1 \text{ m}^2/\text{s}$  (solid lines), and several values of  $n_{\gamma}$ . The case  $D_r = 2 \text{ m}^2/\text{s}$ ,  $n_{\gamma} = 2.5$  is also shown for comparison (dashed line). Figure 13 shows that between 25 and 70% of the electrons originally driven at  $r' \approx a/2$  have

diffused to the central part of the discharge. This provides a sufficient "seed" to start wave damping and tail build-up by low- $n_r$  waves.

Concerning the difference in efficiency between circular and indented plasmas evident in figure 4, it must be noted that the latter usually have a higher electron temperature. Figure 14 shows the efficiency normalized to the central electron temperature. For  $n_{r0} > 2.1$ , it appears clearly that the efficiency depends linearly from the electron temperature, while this is not the case for  $n_{r0} < 2.1$

The following qualitative scenario can illustrate the effect of temperature on the damping and the current drive efficiency. A higher temperature makes the spectral gap smaller, so the required "average weighed" upshift is less for an indented than a circular plasma (higher efficiency).

At lower temperatures, it is not the width of the gap which determines the necessary upshift, but the ability of the wave itself to upshift at all, which is strongly dependent on the plasma shape [13].

## **7. Conclusions.**

In this paper, we present an experimental argument for the upshift of the LH waves, and consequently, a validation of our ray tracing codes. A scenario has been developed in which the upshifted spectrum creates a fast electron tail strong enough to damp part of the power during the first pass through the plasma. The concept of the 'weighted upshift' can give useful information on the formation and sustainment of the fast electron tail.

Since it is found that the spectral gap can be bridged with the help of the residual electric field, current profile control appears to be possible in experiments with moderately low electron temperature. The current drive efficiency is found to be proportional to  $T_e$ .

For faster waves (lower  $n_r$ ), for which the spectral gap is too wide, only upshifted waves damp. The damping location is fairly independent of  $n_r$ , and the efficiency is only marginally dependent from  $T_e$ .

A method for calculating the experimental current drive efficiency with residual inductive electric field has been presented. As a by-product, the hot conductivity can also be determined.

These results confirm that LH waves can be a powerful tool for current profile control over a wider range of plasma parameters than originally anticipated. This is because localized RF power absorption can occur in discharges where the electron temperature is too low for first-pass damping. In all cases, it is desirable to have coupling systems which give a good and wide control of the  $n$ -spectrum.

## **Appendix: approximate Green's function for radial diffusion**

The steady-state kinetic equation for the superthermal distribution  $f(v_r, v_r, r)$  is given by:

$$\frac{1}{r} \frac{\partial}{\partial r} r D_r \frac{\partial f}{\partial r} + \hat{C}f = S(r)$$

where  $\hat{C}$  is the collision operator and the source term  $S(r)$  is the quasilinear term

$$S(r) = -\frac{\partial}{\partial v_{\parallel}} D_{LH} \frac{\partial}{\partial v_{\parallel}} (f + f_{Maxw.})$$

or more generally, the sum of the quasilinear diffusion term and the convective term due to the dc electric field. The following approximations are used:

- 1)  $S(r)$  is treated as a known term (its radial structure can be accurately evaluated by ray-tracing codes).
- 2) A simplified collision term is used:

$$\hat{C}f = -\nu(v)f,$$

where

$$\nu(v) = \nu_0 \left( \frac{v_{th}}{v} \right)^3,$$

and  $\nu_0$  is the collision frequency of thermal electrons.

- 3) We assume that  $D_r$  and  $n_e$  (thus  $\nu$ ) are radially constant. This is a good approximation for this particular problem, since we are interested in inward diffusion, and the density profile is rather flat between  $0 < r < a/2$ .

With these approximations, one can analytically evaluate the radial Green's function  $g(r, r')$ , i.e., the solution of:

$$\frac{D}{r} \frac{\partial}{\partial r} r \frac{\partial g}{\partial r} - \nu g = \frac{\delta(r - r')}{2\pi r}$$

with the boundary conditions:



$$\left. \frac{\partial g}{\partial r} \right|_{r=0} = 0 \quad g(r=a) = 0$$

The result is:

$$g(r, r') = \begin{cases} \left[ \frac{K_0(ha)}{I_0(ha)} - \frac{K_0(hr')}{I_0(hr')} \right] \frac{I_0(hr)I_0(hr')}{2\pi D_r} & 0 \leq r < r' \\ \left[ \frac{K_0(ha)}{I_0(ha)} - \frac{K_0(hr)}{I_0(hr)} \right] \frac{I_0(hr)I_0(hr')}{2\pi D_r} & r' < r \leq a \end{cases}$$

$h = \sqrt{v/D_r}$  and  $I_n$ ,  $K_n$  are modified Bessel functions. From  $g(r, r')$  and the source term  $S$  the distribution function  $f(r)$  can be evaluated:

$$f(r) = 2\pi \int_0^a dr' r' g(r, r') S(r')$$

but  $g$  itself already provides information on the inward diffusion of fast electrons originally located at  $r = r'$ .

## **Aknowledgments.**

The authors gratefully aknowledge enlightening discussions with Dr. Emilia Barbato and encouragement from Dr. F. Santini.  
Work supported by USDOE under contract AC02-76-CH0-3073

## **References.**

- [1] R. Bell, N. Asakura, S. Bernabei, M. Chance, P. Duperrex, R. Fonck, G. Gammel, G. Greene, R. Hatcher, A. Holland, S. Jardin, T. Jiang, R. Kaita, S. Kaye, C. Kessel, H. Kugel, B. LeBlanc, F. Levinton, M. Okabayashi, M. Ono, S. Paul, E. Powell, Y. Qin, D. Roberts, N. Sauthoff, S. Sesnic, H. Takahashi, Phys. Fluids B 2, 1271 (1990).
- [2] N. Greenough, S. Bernabei, M. Norris, S. Schweitzer, R. Schwartz, 14<sup>th</sup> IEEE Symp. on Fus. Eng. S. Diego (Oct 1991) vol. 1 p. 126.
- [3] N. J. Fisch, Rev. of Modern Phys., 59 1 (1987) p. 175
- [4] C. F. F. Karney and N. J. Fisch, Phys. Fluids Vol. 29, No. 1 (1986) 181
- [5] S. Bernabei and D. W. Ignat, Nucl. Fus., vol. 22, 6, (1982) p. 735.
- [6] N. J. Fisch, Phys. Rev. Lett. 41 (1978) 873.
- [7] S. von Goeler, S. Jones, R. Kaita, S. Bernabei, W. Davis, H. Fishman, G. Gettelfinger, D. Ignat, F. Paoletti, G. Petravich, F. Rimini, P. Roney, J. Stevens, W. Stodiek, Rev. Sci. Instrum. 65 (5) May 1994, p. 1621
- [8] S. E. Jones, S. von Goeler, S. Bernabei, D. Ignat, J. Kesner, R. Kaita, S. Luckhardt, F. Paoletti, F. Rimini and the PBX-M Group, Plasma Phys. Control. Fusion 35, 1003-1017 (1993)
- [9] S. Sesnic, S. Batha, R. Bell, S. Bernabei, L. Blush, R. Cesario, M. Chance, T. K. Chu, R. Conn, M. Corneliussen, W. Davis, E. de la Luna, R. Doerner, J. Dunlap, A. England, H. Fishman, I. Garcia, G. Gettelfinger, T. Gibney, N. Greenough, J. Harris, R. Hatcher, G. Henkel, H. Hermann, Y. Hirooka, S. Hirschman, D. Ignat, R. Isler, S. Jardin, S. Jones, R. Kaita, S. Kaye, J. Kesner, C. Kessel, T. Kozub, H. Kugel, L. Lagin, B. LeBlanc, D. Lee, F. Levinton, S. Luckhardt, J. Manickam, M. Okabayashi, H. Oliver, M. Ono, F. Paoletti, S. Paul, G. Petravich, S. Preische, F. Rimini, P. Roney, J. Sanchez, N. Sauthoff, L. Schmitz, S. Schweitzer, T. Seki, Y. Sun, H. Takahashi, W. Tighe, G. Tynan, E. Valeo, S. von Goeler, K. Voss, P. Woskow,

A. Zolfaghari, in Proc. 15th Int. Conf. on Plasma Phys. and Contr. Nucl. Fus. Res. Seville Spain (26 Sept-1 Oct 1994) IAEA-CN-60/A3/5-P-12

[10] S. Bernabei, R. Bell, M. Chance, T. K. Chu, M. Corneliussen, W. Davis, G. Gettelfinger, N. Greenough, R. Hatcher, H. Hermann, D. Ignat, S. Jardin, R. Kaita, S. Kaye, C. Kessel, T. Kozub, H. Kugel, L. Lagin, B. LeBlanc, J. Manickam, M. Okabayashi, H. Oliver, M. Ono, S. Paul, S. Preische, P. Roney, N. Sauthoff, S. Schweitzer, S. Sesnic, Y. Sun, H. Takahashi, W. Tighe, E. Valeo, S. von Goeler, K. Voss, M. Mauel, G. Navratil, R. Cesario, S. Batha, F. Levinton, F. Rimini, N. Asakura, S. Jones, J. Kesner, S. Luckhardt, F. Paoletti, P. Woskow, A. Zolfaghari, T. Seki, J. Bell, J. Dunlap, A. England, D. Greenwood, J. Harris, G. Henkel, S. Hirschman, R. Isler, D. Lee, L. Blush, R. Conn, R. Doerner, Y. Hirooka, R. Lehmer, L. Schmitz, G. Tynan, Phys. Fluids B 5, 2562-2570 (July 1993)

[11] S. E. Jones, J. Kesner, S. Luckhardt, F. Paoletti, S. von Goeler, S. Bernabei, R. Kaita, G. Giruzzi and the PBX-M Team, Phys. Plasmas 2, 1548-1560 (1995).

[12] D. W. Ignat, E. Valeo, S. Jardin, Nucl. Fusion 34 (1994) 837

[13] F. Paoletti, D. Ignat, J. Kesner, S. Bernabei, R. Kaita, B. LeBlanc, F. Levinton, S. Luckhardt, Nucl. Fus. Vol. 34, No. 6 (1994) 771.

[14] S. von Goeler, H. Fishman, D. Ignat, S. Jones, P. Roney, J. Stevens, S. Bernabei, W. Davis, R. Kaita, F. Paoletti, G. Petravich, F. Rimini, Phys. Plasmas 2 (1) (Jan 1995) 205.

[15] M. Brambilla, A. Cardinali, Plasma Physics and Controlled Nuclear Fusion, 24 1187 (1982).

[16] M. Shoucri, I. Shkarofsky, Comput. Phys. Commun., **82** 287 (1994).

[17] Simulation of PBX-M Plasmas with Lower Hybrid Current Drive Using the TSC/LSC Model

D. W. Ignat, R. Kaita, S. C. Jardin, and M. Okabayashi

Princeton Plasma Physics Laboratory Report PPPL-3158 (January 1996), accepted for publication to Nuclear Fusion.

[18] Current Profile Modification During Lower Hybrid Current Drive in the Princeton Beta Experiment-Modification

R. Kaita, S. H. Batha, R. Bell, S. Bernabei, S. Hirshman, D. Ignat,

S. Jardin, S. Jones, S. Kaye, J. Kesner, H. Kugel, B. LeBlanc,

F. Levinton, S. Luckhardt, J. Manickam, M. Okabayashi, M. Ono,

F. Paoletti, S. Paul, N. Sauthoff, S. Sesnic, H. Takahashi, W. Tighe, and

S. von Goeler

Princeton Plasma Physics Laboratory Report PPPL-3169 (January 1996),  
submitted for publication to Nuclear Fusion.

## **Figure captions.**

Fig 1 a. Loop voltage versus time: top trace without RF power, lower trace with RF power.  
b. RF power pulse; power = 231 kW, B=15.3 kG,  $I_p = 187$  kA,  $\tilde{n}_e = 1.3 \times 10^{13} \text{cm}^{-3}$ , bean configuration.  
Note that at  $t \approx 490$  msec, MHD instability eliminates current drive.

Fig 2a Relative loop voltage drop  $[-(V_{rf} - V_{oh})/V_{oh}]$  vs. RF power, normalized to average density  $\tilde{n}_e$ , major radius R, plasma current  $I_p$ .  
The top scale is the inverse of the bottom one.  
full dots = LH in bean plasma configuration  
circles = LH in circular plasma  
All the data for  $n_{injected} = 2.1$   
The values at  $-\Delta V/V_{oh} = 1$  [ $V_{rf} = 0$ ] are obtained from figure 4.

Fig 2b Fit to the data of figure 2a obtained with the quasilinear Fokker-Planck code.

Fig 3  $P_{el}/P_{abs}$  vs. phase velocity normalized to runaway velocity: the various symbols refer to different values of  $n_{injected}$ , density, current, field; lines are the theoretical value from Karney-Fisch for  $Z_{eff} = 1, 2$  and 5.  
For all the cases  $\alpha = 0.65$ ,  $\beta$  is variable.

Fig 4 Current drive efficiency ( $InR/P$ ) at  $V_{loop} = 0$  vs. injected value of  $n$ .  
full dots = LH in bean plasma configuration  
circles = LH in circular plasma.

Fig 5  $n$ - absorbed vs.  $n$ - injected for the data of figure 4.

Fig 6 Analytic fit to the data of figure 2a, with the hot conductivity.

Fig 7 Radial location of the maximum of the HXR intensity, versus  $n_{injected}$ . The line is the maximum accessibility of the wave at the

first pass from LSC code. Plasma conditions are:  $I_p=180$  kA, average density =  $2.2 \times 10^{13}$  cm<sup>-3</sup>,  $B=1.7$  T,  $P_{RF} \approx 260$  kW, bean plasma configuration.

Fig 8 Photon temperature vs.  $n_{injected}$  for the same shots of figure 6.

Fig 9 Photon temperature vs.  $n_{absorbed}$ .

Fig 10 Launched  $n_{\nu}$ -spectrum (continuous line) and absorbed spectra (broken lines) as obtained from the quasilinear Fokker-Planck code. The ordinate is the power per unit  $n_{\nu}$ , normalized to the total RF power.

Fig 11 Comparison of the experimental efficiency (dots) vs.  $n_{injected}$  with the calculated one (crosses).

Fig 12 Electron distribution function vs.  $v_{\nu}$  for  $n_{injected} < 2$  (a) and  $n_{injected} > 2$  (b).

Fig 13 Electron populations vs.  $r/a$ , normalized to the value at  $r/a=0.5$

Fig 14 Data of figure 4: efficiency normalized to peak electron temperature (squares=bean plasma, dots=circular plasma).