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ION CYCLOTRON EMISSION DUE TO THE NEWLY-BORN FUSION PRODUCTS INDUCED FAST ALFVEN WAVE RADIATIVE INSTABILITIES IN TOKAMAKS

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ABSTRACT

The velocity distribution functions of the newly born (t = 0) charged fusion products (protons in DD and alpha particles in DT plasmas) of tokamak discharges can be approximated by a monoenergetic ring distribution with a finite $v_{||}$ such that $v_\perp \approx v_{||} \approx V_j$ where $(M_j V_j^2/2) = E_j$, the directed birth energy of the charged fusion product species j of mass M_j. As the time t progresses these distribution functions will evolve into a Gaussian in velocity (i.e., a drifting Maxwellian type) with thermal spreadings given by the perpendicular and parallel temperatures $T_{\perp j}(t) = T_{||j}(t)$ with $T_j(t)$ increasing as t increases and finally reaches an isotropic saturation value of $T_{\perp j}(t \approx \tau_j) = T_{||j}(t \approx \tau_j) = T_j(t \approx \tau_j) \approx [M_j T_d E_j/(M_j+M)]^{1/2}$, where T_d is the temperature of the background deuterium plasma ions, M is the mass of a triton or a neutron for j = protons and alpha particles, respectively, and $\tau_j \approx \tau_{sj}/4$ is the thermalization time of the fusion product species j in the background deuterium plasma and τ_{sj} is the slowing-down time. For times t of the order of τ_j their distributions can be approximated by a Gaussian in their total energy (i.e., a Brysk type). Then for times $t \geq \tau_{sj}$ the velocity distributions of these fusion products will relax towards their appropriate slowing-down distributions. Here we will examine the radiative stability of all these (i.e., a monoenergetic ring, a Gaussian in velocity, a Gaussian in energy, and the slowing-down) distributions.

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1. INTRODUCTION

Quantitative measurements of ion cyclotron emission (ICE) from energetic ions produced by fusion reactions or neutral beam injection promises to be a useful diagnostic tool in the large tokamak fusion devices that are entering the reactor regime of operation such as TFTR, JT-60, and JET. These energetic fusion products of the primary deuterium-deuterium (DD) reaction are given by

$$D + D = \begin{cases} {}^{3}He(0.82\ MeV) + n(2.5\ MeV) \\ T(1.0\ MeV) + p(3.0\ MeV) \end{cases} \tag{1}$$

and the secondary reaction are given by

$$ {}^{3}He + D = p(14.7\ MeV) + {}^{4}He(3.7\ MeV) \tag{2}$$

$$ T + D = {}^{4}He(3.6\ MeV) + n(14.7\ MeV). \tag{3}$$

For the L and H mode[1] operating conditions in both TFTR and JET, the equally spaced line structured ion cyclotron harmonic emission occurs primarily from the fusion product 3.0 MeV protons in the DD reaction dominated plasmas and from the fusion product 3.6 MeV ${}^{4}He$ (i.e., the alpha particles) in the deuterium-tritium (DT) reaction dominated plasmas. The basic reason for this is that it is only the protons and the alpha particles that have large enough energies so as to satisfy the condition $\lambda_j = k_\perp \rho_j > 1$, a necessary condition for any charged particle to emit equally intense electromagnetic (e.m.) radiation at all harmonics of its cyclotron frequency $\omega = m\,\omega_{cj} < \omega_{LH}$, where ω_{LH} is the conventional lower hybrid frequency of the background deuterium plasma and m is the harmonic number. Here $k_\perp$ is the component of the wave vector that is perpendicular to the confining tokamak magnetic field $B \propto R^{-1}$, ρ_j is the appropriate Larmor radius of the charged fusion product ionic species, and R is the major radius of the torus. Further, for these plasma conditions in TFTR and JET, the burnup fraction of 1 MeV tritons is approximately 1%. This implies that in DD reaction dominated plasmas, the alpha particle density $n_\alpha \approx 10^{-2} n_p$, and this, in turn, implies that in these DD plasmas about 1% contribution to the ICE power P_{ICE} will come from the alpha particles' cyclotron harmonic emission while the main contribution to P_{ICE} of DD plasmas is from the protons' cyclotron harmonic emission. Also, since the two primary reactions of Eq. (1) occur with equal probability, in these DD reaction dominated plasmas the fusion product protons, tritons, ^3He, and neutrons are all of equal number density [i.e., $n_p = n_t = n_{He3} = n_n$] and, consequently, there will be some cyclotron harmonic emission from tritons and ^3He since their $k_\perp \rho_j$ is only marginally less than unity for these plasma conditions in TFTR and JET. Again, since for these plasma conditions in TFTR and JET the ratio of the cross-sections for the reaction DT to those for the reaction DD is approximately equal to 10^2, in DT fusion reaction dominated plasmas the main contribution to P_{ICE} is from alpha particles with about 1% contribution from the protons while in the DD case the main contribution to P_{ICE} is from protons with about 1% contribution from the alpha particles.

Earlier experimental measurements of ICE from fusion products in L and H mode discharges both in TFTR and JET and their theoretical interpretations in terms of Trubnikov "dressed test particle cyclotron harmonic emission theory" in conjunction with Stringer's radial profile analysis of fusion products subject to the requirements of the Stix-Golant lower hybrid accessibility condition have been presented elsewhere.[2,3] These early measurements were in the nature of a steady-state measurement. However, the very recent experimental studies[4] of ICE have been made as a function of time in "supershot regimes"[5,6] in TFTR when either deuterium (D), tritium (T) or both (D-T) beams are injected into the background deuterium plasma. This time-behavior of ICE is somewhat of a transient in nature, but the total time evolution study of ICE in TFTR yield the following experimental observations:[4] (1) ICE at the harmonics of the cyclotron frequency of the fusion products species are observed immediately after the beams are turned on (i.e., at the beam injection time $t \approx 0$). (2) The time duration of these fusion product ICE peaks is $0 \le t \le \tau_{ice} \approx 100$ to 250 ms; while the fusion product alpha particle slowing-down time from plasma edge to center is[6] $\tau_{s\alpha} \approx 130$ to 650 ms. (3) For times $t > \tau_{ice}$ these fusion product ICE subsides and are now replaced by ICE at the harmonics of the cyclotron frequency of the injected beam species and this (beam species) emission persists until the beams are turned off. (4) In all cases the observed fundamental cyclotron frequency corresponds to the value of the B field *evaluated at the outer midplane plasma edge on the low field side of the torus.*

We now wish to point out that according to the TRANSP code plasma analysis of reference 6, the TFTR supershots edge background plasmas consists of about 50% deuterium ions and 50% fully ionized carbon impurity ions. The charge to mass ratio of these fully ionized carbon ions is equal to that of the deuterium ions and thus their cyclotron frequencies are degenerate (i.e., $\omega_{cc} = \omega_{cd}$). Also in this plasma edge region, the temperatures of the carbon impurity ions and the deuterium ions are equal to each other. That is, $k_\perp \rho = k_\perp v_\perp/\omega_c$ of these fully ionized carbon $\approx (1/12)^{1/2}\, k_\perp \rho$ of deuterium. Thus the damping of the fast Alfven waves at $\omega \approx m\omega_{c\alpha} \approx m\omega_{cd} \approx m\omega_{cc}$ in the DT reaction dominated discharges and at $\omega \approx m\omega_{cp} \approx 2m\omega_{cd} \approx 2m\omega_{cc}$ in the DD reaction dominated discharges by the fully ionized carbon $\approx (1/12)^m$ and $(1/12)^{2m}$, respectively, of that by deuterium. Hence, for our purposes here, without any loss of generality, we can take the entire background edge plasma as consisting of deuterium ions only but with half the edge population, since for $k \approx k_\perp$ the damping contribution by carbon can be neglected in comparison to that from deuterium. Furthermore, this TRANSP code analysis shows that the early time evolution of the fusion products velocity distribution function can be approximated by a drifting Maxwellian with a fractional width $(\Delta E_j/E_j) = 2(\Delta V_j/V_j)$ increasing almost linearly with time, starting from the initial value of zero and reaching a value of 0.5 in about two-third the slowing-down time at the plasma center. Eventually for $t \approx \tau_{sj}$, this TRANSP code analysis indeed shows that the fusion products velocity distribution is the usual slowing-down distribution.[7] Also, according to this TRANSP code analysis one finds that once this fractional width has increased beyond 0.3 it not very meaningful to approximate these distributions by a drifting Maxwellian types, since they are already well on their way toward slowing-down type.

It is our aim in this paper to examine whether or not one can have cyclotron harmonic fast Alfven wave radiative instabilities for frequencies $\omega \approx m\omega_{cj}$ due to the newly born fusion product species j during the early phase (for $0 \leq t < \tau_{sj}$) of beam injection. Such instabilities can give rise to the experimentally observed unstable fusion product ICE in the early phase of beam injection. Indeed, we will show that it is much easier for the fraction of the fusion products that are "marginally mirror-trapped" [i.e., "the trapped-passing boundary particles" in the $(v_\perp, v_{||})$-plane] to drive this instability than those fractions that are fully trapped or fully circulating. There are two sources of *free energy* to drive this instability, namely: one coming from the $(\perp, ||)$-temperature anisotropy, and the other coming from (the inverse cyclotron damping due to) the fusion product's directed birth velocity along **B** (i.e., $V_{||j}$). For these early times the fusion product velocity space distribution function $f_j(\mathbf{v})$ remains narrow enough to drive the system unstable. This cyclotron two-stream instability (or overstability) is a consequence of the fact that the growth rate of the cyclotron harmonic fast Alfven waves due to the newly born fusion products (via the inverse cyclotron damping) exceeds the damping rate of these waves by the background deuterium plasma ions (via the coventional cyclotron damping). However, as time t progresses $f_j(\mathbf{v})$ broadens, first due to thermalization for $0 \leq t \leq \tau_j$ and then due to slowing down for $\tau_j \leq t < \tau_{sj}$, and eventually for $t > \tau_{sj}$, $f_j(\mathbf{v})$ will totally relax towards the appropriate slowing-down distribution for this fusion product species j. When $f_j(\mathbf{v})$ broadens to sufficiently large value one finds that this instability is turned off, thus quenching the fusion product ICE, and indeed we find that all slowing-down distributions are totally stable for any emission and absorption of the cyclotron harmonic fast Alfven waves, i.e., their net absorption does exceed their net induced emission.

The method of analysis used here is the well-established standard technique of the "master equation approach" of non-equilibrium quantum statistical mechanics and is based on the Einstein A and B coefficients and the *principle of detailed balance.* In the literature other authors[8-10] have analyzed this problem using the standard techniques of classical plasma kinetic theory and/or plasma dispersion theory. Coppi *et.al*,[8] Gorelenkov and Cheng[9] have examined the fast Alfven cyclotron instability (ACI) resulting from the resonant cyclotron interaction of the marginally mirror-trapped fusion alphas with the collective eigenmodes of the background deuterium tokamak plasma in the large aspect ratio limit. Dendy *et.al.*[10] have studied the magnetoacoustic cyclotron instability (MCI) resulting from this resonant cyclotron interaction of the marginally trapped fusion alphas with the modes given by the local dispersion relation for a uniform plasma with conditions comparable to those in the plasma edge. All these analysis[8-10] are based on the classical kinetic and/or dispersion theory while our analysis here is based on particle-orbit theory. In general, the particle-orbit theory is much more of a physical approach and provides a better insight into the fundamental physical processes involved in a particular problem. However, the kinetic description on the basis of the Vlasov equation provides a much more rigorous treatment of complex problems which may not be easily accessible to the analysis of the particle-orbit theory. The equivalence of the particle-orbit theory and the kinetic description on the basis of the Vlasov equation was first demonstrated by Jeans and is usually referred to as the Jeans theorem in the astrophysical literature.

The paper is organized as follows: In section 2 we present the various possible velocity distribution functions for the fusion products of species j. Here we follow their distributions from their time of birth at $t = 0$ when they are monoenergetic, and then these monoenergetic distributions thermalize towards approximately to the conventional drifting Maxwell-Boltzmann type distributions in the time interval $0 \leq t \leq \tau_j$, then for times t of the order of τ_j they change into a Brysk type distribution[11] (i.e., a Gaussian in energies), and finally in the time interval $\tau_j \leq t \leq \tau_{sj}$ they evolve towards the appropriate slowing-down distributions. Here the thermalization time $\tau_j \approx \tau_{sj}/4$ and τ_{sj} is the slowing-down time. Section 3 summarizes the theoretically expected spatial (i.e., radial) distribution of the fusion products. In section 4 we examine the conditions for radiative instabilities, i.e., the conditions under which the induced emission exceeds the absorption. Indeed, we will show that the cyclotron harmonic fast Alfven waves of frequency $\omega \approx m\omega_{cd} = m\omega_{c\alpha}$ in the DT reaction dominated plasmas and/or $\omega \approx m\omega_{cp} = 2m\omega_{cd}$ in the DD reaction dominated plasmas are always damped by the background deuterium plasma (via the coventional cyclotron damping); but they can be driven unstable and made to grow by the newly born fusion products (i.e., protons in DD and alpha particles in DT) by the inverse cyclotron damping (or equivalently, by the conventional cyclotron overstability mechanism). That is, we investigate the conditions under which the growth rate γ_j coming from these fusion products exceeds the damping rate γ_d coming from the background deuterium plasma, and thus giving rise to the unstable ICE. Section 5 deals with some numerical estimates and comparison with the experimental observations. Finally in section 6 we present our conclusions and summary.

II. FUSION PRODUCTS' VELOCITY DISTRIBUTION FUNCTIONS

The velocity distribution function (at the time of birth) of these newly born fusion product of species j in tokamak discharges can be well approximated by a monoenergetic ring distribution with a finite $v_{||}$ such that $v_\perp \approx v_{||} \approx V_j$ where $M_jV_j^2/2 = E_j$, the directed birth energy of the fusion product species of mass M_j. For example, for (DD plasmas), j = protons, E_p = 3.0 MeV; and for (DT plasmas), j = alpha particles, E_α = 3.6 MeV. That is, at the time of birth, the jth species velocity distribution function may be written

$$f_j(v) = (4\pi V_j^2)^{-1}\,\delta(v - V_j) = f_j(v_\perp, v_{||}) \approx f_{j\perp}(v_\perp)\, f_{j||}(v_{||})$$

$$= (2\pi V_j)^{-1}\,\delta\,(v_\perp - V_j)\,\delta(v_{||} - V_j), \tag{4}$$

where we have adopted an approximate decomposition $f(v) \approx f_\perp(v_\perp)\, f_{||}(v_{||})$ so as to retain the $(\perp, ||)$ anisotropy. That is, we have approximated a spherically symmetric thin shell distribution by a cylinderically symmetric drifting thin ring distribution. This monovelocity

velocity space distribution of Eq. (4) is also equivalent to the monoenergetic energy distribution $f_j(E) = \delta(E - E_j) = f_j(E_\perp, E_{||}) \approx f_{j\perp}(E_\perp) f_{j||}(E_{||}) = \delta(E_\perp - E_j)\ \delta(E_{||} - E_j)$.

However, as the time t progresses these fusion products' monoenergetic and/or monovelocity distributions will develop thermal spreadings given by the perpendicular and parallel temperatures $T_{\perp j}(t)$ and $T_{||j}(t)$, respectively. These thermal spreadings $T_{\perp j}(t)$ and $T_{||j}(t)$ will keep on increasing as t increases and finally they will reach a common isotropic saturation value of[11] $T_{\perp j}(t \approx \tau_j) = T_{||j}(t \approx \tau_j) = T_j(t \approx \tau_j) \approx [M_j T_d E_j/(M_j + M)]^{1/2}$, where T_d is the temperature of the background deuterium plasma ions, M is the mass of a triton or a neutron for j = protons [see Eq. (1)] and j = alpha particles [see Eq. (3)], respectively, and τ_j is the thermalization time of the fusion product species j in the background deuterium plasma. This saturation value of the mean temperature in the center-of-mass system $T_j(t \approx \tau_j)$ is basically a consequence of the conservation laws of energy and the fact that the net momentum in the center-of-mass system is always zero.[11] It may also be noted that $T_j(t \approx \tau_j)$ is essentially the geometric mean of the fusion products birth energy and the background deuterium plasma thermal energy [i.e., $T_j(t \approx \tau_j) \propto \sqrt{(T_d E_j)}$] weighted by the reduced mass factor $[M_j/(M_j + M)]^{1/2}$. According to Brysk[11] the distribution functions of the fusion products can be well approximated by a Gaussian distribution in their energies, i.e.,

$$f_j(E) = [2\sqrt{\pi}\ \kappa T_j(t \approx \tau_j)]^{-1} \exp\{-(E - E_j)^2/4\ [\kappa T_j(t \approx \tau_j)]^2\} \quad \text{for } 0 \le E \le \infty, \qquad (5)$$

where $\kappa T_j(t \approx \tau_j) << E_j$. An approximate $(\perp, ||)$-decomposition of $f_j(E)$ of Eq. (5) may be written as

$$f_j(E) = f_j(E_\perp, E_{||}) \approx f_{j\perp}(E_\perp)\ f_{j||}(E_{||})$$

$$\approx ([2\sqrt{\pi}\ \kappa T_j(t \approx \tau_j)]^{-1} \exp\{-(E_\perp - E_j)^2/[4\kappa T_j(t \approx \tau_j)]^2\})$$

$$([2\sqrt{\pi}\ \kappa T_j(t \approx \tau_j)]^{-1}\ \exp\{-(E_{||} - E_j)^2/[4\kappa T_j(t \approx \tau_j]^2\}), \tag{6}$$

where we have neglected the ($\perp$, $||$) cross-coupling term. Since $\kappa T_j << E_j$, It may be noted that in Eqs. (5) and (6) when $T_j \to 0$, $f_j(E) \to \delta(E - E_j) \approx \delta(E_\perp - E_j)\ \delta(E_{||} - E_j) \propto \delta(v - V_j) \approx \delta(v_\perp - V_j)\ \delta(v_{||} - V_j)$ in agreement with Eq. (4) as it should.

Furthermore, for the sake of analytical simplicity and for the purpose of illustrating the similarity to (and for comparing with) the familiar cyclotron-overstability terms of the conventional hot plasma theory,[12] we will also examine the perpendicular and parallel velocity space distribution functions of these fusion products which can be represented by drifting Maxwell-Boltzmann distributions with temperatures $T_{\perp j}(t)$ and $T_{||j}(t)$, respectively, for times $t \leq \tau_j$. That is, we will assume that for $t \leq \tau_j$, $f(\mathbf{v})$ may be written

$$f(\mathbf{v}) = f(E_\perp, v_{||}) = [1 - \Sigma_j\ \eta_j]f_d(E_\perp, v_{||}) + \Sigma_j\ \eta_j f_j(E_\perp, v_{||}), \tag{7}$$

where the background deuterium plasma ion distribution is

$$f_d(E_\perp, v_{||}) = f_{d\perp}(E_\perp)\ f_{d||}(v_{||}) = (\kappa T_{\perp d})^{-1}[\exp(-E_\perp/\kappa T_{\perp d})]$$

$$[(M_d/2\pi\kappa T_{||d})^{1/2}\ \exp\{-M_d(v_{||} - V_d)^2/2\kappa T_{||d}\}], \tag{8}$$

the jth fusion product ionic species distribution is

$$f_j(E_\perp, v_{||}) = f_{j\perp}(E_\perp)\ f_{j||}(v_{||}) = (\kappa T_{\perp j})^{-1}\ \exp\{-(E_\perp - E_j)/\kappa T_{\perp j}\}$$

$$(M_j/2\pi\kappa T_{||j})^{1/2}\ \exp\{-\ (M_j/2\kappa T_{||j})[v_{||} - V_j]^2\}, \tag{9}$$

and the total fusion products' fractional population $\eta = \Sigma_j \eta_j$, where j = protons, alpha particles, tritons and ^{3}He. Here for the fusion product species j, $T_{\perp j} = T_{\perp j}(t)$, $T_{||j} = T_{||j}(t)$, the background deuterium plasma ions' parallel drift velocity is V_d (if any), and the fractional population of the fusion product species j is $\eta_j = n_j/n = n_j/\Sigma_i n_i = n_j/(n_d + \Sigma_j n_j)$ where n is the total density of ionic species (i.e., the background deuterium plasma ions plus all the newly born fusion product ions) in the plasma. In Eq. (9), since $T_{\perp j}$ and $T_{||j}$ are such that the widths $(2\kappa T_j/M_j)^{1/2} << V_j$, the Gaussian distribution $f_j(v_\perp, v_{||})$ of Eq. (9) reduces smoothly and analytically continously to the monoenergetic distribution of Eq. (4) when $T_j \to 0$. One may recall that, from a statistical thermodynamic point of view, the drifting Maxwell-Boltzmann distributions of Eqs. (8) and (9) are the most probable ones that are likely to occur naturally since, in general, these Gaussian type distributions are the ones that correspond to *minimum entropy production states.*

We pointed out earlier that according to Brysk, due to the center-of-mass thermalization with the background deuterium plasma, these initial δ function fusion products' velocity space birth distribution functions will in time acquire thermal spreads of order $T_j(t \approx \tau_j)$. However, according to Rome, *et. al.*[7] these are only quasisteady state thermal spreadings of the velocity distribution functions. Then for times $t > \tau_{sj}$, these fusion products' velocity distribution functions will relax toward the corresponding slowing-down distribution functions of the form

$$f_j(v) = \begin{cases} (A_{oj}/v_{cj}^3)[\,1 + (v/v_{cj})^3]^{-1} & \text{for } v \le V_j = (2E_j/M_j)^{1/2} \\ 0 & \text{for } v > V_j, \end{cases} \qquad (10)$$

where $v_{cj} = (3\sqrt{\pi}/4)^{1/3}(Z_{eff}\, M_e/M_j)^{1/3}\, v_e$, and $A_{oj} = 3/\{4\pi\, \ln[1 + (V_j/v_{cj})^3]\}$ since $\int d\mathbf{v} f_j(v) = \int 4\pi v^2\, dv f_j(v) = 1$. Here, $v_e = (2\kappa T_e/M_e)^{1/2}$ is the electron thermal speed, $Z_{eff} = (\Sigma_i\, n_i Z_i^2 / \Sigma_i\, n_i Z_i)$ is the effective value of the ionic charge, and the *slowing-down time* $\tau_{sj}(v)$

of the fast charged *test* ionic species j of speed v (by collisions with the background plasma electrons, i.e., the *field particles*) is given by[13]

$$\tau_{sj}(v) = (v_e^2\ v)/\ [(1 + M_j/M_e)\ A_D\ \psi(v/v_e)], \qquad (12)$$

where the *dynamical friction parameter* $A_D = 8\pi\ n_e\ e^4\ Z_j^2\ Z_e^2\ \ln\Lambda\ /\ M_j^2$, the electronic Z_e = 1, $\psi(x) \approx 2x/3\sqrt{\pi}$ for $x << 1$, and $\psi(x) \approx 1/2x^2$ for $x >> 1$. Also for these tokamak plasmas $V_j << v_e$, and since $M_e << M_j$, Eq. (12) yields that $\tau_{sj} \approx (3\ \sqrt{\pi}\ /\ 2)\ (M_e v_e^3\ /\ M_j A_D) = (M_j/M_e)\ \tau_e$, where τ_e is the usual Spitzer *electron collision time* [14]

It should be noted that according to Rome *et. al's.* result of Eq. (10), $f_j(v > V_j) = 0$. Obviously, this result cannot be exactly true since, due to their collisions with the background plasma electrons, these monoenergetic charged ionic species of Eq. (4) will undoubtedly first undergo a somewhat symmetric thermal spreading around their birth velocity V_j and first evolve to a Gaussian in velocity as given approximately by Eq. (9), and at later times evolve to the Brysk's Gaussian distribution in their energies with a somewhat symmetrical thermal spreading around their birth energy E_j as given by Eq. (5). That is, due to the energy or the heat exchange between the fusion products and the background plasma electrons, these fusion products initial monoenergetic distributions of Eq. (4) with $T_j(t = 0) \approx 0$ will first undergo a thermal spreading and evolve towards a distribution with a finite value of $T_j(t > 0) > 0$. Consequently, there will be some particles with $v > V_j$ and $E > E_j$ as pointed out by Brysk and given approximately by Eq. (5). The *thermalization time* τ_j is also the *heat or energy exchange time* and is given by

$$\tau_j \approx (M_j\ /\ 4M_e)\ \tau_e = (\tau_{sj}/4). \qquad (12)$$

The fact that τ_{sj} is always larger than τ_j as in Eq. (12) can be understood in the following way: The slowing down time of a test particle of velocity u is usually defined as $\tau_s = -< u/(du/dt)>$

where du/dt is the change of velocity due to collisions with the field particles (which are usually taken to have a Maxwellian distribution of velocities v) and the corresponding heat or the energy exchange time of this test particle is usually defined as $\tau = - < E^2/\{d(\Delta E)^2/dt\} > = - <(M^2u^4/4) \;/[d\{M^2(v^2 - u^2)^2/4\}/dt]> \approx - < u/4(du/dt)> = \tau_S/4$.

Hence, for the sake of completeness, we wish to point out that according to Brysk, for times $t > \tau_j$ the Rome *et. al's* slowing-down distribution of Eq. (10) has to be modified into the *piece-wise continous thermalized-slowing-down distribution* of the form

$$f_j(v) = (A_{oj}'/v_{cj}^3)[1 + (v/v_{cj})^3]^{-1} \quad \text{for } v \le V_j = (2E_j/M_j)^{1/2} \text{ and}$$

$$f_j(E) = (A_{oj}'/v_{cj}^3)[1 + (V_j/v_{cj})^3]^{-1}\exp\{-(E - E_j)^2/4\,[\kappa T_j(t \approx \tau_j)]^2\} \quad \text{for } E \ge E_j, \quad (13)$$

where

$$A_{oj}' = \{(4\pi/3)\ \ln[1 + (V_j/v_{cj})^3] + (2\,\pi^{3/2})(V_j 2\kappa T_j/M_j v_{cj}^3)\ [1 + (V_j/v_{cj})^3]^{-1}\}^{-1}$$

$$= \{ A_{oj}^{-1} + (2\,\pi^{3/2})(V_j v_j^2/v_{cj}^3)\ [1 + (V_j/v_{cj})^3]^{-1}\}^{-1}\ . \quad (14)$$

Here $v_j^2 = 2\kappa T_j/M_j$, f_j is normalized so that $\int (4\pi v^2)dv\ f_j(v)$ for $0 \le v \le V_j + \int dE\ f_j(E)$ for $E_j \le E \le \infty \;= 1$, and we have matched the two pieces of Eq. (13) so that $f_j(E_j) = (4\pi V_j^2\, dV_j/dE_j)f_j(V_j) = (4\pi V_j/M_j)f_j(V_j)$.

III. SPATIAL DISTRIBUTION OF THE FUSION PRODUCTS

Thus far we have examined the most probable zero-order forms of the velocity space distribution functions of the fusion products. In higher orders we must of course correct these distributions so as to take full account of the toroidal magnetic field effects of the tokamak geometry of the fusion devices under study such as, for example, the magnetic mirror trapping and the resulting loss cone distributions, etc. We have seen that initially newly born fusion products with approximately monoenergetic (velocity or energy) distributions, in the course of time, tend to simultaneously thermalize and slow down and eventually reach a steady-state distribution of the form given in Eq. (13). It is now of interest to examine the most probable spatial (and in particular the radial) distribution of these fusion products.

According to Glasstone and Lovberg[15] the rate of production of alpha particles in a DT and the protons in a DD plasma may be approximated as

$$n_\alpha(r) \approx 3.7 \times 10^{-12}\ (n_d n_t/T^{2/3})\ \exp(-20/T^{1/3})\ \text{cm}^{-3}\ \text{sec}^{-1},$$

and

$$n_p(r) \approx 2.3 \times 10^{-14}\ (n_d{}^2/2\ T^{2/3})\ \exp\ (-19/T^{1/3})\ \text{cm}^{-3}\ \text{sec}^{-1}, \qquad (15)$$

respectively, where T is the ion temperature in keV, n_d, n_t are the deuteron and triton densities in cm^{-3}, and the factor 1/2 in $n_d{}^2/2$ for DD instead of $n_d n_t$ for DT is introduced so that the interaction between identical nuclei should not be counted twice. If we now assume that n_d, n_t, and T all vary as $[1 - (r/a_p)^2]$ where a_p is the plasma radius, then Eq. (15) yield the radial birth profiles $n_j(r)$ of fusion products in tokamak geometries as

$$n_j(r) \approx n_{jo}\ [1 - (r/a_p)^2]^{4/3}\ \exp\{-20/[T_0(1 - r^2/a_p{}^2)]^{1/3}\}\ , \qquad (16)$$

where the central density $n_j(0) = n_{jo}\ \exp\{-20/\ T_0{}^{1/3}\}$, and the central ion temperature $T(0) = T_0$. For $r < a_p$, Eq. (16) may be approximated as

$$n_j(r) \approx n_j(0)[1 - (r/a_p)^2]^{4/3}\ \exp[-(20/3T_0{}^{1/3})(r/a_p)^2]. \qquad (17)$$

Thus we see that the fusion products birth profiles [whose radial variation is dominated by the exponential factors of Eqs. (16) and (17)] are centrally peaked on axis of the torus, i.e., the maximum value $\eta_{jmax} = \eta_j(r = 0)$. However, accordinger to Stringer,[16] for typical tokamak parameter conditions, about half the fusion products are formed with pitch angles in velocity space such that they are magnetically mirror traped (i.e., with $v_\perp \geq v_{||}/\sqrt{\varepsilon}$, where $\varepsilon = \Delta B/B \approx r/R$ is the mirror ratio). These trapped particles make radial excursions[17] of up to $\varepsilon^{1/2}\rho_\theta$, where $\rho_\theta = M_j v_\perp / qB_\theta$ is the Larmor radius in the poloidal field, and these excursions are very much larger than the widths of the production profile. Using Stringer's theory, very recent numerical calculations for JET by Cottrell *et. al.*[18] reveal a class of centrally born fusion products (i.e., approximately 10% centrally born within a narrow range of pitch angles just beyond the trapped-passing boundary) which make large radial excursions, sufficient to reach the outer midplane edge where the experimentally observed ICE seems to originate. Here, for convenience, we will call them the "marginally mirror-trapped" or the "trapped-passing boundary" particles, and for these particles $v_\perp \approx v_{||}/\sqrt{\varepsilon}$, where $\varepsilon \approx a_p/R_p$ for these boundary particles that make radial banana drift excursions large enough to reach the outer midplane plasma edge on the low field side of the torus, a_p and R_p are the plasma minor and major radius, respectively. It should be noted that these particles make drift excursions *only* to the low field side edge near the torus midplane, and they are also the ones with the fattest banana orbits.

IV. CONDITIONS FOR RADIATIVE INSTABILITIES

Let us first consider the cyclotron emission from a *dressed test charged particle* (i.e., of charge q, mass M) in a static confining magnetic magnetic field $\mathbf{B} = B\mathbf{i}_{||}$. The *field particles* are the ions and electrons of the background deuterium plasma. We are interested in examining the mth cyclotron harmonic emission from the charged fusion products of species j (i.e., primarily, protons of DD plasmas and alpha particles of DT plasmas) at and around frequencies $\omega \approx m\omega_{cj} < \omega_{LH}$, where ω_{LH} is the lower hybrid frequency of the background deuterium plasma and ω_c = qB/Mc. For these range of frequencies ($0 \le \omega \le \omega_{LH}$) the allowed electromagnetic waves are the fast Alfven waves and our interest, in particular, is in the resonances of these fast Alfven waves of the background deuterium plasma with the mth cyclotron harmonic of the energetic fusion ion products (such as the protons, alpha particles, tritons, and ^3He). Hence the background index of refraction μ (= $K^{1/2}$, where K is the corresponding dielectric coefficient) appropriate for *dressing the bare particle emission* is that corresponding to the fast Alfven waves and is given by

$$\mu = ck/\omega = c/V_A = [1 + (4\pi n_d M_d c^2/B^2)]^{1/2}, \tag{18}$$

where V_A is the Alfven wave phase velocity, M_d is the mass of a deuteron, and n_d is the number density of the background deuterium plasma ions. It is shown elsewhere[2] that the Einstein (quantum mechanical) spontaneous emission probability coefficient A(m) for such Trubnikov cyclotron harmonic emission is

$$A(m) = (4\pi^2 q^2/L^3 K h\omega)\ [v_\perp J_m'(\lambda)]^2\ \delta(\omega - m\omega_c - k_{||}v_{||}) \tag{19}$$

for the emission of the extraordinary (X) mode near the mth harmonic, and

$$A(m) = (4\pi^2 q^2/L^3 K h\omega)\ [(\mu^{-1}c\ \cos\theta - v_{||})J_m(\lambda)/\sin\theta]^2\ \delta(\omega - m\omega_c - k_{||}v_{||}) \tag{20 }$$

for the corresponding ordinary (O) mode emission, where $\lambda = k_\perp\rho = k_\perp v_\perp/\omega_c$, $J_m(\lambda)$ is the Bessel function of order m, $J_m'(\lambda) = dJ_m(\lambda)/d\lambda = [J_{m-1}(\lambda) - J_{m+1}(\lambda)]/2$, θ is the angle between **k** and **B**, and L^3 is the plasma volume under study. The O-mode emission is usually very small in comparison with the corresponding X-mode emission and hence we will neglect it in this paper. It should be noted that the *dressing factor* K in Eqs. (19) and (20) comes from an eigenmode Fourier analysis of the electromagnetic (e.m.) wave field energy density inside the whole tokamak plasma, i.e., the volume under consideration.[19] Hence, μ and V_A are to be taken as the average values for the entire plasma column, *regardless* of the location of the resonant emitting layer. This means that even if the resonant layer is in the scrape off region of the plasma where the ion density n_d is extremely low, we must use the average value of μ and K, and not the locally evaluated value of $\mu \propto n_d^{1/2}/B$, for our emission calculations everywhere. Here, we are using a particle orbit analysis, and the global features of the background medium enter via this dressing factor K. In the usual kinetic dispersion theory analysis, the Alfven wave dispersion $\omega = kV_A$ comes naturally as an average global quantity for the eigenmodes of the plasma column. The A(m)'s of Eqs. (19) and (20) are the Einstein spontaneous emission probabilities and are hence independent of the electromagnetic energy density $\varepsilon(\omega,\mathbf{k}) = \hbar\omega N(\omega,\mathbf{k})/L^3$, where $N(\omega,\mathbf{k})$ is the number of photons of frequency ω and wave vector $\mathbf{k}$ in the box of volume L^3 under study. The Einstein B coefficients for describing the induced emission and absorption are then proportional to these A(m)'s, and the probabilities of induced emission and absorption are given by $B(m)N(\omega,\mathbf{k})$ for $\omega \approx m\omega_{cj}$. By the *principle of detailed balance* the rate of increase of photons in the box of volume L^3 is given by

$$dN(\omega,\mathbf{k})/dt = [dN(\omega\mathbf{k})/dt]_{sem} - \{[dN(\omega,\mathbf{k})/dt]_{ab} - [dN(\omega,\mathbf{k})/dt]_{iem}\}, \tag{21}$$

where the suffixes sem, ab, and iem stand for spontaneous emission, absorption, and induced emission, respectively. One can show that

$$[dN(\omega \approx m\omega_c,\mathbf{k})/dt]_{sem} = \int dv_\perp(2\pi v_\perp) \int dv_{||}\, [L^3 nA(m)]\;\; f(v_\perp, v_{||}) = \langle [L^3 nA(m)]\rangle, \tag{22}$$

$$\{[dN(\omega \approx m\omega_c,\mathbf{k})/dt]_{ab} - [dN(\omega \approx m\omega_c,\mathbf{k})/dt]_{iem}\} = \int dv_\perp(2\pi v_\perp) \int dv_{||}\, [L^3 nA(m)]$$

$$\{(m\hbar\omega_c/Mv_\perp)(\partial/\partial v_\perp) + (\hbar k_{||}/M)(\partial/\partial v_{||})\}\ f(v_\perp, v_{||})\ N(\omega \approx m\omega_c,\mathbf{k})$$

$$= \langle [L^3 nA(m)]\ \{(m\hbar\omega_c/Mv_\perp)(\partial/\partial v_\perp) + (\hbar k_{||}/M)(\partial/\partial v_{||})\} \rangle\ N(\omega \approx m\omega_c,\mathbf{k})$$

$$= -\,2\gamma(\omega \approx m\omega_c,\mathbf{k})\ N(\omega \approx m\omega_c,\mathbf{k}), \tag{23}$$

where the angular brackets $< \cdots >$ refer to the statistical average over $f(v_\perp, v_{||})$, and γ is the damping rate. Thus it is seen from Eqs. (21) - (23) that the most general condition for linear radiative instability is

$$dN(\omega,\mathbf{k})/dt = \langle [L^3 nA(m)] \rangle - 2\gamma(\omega,\mathbf{k})\ N(\omega,\mathbf{k}) \geq 0. \tag{24}$$

This means that the most general condition for linear radiative instability requires that the spontaneous emission exceeds the absorption *minus* the stimulated emission. However, in classical plasma kinetic theory it is extremely difficult if not impossible to calculate this spontaneous emission, and thus one usually assumes that the intensity or the energy density of the radiation field in the box of volume L^3 under consideration is large enough that one can always neglect the spontaneous emission terms. Then, for linear radiative instability in classical plasma kinetic theory we only require that the induced emission exceeds the absorption, i.e., $\gamma(\omega,\mathbf{k}) \leq 0$. That is,

$$-2\gamma = \langle [L^3 nA(m)]\ \{(m\hbar\omega_c/Mv_\perp)(\partial/\partial v_\perp) + (\hbar k_{||}/M)(\partial/\partial v_{||})\} \rangle$$

$$= \langle [L^3 nA(m)]\ \{(m\hbar\omega_c)(\partial/\partial E_\perp) + (\hbar k_{||}/M)(\partial/\partial v_{||})\} \rangle$$

$$= \langle [L^3 nA(m)]\ \{(m\hbar\omega_c)(\partial/\partial E_\perp) + (\hbar k_{||}v_{||})(\partial/\partial E_{||})\} \rangle \geq 0. \tag{25}$$

Here again the angular brackets refer to an average over the distribution functions. It may be

pointed out that this damping rate γ of Eq. (25) is simply proportional to the anti-Hermitian part of the hot plasma dielectric tensor[20,12] appropriate to the total system (i.e., the background deuterium plasma ions plus all the charged fusion products) under study. Thus, Eq. (25) represents the necessary and sufficient condition for "a kinetic or a causal instability."[2] In the early literature other authors[21] have examined the corresponding "hydrodynamic instabilities"[2] resulting from the Hermitian part of the hot plasma dielectric tensor via the conventional dispersion relation. This instability condition of Eq. (25) is sometimes referred to as the cyclotron overstability condition in the classical plasma physics literature.[12] Under these conditions the system will behave as an ion cyclotron harmonic laser or maser for the fast Alfven waves of frequency $\omega \approx m\omega_c$ and wave vector $\mathbf{k}$. As seen from Eq. (24), the energy density of these unstable fast Alfven waves in the neighborhood of marginal stability is $\varepsilon(\omega,\mathbf{k}) = \hbar\omega N(\omega,\mathbf{k})/L^3 \approx \{(\hbar\omega/L^3) < [L^3 nA(m)] > / 2\gamma(\omega,\mathbf{k})\}$. That is, in this linear instability theory, the wave energy density in the neighborhood of marginal stability is inversely proportional to the wave linear damping rate and diverges linearly as $1/\gamma$. For growing waves the induced emission exceeds the absorption, i.e., $\gamma \leq 0$, and the system behaves as a negative temperature laser. However, when $\gamma \leq 0$, Eq. (24) is no longer adequate to determine the radiative steady state value of $\varepsilon(\omega,\mathbf{k})$, and one must modify Eq. (24) so as to include all the other nonlinear processes such as the nonlinear Landau or cyclotron damping, nonlinear mode coupling, linear (i.e., parametric) and nonlinear decay interactions, linear and nonlinear mode conversion and/or mode transformation, bounce frequency effects due to particle trapping, Karplus-Schwinger nonlinear resonance broadening, Dupree's turbulent velocity-space diffusion broadening, etc..[2,22]

From Eqs. (4) - (10), one can show that $[(m\omega_c/Mv_\perp)(\partial/\partial v_\perp) + (k_{||}/M)(\partial/\partial v_{||})]f(v_\perp, v_{||}) = [m\omega_c(\partial/\partial E_\perp) + (k_{||}/M)(\partial/\partial v_{||})]f(E_\perp, v_{||}) = [m\omega_c(\partial/\partial E_\perp) + (k_{||}v_{||})(\partial/\partial E_{||})]f(E_\perp, E_{||})$ is equal to

$+[(1/M_j)\ (m\omega_{cj} + k_{||}v_{||})]\ (1/v)[\partial f_j(v)/\partial v]$ for Eq. (4)

$-\ [2(E - E_j)(m\omega_{cj} + k_{||}v_{||})\{2\kappa T_j(t \approx \tau_j)\}^{-2}]\ f_j(E)$ for Eq. (5),

$-\ [\{2(E_\perp - E_j)m\omega_{cj} + 2(E_{||} - E_j)k_{||}v_{||}\}\{2\kappa T_j(t \approx \tau_j)\}^{-2}]\ f_j(E)$ for Eq. (6),

$-\ [(m\omega_{cd}/\kappa T_{\perp d}) + (k_{||}v_{||} - k_{||}V_d)/\kappa T_{||d}]\ f_d(E_\perp, v_{||})$ for Eq. (8),

$-\ [(m\omega_{cj}/\kappa T_{\perp j}) + (k_{||}v_{||} - k_{||}V_j)/\kappa T_{||j}]\ f_j(E_\perp, v_{||})$ for Eq. (9),

$-\ [(3/M_j)\{v/(v^3 + v_{cj}{}^3)\}(m\omega_{cj} + k_{||}v_{||})]\ f_j(v)$ for Eqs. (10) and (13) with $v \le V_j$,

and

$-[2(E - E_j)(m\omega_{cj} + k_{||}v_{||})\{2\kappa T_j(t \approx \tau_j)\}^{-2}]f_j(E)$ for Eq. (13) with $E \ge E_j$, (26)

respectively. Here, since the emission probability coefficients A(m) of Eqs. (19) and (20) are proportional to $\delta(\omega - m\omega_c - k_{||}v_{||})$, taking the average value of Eq. (25) over the parallel velocity distribution function will simply result in the selection of the value of Eq. (26) at the value of $v_{||}$ that is resonant with the jth species cyclotron phase velocity, i. e., $v_{||} = V_{jcph} = [(\omega - m\omega_{cj})/k_{||}]$.

We recall that the cyclotron frequencies of the protons (ω_{cp}), the deuterons (ω_{cd}), alpha particles ($\omega_{c\alpha}$), the tritons (ω_{ct}), and the helium-3s (ω_{cHe3}) in a given **B** field are connected by the relation $\omega_{cp} = 2\omega_{cd} = 2\omega_{c\alpha} = 3\omega_{ct} = (3/2)\omega_{cHe3}$. That is, when $\omega = m\omega_{cp}$, then $\omega = 2m\omega_{cd}$, $\omega = 2m\omega_{c\alpha}$, $\omega = 3m\omega_{ct}$, and $2\omega = 3m\omega_{cHe3}$. In other words, any given frequency can be in simultaneous resonance with different cyclotron harmonics of all the fusion products (say j = protons, alpha particles, tritons and ^{3}He) and those of the parents (deuterons and tritons).

Thus in Eq. (25), the contributions from the different fusion products and those of the parent particles will in general have different signs (some positive and some negative coming from different cyclotron harmonics that are resonant with the frequency under consideration). If their induced emission exceeds the absorption, their contribution to Eq. (25) is negative and leads to the growth of the instability; while if their absorption exceeds the induced emission, their contribution to Eq. (25) is positive and leads to the damping of the fast Alfven waves of frequency ω under study. For example, as seen from Eqs. (7) and (25), the newly born fusion product alpha particles induced radiative instabilities will occur in the neighborhood of frequencies $\omega \approx m\omega_{c\alpha} = m\omega_{cd}$, when the growth rate of these waves coming from the fusion alphas exceeds the corresponding damping rate of these same waves coming from the background deuterium plasma. In essence the instability under study is a "two-stream cyclotron harmonic instability or overstability" induced by the fast streaming newly born fusion products relative to the stationary background deuterium plasma ions. This is the simple physical picture of the instability or the overstability mechanism which we are studying here.

For the sake of analytical simplicity we will consider the emission from the fusion product species, <u>only one at a time</u>. Let us first consider a DD reaction dominated plasma. Here the background plasma is of deuterium ions and its velocity space distribution is a drifting Maxwellian of Eq. (8). Let us now choose to study the unstable ICE from the fusion product protons and take their velocity space distribution as given by Eq. (9) with j = p for protons. Then, since $m\omega_{cp} = 2m\omega_{cd}$, on making use of Eqs. (7) - (9), (19), and (26) in Eq. (25), we obtain the radiative instability and/or the overstability condition for copious unstable ICE at $\omega \approx m\omega_{cp}$ as

$$\{(1 - \eta_p)\ [v_{\perp d} J_{2m}'(\lambda_d)]^2\ [(2m\omega_{cd}/\kappa T_{\perp d}) + (\omega - 2m\omega_{cd} - k_{||}V_d)/\kappa T_{||d}]\}\ \{|k_{||}|^{-1} f_{d||}(v_{||}$$

$$= V_{pcph})\} + \{\eta_p\ [V_p J_m'(\lambda_p)]^2\ [(m\omega_{cp}/\kappa T_{\perp p}) + (\omega - m\omega_{cp} - k_{||}V_p)/\kappa T_{||p}]\}$$

$\{|k_{||}|^{-1} f_{p||}(V_{||} = V_{pcph})\} \leq 0,$

where the proton and/or the deterium ion cyclotron phase velocity $V_{pcph} = (\omega - m\omega_{cp})/k_{||} \approx V_{dcph} = (\omega - 2m\omega_{cd})/k_{||}$, since $m\omega_{cp} = 2m\omega_{cd}$. That is,

$$[\eta_p/(1 - \eta_p)] \geq - [v_{\perp d} J_{2m}'(\lambda_d)/V_p J_m'(\lambda_p)]^2 \ [\{|k_{||}|^{-1} f_{d||}(v_{||} = V_{pcph})\}/\{|k_{||}|^{-1} f_{p||}(v_{||} = V_{pcph})\}] \{[(m\omega_{cp}/T_{\perp d}) + (\omega - m\omega_{cp} - k_{||}V_d)/T_{||d}] / [(m\omega_{cp}/T_{\perp p}) + (\omega - m\omega_{cp} - k_{||}V_p)/T_{||p}]\} \geq 0. \quad (27)$$

Note, that for marginal stability, one must take the equality sign of the first inequality of Eq. (27) and, of course, Eq. (27) must yield a corresponding positive definite marginal-stability value for η_p. If the right hand side of the first inequality of Eq. (27) comes out to be negative, it simply means that one cannot have any radiative instability since the fast Alfven waves under study are damped by <u>both</u> the newly born fusion products ions and the background deuterium plasma ions. Thus, the last inequality of Eq. (27) ensures that the requirements of this Eq. (27) is a <u>necessary and sufficient condition for the radiative instability.</u> Here we have used the fact that bulk of the ICE is due to the emission in the extraordinary mode as given by Eq. (19). There is always a very small contribution to the ICE power from the ordinary mode emission of Eq. (20) and this will show up as a small double-humped structure in the region very near the peak of the observed ICE lines.[2] Equation (27) gives the most general condition for unstable ICE from the fusion product protons in the DD fusion reaction dominated plasmas. It is clear that the fractional proton density has to exceed a certain critical threshold value for this unstable ICE to occur. Here, there are two sources of *free energy* that are driving this instability, namely: one coming from the ($\perp$, $||$)-temperature anisotropy and the other coming from the fusion product's directed birth velocity along the confining magnetic field (i.e., the inverse cyclotron damping). However, for example, if we now make the reasonable assumption that $T_{\perp d} = T_{||d} = T_d$, then Eq. (27) yields the instability condition for copious ICE as

$$[\eta_p/(1 - \eta_p)] \geq [v_{\perp d} J_{2m}'(\lambda_d)/ V_p J_m'(\lambda_p)]^2 \; [\{|k_{||}|^{-1} f_{d||}(v_{||} = V_{pcph})\} / \{|k_{||}|^{-1} f_{p||}(v_{||}$$

$$= V_{pcph})\}] \, (T_{||p}/T_d)(V_A - k_{||}V_d/k) \; / \; \{(k_{||}/k)[V_p - V_{pcph}] - (m\omega_{cp} T_{||p}/kT_{\perp p})\} \geq 0, \quad (28)$$

where $V_A = \omega/k$ is the fast Alfven wave phase velocity. Note that in Eq. (28) usually $V_A >> k_{||}V_d/k$. Hence for the instability to occur <u>not only</u> η_p has to exceed the threshold critical value given by the right hand side of the first inequality sign of this Eq. (28) <u>but also</u> the denominator of this equation has to be a positive definite quantity, i.e., $(k_{||}/k)\ (V_p - V_{pcph}) \geq (m\omega_{cp}/k)\ (T_{||p}/T_{\perp p}) \approx V_A\ (T_{||p}/T_{\perp p})$. That is, for this instability to occur both η_p and V_p has to simultaneously exceed their respective critical threshold values. Equation (28) can also be used to determine the range of $k_{||}/k$ for which the fast Alfven waves under study are unstable.

In a similar way, assuming that $V_d = 0$ and $T_{\perp d} = T_{||d} = T_d$, one can show that the approximate instability condition for copious ICE at $\omega \approx m\omega_{cp}$ may be written

$$[\eta_p/(1 - \eta_p) \geq - \mathbf{(}[v_{\perp d} J_{2m}'(\lambda_d)]^2 \, / \, \partial\{[V_p J_m'(\lambda_p)]^2/V_p\} \, / \, \partial V_p\mathbf{)} \; (M_p/\kappa T_d) \; [\{f_{d||}(v_{||} =$$

$$V_{pcph})\} / \{\delta(V_{pcph} - g_\delta V_p)\}] \geq 0$$ for the distribution of Eq. (4) with $g_\delta \approx 1/3$ for $\delta(v - V_j)$ and $g_\delta \approx 1$ for $\delta(v_\perp - V_j)\delta(v_{||} - V_j)$,

$$[\eta_p/(1 - \eta_p)] \geq [v_{\perp d} J_{2m}'(\lambda_d)/ g_e V_p J_m'(g_e\lambda_p)]^2 \; [\{f_{d||}(v_{||} = V_{pcph})\} / \{f_j(E_\perp \approx g_e{}^2 E_j,\ E_{||}$$

$$= M_p V_{pcph}{}^2/2)\}] \; (T_p/T_d) \; / \; \{[(V_p{}^2 - V_{pcph}{}^2)/2v_p{}^2] - g_e{}^2\} \geq 0$$ for the distribution of Eq.

(5) with $g_e^2 \approx 1$,

$[\eta_p/(1 - \eta_p)] \geq [v_{\perp d} J_{2m}'(\lambda_d)/(V_p J_m'(\lambda_p)]^2$ $[\{f_{d||}(v_{||} = V_{pcph})\} / \{f_{p||}(E_{||} =$

$M_p V_{pcph}^2/2)\}]$ $(T_p\omega/T_d m\omega_{cp})$ / $\{[(V_p^2 - V_{pcph}^2)/2v_p^2]\ [(\omega/m\omega_{cp}) - 1] - (1/\sqrt{\pi})\} \geq 0$

for the distribution of Eq. (6), and

$[\eta_p/(1 - \eta_p)] \geq - [v_{\perp d} J_{2m}'(\lambda_d)/g_s V_p J_m'(g_s\lambda_p)]^2$ $[f_{d||}(v_{||} = V_{pcph})]$ $(M_p/3\pi A_{op} M_d)$

$[v_{cp}^3 + (g_s^2 V_p^2 + V_{pcph}^2)^{3/2}]^2 / [v_d^2 g_s^2 V_p^2 (g_s^2 V_p^2 + V_{pcph}^2)^{1/2}] \geq 0$, for the

distributions of Eqs. (10)and (13) with $g_s < 1$ and $V_{pcph} \leq V_p$, (29)

where $v_p^2 = 2\kappa T_p/M_p$, $v_d^2 = 2\kappa T_d/M_d$. Here, the numerical factors g are somewhat less than unity. The integrals over the perpendicular velocity and/or perpendicular energy in evaluating the angular bracket of Eq. (25) for the distribution functions of Eqs. (5), (10) and (13) has to be done numerically. Physically, such a numerical integration will yield $< v_\perp > \approx gV_j$, and $< E_\perp > \approx g^2 E_j$, where $g < 1$.

It is interesting to note from Eq. (29) that the right hand side of this equation is always negative for the slowing-down distributions of Eqs. (10) and (13). This implies that the absorption always exceeds the induced emission and consequently the cyclotron harmonic fast Alfven waves can never become unstable for these slowing-down distributions. Further, by comparing Eqs. (27) or (28) with Eq. (29), it is apparent that it is easier to make the fast Alfven waves under study become unstable with the Brysk's type distributions of Eq. (5) than with the drifting Maxwellian of Eq. (9). However, it is interesting to note that when an approximate ($\perp$, $||$)-decomposition is made of this Brysk's distribution, the resulting ($\perp$, $||$)-decoupled distribution of Eq. (6) cannot give rise to any radiative instability at all. From now on we will simply concentrate mostly on the drifting Maxwellian type distributions of Eqs. (8) and (9), since they are the most probable ones from the point of view of statistical thermodynamics (i.e., they correspond to the states of minimum entropy production).

It is relatively easy to show that approximately the same instability conditions of Eqs. (27) - (29) apply for copious ICE from any of the fusion product species j in both the DD and the DT reaction dominated plasmas with η_p, $\lambda_p = k_\perp V_p/\omega_{cp}$, T_p, and V_p replaced by the appropriate values of η_j, λ_j, T_j, and V_j, respectively, for the species j (= protons, or alpha particles, or tritons, or He-3s). For example, for unstable alpha particle ICE from the DT fusion reaction dominated plasmas, Eqs. (27) and (28) become

$$[\eta_\alpha/(1 - \eta_\alpha)] \geq - [v_{\perp d} J_m{}'(\lambda_d)/ V_\alpha J_m{}'(\lambda_\alpha)]^2 \; [\{|k_{||}|^{-1} f_{d||}(v_{||} = V_{\alpha cph})\} /$$

$$\{|k_{||}|^{-1} f_{\alpha||}(v_{||} = V_{\alpha cph})\}] \; \{[m\omega_{c\alpha}(T_{\perp d}{}^{-1} - T_{||d}{}^{-1}) + (\omega - k_{||}V_d)/T_{||d}] / [m\omega_{c\alpha}(T_{\perp\alpha}{}^{-1}$$

$$- T_{||\alpha}{}^{-1}) + (\omega - k_{||}V_\alpha)/T_{||\alpha}]\} \geq 0, \qquad (30)$$

and

$$[\eta_\alpha/(1 - \eta_\alpha)] \geq [v_{\perp d} J_m{}'(\lambda_d)/ V_\alpha J_m{}'(\lambda_\alpha)]^2 \; [\{|k_{||}|^{-1} f_{d||}(v_{||} = V_{\alpha cph})\} / \{|k_{||}|^{-1} f_{\alpha||}(v_{||}$$

$$= V_{\alpha cph})\}](T_{||\alpha}/T_d)(V_A - k_{||}V_d/k) \; / \; \{(k_{||}/k)[V_\alpha - V_{\alpha cph}] - (m\omega_{c\alpha}T_{||\alpha}/kT_{\perp\alpha})\} \geq 0, \quad (31)$$

respectively, where we have used the fact that $\omega_{cd} = \omega_{c\alpha}$, and Eq. (31) applies only when there is a complete ($\perp$, $||$)-isotropy in the temperature of the background deuterium plasma. Again from Eq. (31) one can determine the range of $k_{||}/k$ for which the cyclotron harmonic fast Alfven waves of frequency $\omega \approx \omega_{c\alpha} = \omega_{cd}$ are unstable when $T_{\perp d} = T_{||d} = T_d$ and $V_d = 0$. It is interesting and physically instructive to examine the $k_{||} \rightarrow 0$ limit of Eq. (30). In this limit $\{|k_{||}|^{-1} f_{d||}(v_{||} = V_{\alpha cph})\} \approx \{|k_{||}|^{-1} f_{\alpha||}(v_{||} = V_{\alpha cph})\} \rightarrow \delta(\omega - m\omega_{c\alpha})$. Thus for $k_{||} \approx 0$, Eq. (30) becomes

$$[\eta_{\alpha}/(1 - \eta_{\alpha})] \geq - [v_{\perp d} J_m'(\lambda_d)/ V_{\alpha} J_m'(\lambda_{\alpha})]^2 \ (T_{||\alpha}/T_{||d}) \ \{[(T_{||d} / T_{\perp d}) - 1 +$$

$$(\omega/m\omega_{c\alpha})] / [(T_{||\alpha} / T_{\perp \alpha}) - 1 + (\omega/m\omega_{c\alpha})]\} \geq 0. \qquad (32)$$

It is clear from Eq. (32) that if its right hand side is positive definite then $k_{||} \approx 0$ cyclotron harmonic fast Alfven waves will be unstable for sufficiently large values of the fusion product alpha particle fractional density η_{α}. From Eq. (32) we find that $[\eta_{\alpha}/(1 - \eta_{\alpha})] \geq 0$ if either $\omega/m\omega_{c\alpha} \leq 1 - T_{||d}/T_{\perp d}$ (which in turn implies that $\omega \leq m\omega_{c\alpha}$ and $T_{\perp d} \geq T_{||d}$) or $\omega/m\omega_{c\alpha} \leq 1 - T_{||\alpha}/T_{\perp \alpha}$ (which in turn implies that $\omega \leq m\omega_{c\alpha}$ and $T_{\perp \alpha} \geq T_{||\alpha}$). Thus, from Eqs. (30) - (32) we find that with sufficient values of the temperature anisotropy and with sufficient values of the fusion products birth drift velocities along the confining magnetic field, one can in principle have unstable ICE from these charged fusion products. Of course, simultaneously, their fractional density η_{α} should also exceed the corresponding threshold value.

It should be noted from Eqs. (19) and (20) that the fundamental emission probability coefficient $A(m) \propto \delta(\omega - m\omega_c - k_{||}v_{||})$. That is, the particle-wave resonance interaction occurs only when the charged ionic species parallel velocity $v_{||}$ exactly matches the fast Alfven wave cyclotron phase velocity $V_{cph} = (\omega - m\omega_c)/k_{||}$. Thus, in the radiative instability condition of Eq. (25), it is only the term that is proportional to the slope of the parallel velocity distribution function (i.e., the term $\propto k_{||}v_{||} \ \partial/\partial E_{||}$) whose sign will depend on the sign of $(\omega - m\omega_c)$. There is, however, no particle-wave resonance interaction anywhere in the perpendicular velocity space, and consequently the result of Eq. (25) is not at all sensitive to the sign of the slope of the perpendicular velocity space distribution function anywhere; and indeed by performing a simple partial integration over $dE_{\perp}$ one can easily show that the instability condition of Eq. (25) depends only on the various perpendicular velocity (or perpendicular energy) moments of the distribution function (i.e., in dimensionless parameter it is proportional to $T_\perp/T_{||}$). Hence, in particular, any perpendicular velocity space distribution however anisotropic (say, for example, $T_x \neq T_y$) and non-monotonic it may be in any range of values of $v_\perp$, it certainly cannot provide the necessary free energy to drive the cyclotron harmonic fast Alfven waves under study to become unstable since there exists no perpendicular velocity at which a particle-wave resonance can occur, i.e., there is no coupling whatsoever between the waves and the particles' perpendicular velocity distribution function anywhere in the range $0 \leq v_\perp \leq \infty$. However, we should point out that in Eqs. (19) and (20) we have neglected the effects of magnetic curvature drifts. Since the δ functions of these equations are a consequence of the total energy and only the parallel (but not the perpendicular) momentum conservation, it is relatively easy to show[8,2] that taking account of the magnetic curvature drifts in these equations will result in the replacement of $\delta(\omega - m\omega_c - k_{||}v_{||})$ by $\delta(\omega - m\omega_c - k_{||}v_{||} - \omega_{mcd})$, where $\omega_{mcd} = \mathbf{k}\cdot\mathbf{v}_{mcd} = k_\perp v_{mcd} = k_\perp(c\kappa T_{||}/qBR)$ is the magnetic curvature drift frequency corresponding to the magnetic curvature drift velocity v_{mcd}. That is, taking account of the effects of magnetic curvature drifts in our radiative instability analysis of ICE will simply result in the replacement of ω by $(\omega - \omega_{mcd})$ in Eqs. (27), (29), and (32), or equivalently, the approximate frequencies of the ICE lines are given by $\omega \approx m\omega_{cj} + \omega_{mcd}$; i.e., all the cyclotron harmonic emission lines have their frequencies upshifted by the constant amount equal to ω_{mcd}. Since, $\omega_{mcd} = k_\perp(2\kappa T_{||}/M)\ (qB/Mc)^{-1}$ $(1/2R) = k_\perp v_t^2/2\omega_c R = (k_\perp\rho)\ (\rho/2R)\omega_c \ll \omega_c$, this constant frequency-upshift due to magnetic curvature drift effects is negligibly small. The diamagnetic drift frequency rotation of the plasma $\omega_* = k_\perp(c\kappa T/qBL_n)$ occurs only in one direction (either right or left handed depending on the sign of the charge) and these effects do not affect the δ functions of Eqs. (19) and (20). Here, $L_n = d(\ln n)/dr$ is the density gradient scale length. The diamagnetic effects alter the equilibrium distribution function via the guiding center perpendicular canonical momentum.[8,2] Since our radiative instability condition of Eq. (25) depends only on the various perpendicular velocity or perpendicular energy moments of the distribution function and, in particular, is very insensitive to the shape and slope of the perpendicular velocity distribution, it appears that these diamagnetic effects will not alter our instability conditions in any significant way.

In tokamak geometries a certain fraction of the contained charged particles will be magnetically trapped between the mirrors (since the magnetic moment of the particle is an adiabatic invariant). Indeed, the charged particles with $v_\perp \geq v_{||}/\sqrt{\varepsilon}$ are magnetically mirror trapped, and in the banana regime the fraction of particles that are magnetically trapped $\approx \sqrt{\varepsilon}$, where $\varepsilon \approx \Delta B/B \approx r/R$ for tokamaks is a measure of the mirror ratio. We stated earlier that for our present tokamak operating conditions approximately 10% of the centrally born fusion products within a narrow range of pitch angles just near the trapped-passing boundary (i.e., those with $v_\perp \approx v_{||}/\sqrt{\varepsilon}$) make large radial banana excursions sufficient to reach the outer midplane edge where the experimentally observed ICE seems to originate. These "trapped-passing boundary particles" are the ones with the fattest banana orbits, and consequently, are able to make these large radial excursions to reach the outer midplane plasma edge. For these "trapped-passing boundary particles" since the birth energy $E_j = M_j V_j^2/2 = M_j(\varepsilon + 1)V_{\perp bj}^2/2$, $V_{\perp bj} = V_j/\sqrt{(1+\varepsilon)}$ and $V_{||bj} = V_j/\sqrt{(1+\varepsilon^{-1})}$. Further, since $v_\perp \approx v_{||}/\sqrt{\varepsilon}$, it seems reasonable to take $\varepsilon T_{\perp bj} \approx T_{||bj}$ for these particles. Thus for $0 \leq t < \tau_{sj}$ one can approximate these trapped-passing boundary alphas parallel velocity space distribution function by a drifting Maxwellian of the form

$$f_{||bj}(v_{||}, t) \approx [M_j/2\pi\kappa T_{||bj}(t)]^{1/2} \exp\{-[M_j/2\kappa T_{||bj}(t)]\ [v_{||} - V_{||bj}]^2\}. \qquad (33)$$

It may be noted that when $t \to 0$, $T_{||bj}(t) \to 0$, and $f_{||bj}(v_{||}, t) \to \delta(v_{||} - V_{||bj})$ as it should. See Eq. (4). Thus the drifting Maxwellian is a very natural evolution of an initially monoenergetic (or equivalently, an initially monovelocity) distribution during the period of thermalization of these newly born fusion products with the background deuterium plasma, i.e., for times $0 \le t < \tau_j$. It may be pointed out that because of the relation $\sqrt{\varepsilon}\, v_\perp \approx v_{||}$, this parallel velocity distribution of Eq. (33) suffices to generate the perpendicular velocity distribution, and consequently, the entire distribution $f_{bj}(v_\perp, v_{||}) \propto f_{||bj}(v_{||})$. This is true only for these trapped-passing boundary particles whose $v_\perp \approx v_{||}/\sqrt{\varepsilon}$. Then in Eq. (25), the linear differential operator

$$M^{-1} \{(m\omega_c/v_\perp)(\partial/\partial v_\perp) + k_{||}(\partial/\partial v_{||})\} = M^{-1} \{(\varepsilon m \omega_c/v_{||}) + k_{||}\} (\partial/\partial v_{||}). \tag{34}$$

If we now assert that only these "marginally mirror-trapped portion" of the centrally born fusion products are solely responsible for the experimentally observed ICE by a localized interaction of these fusion products at the outer midplane edge with the background deuterium plasma found there,[8-10,23] then on making use of Eqs. (8), (33) and (34) in Eq. (25), the necessary and sufficient condition for the radiative instability due to these marginally trapped alpha particles (which are very near the trapped-passing boundary so that their $\sqrt{\varepsilon}\, v_\perp \approx v_{||}$) may be written

$$[(b\eta_\alpha/a)/\{(1 - (b\eta_\alpha/a)\}] \ge - [v_{\perp de} J_m'(\lambda_{de})/V_{\perp b\alpha} J_m'(\lambda_{b\alpha})]^2 \ [\{|k_{||}|^{-1} f_{d||}(v_{||} = V_{\alpha cph})\}$$

$$/ \{|k_{||}|^{-1} f_{||b\alpha}(v_{||} = V_{\alpha cph})\}]([m\omega_{c\alpha}(T_{\perp de}^{-1} - T_{||de}^{-1}) + (\omega - k_{||}V_{de})T_{||de}^{-1}] /$$

$$[(\varepsilon m \omega_{c\alpha} + \omega - m\omega_{c\alpha})(1 - V_{||b\alpha}/V_{\alpha cph})T_{||b\alpha}^{-1}]) \ge 0, \tag{35}$$

where $\lambda_{b\alpha} = k_\perp V_{\perp b\alpha}/\omega_{c\alpha} = k_\perp V_\alpha/\sqrt{(1 + \varepsilon)}\omega_{c\alpha} = \lambda_\alpha/\sqrt{(1 + \varepsilon)}$, and $V_{||b\alpha} = V_\alpha/\sqrt{(1 + \varepsilon^{-1})}$. Here we have assumed that in this plasma edge region $n_i(r \approx a_p) \approx a n_i(r \approx 0)$ and $n_\alpha(r \approx a_p) \approx b n_\alpha(r \approx 0)$, i.e., $\eta_\alpha(r \approx a_p) \approx [b\eta_\alpha(r \approx 0)/a]$, i.e., $\eta_{\alpha e} \approx (b\eta_\alpha/a)$, and the suffix e stands for the values in this outer midplane plasma edge region. If we further set $T_{\perp de} = T_{||de} = T_{de}$ and $V_{de} = 0$ for the background deuterium plasma, then Eq. (35) may be rewritten as

$$\{\eta_\alpha/[(a/b) - \eta_\alpha]\} \geq [v_{\perp de} J_m'(\lambda_{de})/V_{\perp b\alpha} J_m'(\lambda_{b\alpha})]^2 \; (T_{||b\alpha}/T_{||de}) \; [\{|k_{||}|^{-1} f_{d||}(v_{||} =$$

$$V_{\alpha cph})\} / \{|k_{||}|^{-1} f_{||b\alpha}(v_{||} = V_{\alpha cph})\}] / \varepsilon\{[V\alpha/V_{\alpha cph}\sqrt{(1 + \varepsilon^{-1})}] - 1\} \geq 0 \qquad (36)$$

The physical meaning of the fast Alfven wave radiative instability conditions of Eqs. (35) and (36) is as follows: The fraction $b \approx 0.1$ (i.e.,10%) of the newly born fusion product alpha particles which are marginally trapped with $v_\perp \approx v_{||} / \sqrt{\varepsilon} \approx v_{||}(R_p/a_p)^{1/2}$ make banana excursion orbits sufficient to reach the outer midplane plasma edge on the low field side of the torus and for these particles the induced emission of the cyclotron harmonic fast Alfven wave of frequency $\omega \approx m\omega_{c\alpha}$ exceeds their absorption, and consequently, leads to the growth of these waves; while for this edge region background deuterium plasma the absorption exceeds the induced emission of these waves of frequency $\omega \approx m\omega_{c\alpha} \approx m\omega_{cd}$ and thus leading to the damping of these waves. The instability condition of Eqs. (35) and (36) simply states that the growth due to the fusion products exceeds the damping coming from the background deuterium plasma. This simple physical picture is made possible only because the alpha particle and the deuteron cyclotron frequencies are degenerate, i.e., $m\omega_{c\alpha} = m\omega_{cd}$ for all the harmonics m. For the proton cyclotron harmonic emission in the background deuterium plasma this degeneracy is such that $m\omega_{cp} = 2m\omega_{cd}$, and for such ICE from DD reaction dominated plasmas, Eq. (36) becomes

$$\{\eta_p/[(a/b) - \eta_p)] \geq [v_{\perp de} J_{2m}'(\lambda_{de})\sqrt{(1 + \varepsilon)}/V_p J_m'(\lambda_{bp})]^2 (T_{||bp}/T_{||de}) \; [\{ |k_{||}|^{-1} f_{d||}(v_{||}$$

$$= V_{pcph})\} / \{|k_{||}|^{-1} f_{||bp}(v_{||} = V_{pcph})\}] / \varepsilon\{[V_p/V_{pcph}\sqrt{(1 + \varepsilon^{-1})}] - 1\} \geq 0. \qquad (37)$$

V. SOME NUMERICAL ESTIMATES AND RELEVANT DISCUSSIONS

We now wish to make some approximate numerical estimates of the fusion products induced cyclotron harmonic fast Alfven wave radiative instability conditions for the typical tokamak parameters found in TFTR. We take the following DT reaction dominated "supershot" plasma parameter conditions at t $\approx$ 200 ms after the heating (tritium) beam injection in TFTR:[5,6]

The background "supershot" plasma is of deuterium ions with the density $n_e = n_i \approx n_d \approx 5 \times 10^{13}$ cm^{-3}; $T_{\perp e} \approx T_{||e} \approx 9$ keV, $T_{\perp d} \approx T_{||d} = T_d \approx 25$ keV (and hence $v_d \approx 1.54 \times 10^8$ cm/s); the directed energy $E_\alpha = M_\alpha V_\alpha^2/2$ of the newly born fusion alphas is $E_\alpha \approx 3.6$ Mev (i.e., their birth velocity $V_\alpha \approx 1.3 \times 10^9$ cm/s) for these DT reaction dominated plasmas; according to Brysk[11] the amount of thermal spread $\kappa T_\alpha(t \approx \tau_\alpha) \approx [M_\alpha T_d E_\alpha/(M_\alpha + M_n)]^{1/2}$ in the directed birth energy of fusion alphas is $T_\alpha \approx 270$ keV; the tokamak major radius $R_0 \approx 2.65$ m, the plasma major radius $R_p \approx 2.45$ m, the plasma minor radius $a_p \approx 80$ cm, the minor radius of the vacuum vessel $a_0 \approx 1.2$ m, the critical major radius of the resonant cyclotron layer from which the experimentally observed ICE seems to originate $R_c \approx R_p + a_p \approx 3.25$ m (since the observed $\omega_{c\alpha}/2\pi = \omega_{cd}/2\pi \approx 27.5$ MHz); at the plasma radius a_p, $n_e \approx n_i \approx 1 \times 10^{12}$ cm^{-3}, $T_e \approx T_i \approx 1$ keV in this "supershot regime", and in the scrape-off plasma the density and temperature profiles are approximately exponential with an e-folding length of about 2 - 3 cm; the confining magnetic field $B \approx 4.45$ T $= 4.45 \times 10^4$ G at $R = R_0$; the approximate fractional population $\eta_\alpha \approx 3 \times 10^{-3}$ for these DT fusion reaction dominated TFTR plasmas;[5] the plasma dielectric coefficient for Alfven waves is $K = \mu^2 \approx 4\pi n_d M_d c^2/B^2 \approx 820$, i.e., the Alfven wave index of refraction $\mu \approx 28.6$ and the Alfven wave phase velocity $V_A = \omega/k = c/\mu \approx 1.05 \times 10^9$ cm/s, i.e., $V_\alpha/V_A \approx 1.25$; if we now assume that $k \approx k_\perp$ (since the intensity of all the observed harmonic ICE are roughly the same), then $\lambda_d \approx kv_d/\omega_{cd} = mkv_d/m\omega_{cd} \approx mv_d/V_A \approx 0.15$m, at the plasma edge $\lambda_{de} \approx 0.15\text{m}/5 \approx 0.03$m since T_d $(r = 0)/$ $[T_d(r = a_p) = T_{de}] \approx 25$ keV/1 keV $= 25$, and similarly $\lambda_\alpha \approx 1.25$m, and $a = n_i(r = a_p) / n_i(r = 0) \approx 0.02$, $b \approx 0.1$; for the trapped-passing boundary alphas $\varepsilon \approx a_p/R_p \approx 0.327$, $\lambda_{b\alpha} \approx \lambda_\alpha/\sqrt{(1 + \varepsilon)} \approx 1.09$m. Also for sufficiently small $k_{||}$ (since $k \approx k_\perp$) $\{|k_{||}|^{-1} f_{d||}(v_{||} = V_{\alpha cph})\} \approx \{|k_{||}|^{-1} f_{||b\alpha}(v_{||} = V_{\alpha cph})\}$

$\approx \delta(\omega - m\omega_{c\alpha})$. For these conditions the alpha particle's slowing-down time $\tau_{s\alpha}$ from the plasma center to edge is[6] $\tau_{s\alpha} \approx 650$ to 130 ms.

For the background deuterium plasma, $\lambda_d \approx 0.15m << 1$ and $\lambda_{de} \approx 0.03m << 1$ for the harmonics of interest to us, and thus $\lambda_d J_m'(\lambda_d) \approx (\lambda_d / 2)^m / (m - 1)!$, and the damping of the cyclotron harmonic fast Alfven waves under study in Eqs. (30) - (32), (35) and (36) which is proportional to $\{(1 - \eta_\alpha) [\lambda_d J_m'(\lambda_d)]^2\}$ for the bulk plasma ineraction and/or $\{(1 - b\eta_\alpha/a)[\lambda_{de} J_m'(\lambda_{de})]^2\}$ for the edge plasma interaction decreases rapidly as $\lambda_d{}^{2m}$ and/or $\lambda_{de}{}^{2m}$ with increasing harmonic number m; while for the newly born fusion alpha particles $\lambda_\alpha \approx 1.25m > 1$ and $\lambda_{b\alpha} = \lambda_\alpha/\sqrt{(1 + \varepsilon)} \approx 1.09m > 1$ for $k \approx k_\perp$, and hence the growth of these fast Alfven waves in these equations which is proportional to $\{\eta_\alpha [\lambda_\alpha J_m'(\lambda_\alpha)]^2\}$ for the bulk plasma interaction and/or $\{(b\eta_\alpha/a)[\lambda_{b\alpha} J_m'(\lambda_{b\alpha})]^2\}$ for the edge plasma interaction is roughly a constant value independent of the harmonic number m for a given η_α. Thus the rapid decrease of the factor $Y_{\alpha m} = [\lambda_d J_m'(\lambda_d)/\lambda_\alpha J_m'(\lambda_\alpha)]^2$ in Eqs. (30) - (32) and $Y_{mb\alpha} = [\lambda_{de} J_m'(\lambda_{de})/\lambda_{b\alpha} J_m'(\lambda_{b\alpha})]^2$ in Eqs. (35) and (36) with increasing harmonic number m for the alpha particles is mainly due to the rapid decrease in the cyclotron damping of the fast Alfven waves by the background deuterium plasma, while the growth rate of these waves is roughly the same value for all m at a given value of η_α. Thus, if one can satisfy the instability conditions of Eqs. (30) - (32), (35) and (36) for m = 1, then they will be automatically satisfied for all the higher harmonics m > 1.

Before proceeding to examine the radiative instability conditions of Eqs. (30) - (32), (35), and (36), it is physically instructive to first understand the behavior of the cyclotron phase velocity $V_{jcph} = (\omega - m\omega_{cj})/k_{||} = \Delta\omega/k_{||}$. For $k_{||} > 0$, $V_{jcph} < 0$ for $\omega < m\omega_{cj}$, $V_{jcph} > 0$ for $\omega > m\omega_{cj}$; and for $k_{||} < 0$, $V_{jcph} > 0$ for $\omega < m\omega_{cj}$, $V_{jcph} < 0$ for $\omega > m\omega_{cj}$; and for $k_{||} \neq 0$, $|V_{jcph}|$ takes very large values when $|\Delta\omega| >> k_{||}$, it takes very small values when $|\Delta\omega| << k_{||}$, and is zero for $\Delta\omega = 0$. That is, V_{jcph} can take any value, positive or negative, arbitrarily small or large, anywhere in the range $-\infty \leq V_{jcph} \leq \infty$. However, for $k_{||} = 0$ and $\Delta\omega = 0$ (simultaneously), V_{jcph} is indeterminate within the framework of the conventional nonrelativistic theories[24] (i.e., the nonrelativistic quantum particle orbit theory used here[20] or, equivalently, the conventional classical hot plasma theory[12]). For example, if we let $k_{||} \to 0$ first and then let $\omega \to m\omega_{cj}$, we find that $|V_{jcph}| \to \propto$; while if we let $\omega \to m\omega_{cj}$ first and then let $k_{||} \to 0$, we find that $|V_{jcph}| \to 0$. Thus in the simultaneous dual limit of $k_{||} \to 0$ and $\omega \to m\omega_{cj}$, V_{jcph} is indeterminate.[24] Presumably, one may have to do a fully relativistic analysis in order to resolve this indeterminate nature of V_{jcph} in this dual limit. However, for some of our purposes we need only $(k_{||}/k)V_{jcph}$. It is relatively easy to show that $|(k_{||}/k)V_{jcph}| << |(\omega/k)| = |V_A|$, since $|\Delta\omega| << |\omega|$. Since $m\omega_{c\alpha} = m\omega_{cd}$, $V_{\alpha cph} = (\omega - m\omega_{c\alpha})/k_{||} = (\omega - m\omega_{cd})/k_{||} = V_{dcph}$, one can show that the dominant wave-particle interaction occurs for this degenerate resonance (i.e., $m\omega_{c\alpha} = m\omega_{cd}$) radiative instability under study when $V_{\alpha cph} \approx V_{dcph} \approx v_d = (2\kappa T_d/M_d)^{1/2}$, since the ratio $[V_{dcph} / (2\kappa T_d/M_d)^{1/2}]$ is the usual argument of the familiar dispersion function of the background deuterium plasma.[12]

Let us first examine the instability condition of Eq. (31). The necessary condition for $\eta_\alpha > 0$ implies that $(k_{||}/k)(V_\alpha - V_{\alpha cph}) > (m\omega_{c\alpha}/k)\ (T_{||\alpha}/T_{\perp\alpha})$, i.e., $k_{||}/k > V_A/V_\alpha$ since for TFTR conditions $T_{||\alpha} \approx T_{\perp\alpha}$, $\omega \approx m\omega_{c\alpha}$, and $V_\alpha - V_{\alpha cph} \approx V_\alpha >> V_{\alpha cph} \approx v_d$ for dominant wave-particle interaction. Hence for the instability of Eq. (31) to occur we need $k_{||}/k > 1/1.25 = 0.8$. However, since the observed ICE is comprised of all harmonics of almost equal intensity we must have $k_\perp\rho_\alpha > 1$ which can only be satisfied with $k \approx k_\perp$. Thus for the present TFTR conditions it is not possible to satisfy the instability condition of Eq. (31). Further, since for our TFTR conditions $T_{||\alpha} \approx T_{\perp\alpha}$ and $T_{||d} \approx T_{\perp d}$, the right hand side of the first inequality of Eq. (32) is a negative quantity, and hence there is no instability. Thus the bulk of the newly born fusion alphas which are fully mirror-trapped or fully circulating cannot excite radiative instabilities inside the main body of the plasma for our present TFTR conditions.

Let us now examine whether or not it is possible to excite these radiative instabilities by the small fraction (i.e., $b \approx 0.1$) of the centrally newly born fusion alphas that have a narrow range of pitch angles just near the trapped-passing boundary, and these "boundary alphas" make large radial excursions sufficient to reach the outer midplane edge where the experimentally observed ICE seems to originate. Since we have isotropic temperature conditions in TFTR we have to use the "boundary alphas instability condition" of Eq. (36). For η_α of Eq. (36) to yield a positive value, the necessary condition for the edge radiative instability is $V_\alpha > \sqrt{(1 + \varepsilon^{-1})}$ $(2\kappa T_{de}/M_d)^{1/2} \approx 2\ (2\kappa T_{de}/M_d)^{1/2} = 2v_{de} \approx 2\ v_d/5$. Since $V_\alpha >> v_d$, this necessary condition for edge radiative instability to be excited by these "boundary alphas" is well satisfied for the present TFTR conditions. $Y_{mb\alpha} < Y_{1b\alpha} \approx (\lambda_{de} / \lambda_{b\alpha})^2 \approx 7.6 \times 10^{-4}$. As we stated earlier, since half the edge population of the TFTR "supershot plasmas" is fully ionized carbon, then the second condition of Eq. (36) for sufficiency to induce this edge radiative instability become $\eta_\alpha \geq (0.015)^{2(m-1)} \times 0.23 \times 10^{-3} [\kappa T_{||b\alpha}(t)/\kappa T_{de}] / \{(1/2)(E_\alpha/\kappa T_{de})^{1/2} - 1\}$ $\approx (0.015\ )^{2(m-1)} \times 2.8 \times !0^{-2} [\kappa T_{||b\alpha}(t)/E_\alpha]$. Initially, at t = 0, the fusion alphas are born monoenergetic and hence $T_{||b\alpha}(t = 0) \approx 0$ { and indeed up to times t such that $[\kappa T_{||b\alpha}(t)/$ $E_\alpha] \leq 0.11$} this instability condition is satisfied for all the harmonics m. But when $[\kappa T_{||b\alpha}(t)/ E_\alpha] > 0.11$, this condition cannot be satisfied for m = 1, but is again satisfied for m ≥ 2. As we stated earlier from the TRANSP code plasma analysis of reference 6, we find that $[\kappa T_{||b\alpha}(t) / E_\alpha] \approx 0.11$ when t ≈ 90 ms. We should point out that strictly speaking the observed alpha particle ICE seems to originate at 4 to 5 cm beyond the limitter in the scrape-off layer. In this scrape-off layer both the density and temperature are e-folding down rapidly to lower values, and hence the actual edge values n_{de} and T_{de} could be much lower than the values used here. This will raise the actual value of $[\kappa T_{||b\alpha}(t) / E_\alpha]$ at which the instability is quenched, and thus will raise the theoretically expected value of τ_{ice} accordingly. The alpha particle slowing-down time from the plasma center to edge[6] $\tau_{s\alpha} \approx 650$ to 130 ms. Thus, theoretically we expect this radiative instability, and consequently. the associated alpha particle ICE to be quenched for times $t > \tau_{s\alpha} \approx 130$ to 650 ms, since these trapped-passing boundary alpha particles making their fattest banana orbits spend part of their time near the plasma center and another part near the plasma edge.

VI. CONCLUSIONS AND SUMMARY

In summary, we have presented a comprehensive analysis of the ion cyclotron emission due to the newly born fusion products induced cyclotron harmonic fast Alfven wave radiative instabilities in tokamaks. In essence the radiative instabilities occur only when the induced emission exceeds the absorption. We find that for the background deuterium plasma the absorption always exceeds the induced emission of these cyclotron harmonic fast Alfven waves of frequency $\omega \approx m\omega_{cd} = m\omega_{c\alpha}$ in the DT reaction dominated plasmas and/or $\omega \approx m\omega_{cp} = 2m\omega_{cd}$ in the DD reaction dominated plasmas. That is, these waves are always damped by the background deuterium plasma, and their damping rate γ_d is proportional to $n_d[v_{\perp d}J_m'(\lambda_d)]^2$. For the newly born fusion product species j, we find that the induced emission can exceed the absorption when their directed parallel birth velocity V_j is sufficiently large so that $k_{||}V_j/k > V_A = \omega/k$, the fast Alfven wave phase velocity (i.e., the conditions appropriate for inverse cyclotron damping where the free energy comes from the directed birth energy of $M_jV_j^2/2$) and/or when their exist a sufficiently large ($\perp$, $||$)-temperature anisotropy for these fusion products in such a direction where $T_{||}$ is sufficiently smaller than $T_\perp$ (i.e., conditions appropriate for cyclotron overstability where the necessary free energy comes from the existing temperature anisotropy). Under these conditions the waves under study are made to grow by these newly born fusion products of species j, and their gowth rate γ_j is proportional to $n_j[V_jJ_m'(\lambda_j)]^2$. Thus the condition for the radiative instability is $(\gamma_j - \gamma_d) \geq 0$. Here γ_j is the inverse cyclotron damping due to the fusion product species j, and γ_d is the conventional cyclotron damping due to the background deuterium plasma ions. It is worth remembering that these γ's have terms of the form $[(m\omega_c/T_\perp) + (\omega - m\omega_c - k_{||}V_j)/T_{||}] = \omega T_{||}^{-1}[(T_{||}T_\perp^{-1} - 1) - \{(k_{||}/k)(V_j/V_A) - 1\}]$ for $\omega \approx m\omega_c$. Note first that the smaller the value of $T_{||}$ the larger is the value of the growth rate γ, and second that the term $(T_{||}T_\perp^{-1} - 1)$ in the square bracket is the free energy driving term due to the temperature anisotropy, and the term

$\{(k_{||}/k)(V_j/V_A) - 1\}$ in this square bracket is the paralell drift free energy driving term for the radiative instability.

In general, we find that for the present TFTR plasma conditions, the newly born fusion products which are fully trapped or fully circulating will not be able to induce cyclotron harmonic fast Alfven wave radiative instabilities within the main body of the plasma. This is primarily due to the fact that there is neither not enough temperature anisotropy nor enough birth velocity to satisfy the condition $(k_{||}V_j/k) > V_A$. However, we find that the radiative instability condition due to the marginally mirror-trapped 10% of the fusion products which are very near the trapped-passing boundary so that their $v_\perp \approx v_{||}/\sqrt{\varepsilon}$ is easily satisfied for the present TFTR conditions. This is primarily due to the fact that at these early times these "boundary alphas" velocity space distribution functions attain only very narrow (possibly anisotropic) thermal spreads $T_{\perp bj}(t)$ and $T_{||bj}(t)$ and their birth velocities V_j highly exceed the cyclotron phase velocities V_{jcph}. Thus the inverse cyclotron damping γ_j due to these trapped-passing boundary fusion products highly exceeds the conventional cyclotron damping γ_d due to the background deuterium edge plasma ions. If the observed ICE is indeed a consequence of this radiative instability due solely to these marginally mirror-trapped particles,[8-10,23] then this may explain why the experimentally observed ICE seems to originate from a very narrow region near the outer midplane plasma edge on the low-field side of the torus.

However, as the time t progresses not only $T_{bj}(t)$ increases but also due to collisions $T_{\perp bj}(t) \to T_{||bj}(t)$, i.e., the collision induced thermal spreading of the fusion products also isotropizes; and both these processes tend to weaken the two sources of free energy that was driving this radiative instability at early times. Thus when these fusion products distributions become sufficiently broad and isotropic, the two sources of free energy become sufficiently weak so that γ_j becomes less than γ_d, and the instability is quenched. Of course, eventually these fusion product distribution functions must reach the slowing-down distribution[7] of Eq. (10). However, the upper velocity truncation in the conventional slowing-down distribution of Rome, *et.al.*[7] is obviously rather nonphysical, and one can and should patch it up at this upper end to take account of the thermalization broadening in a manner shown in Eq. (13) or in any other equivalent manner. Finally, we find that such slowing-down distributions are always very stable for radiative equilibrium and hence cannot yield the observed unstable ICE. Thus the initially unstable ICE will be eventually quenched in times t less than the slowing-down time τ_{sj}. That is, $0 \leq \tau_{ice} < \tau_{sj}$. These theoretical predictions are consistent with the recent experimental observations of ICE in[4] TFTR.

Finally, we should emphasize that we have not done any nonlinear calculation of the saturated level of emission. However, for these fusion alphas $\lambda_\alpha = k_\perp \rho_\alpha > 1$ for all the harmonics. It therefore follows that the linear growth rates γ_m of Eq. (25) for $\omega \approx m\omega_{c\alpha}$ due to these fusion alphas will be roughly the same value for all the harmonics m. If the nonlinear saturated levels are proportional to the linear growth rates, then the experimentally observed equal amplitude harmonic ICE is consistent with our theoretical expectations. For example, if the nonlinear mode coupling is the dominant nonlinear saturation mechanism, then it is shown elsewhere[25,8] the saturated wave energy $\varepsilon^\infty(\omega \approx m\omega_{c\alpha},\mathbf{k})$ at $t \to \infty$ satisfies an equation of the type

$$2\gamma(\omega,\mathbf{k})\, \varepsilon(\omega,\mathbf{k}) - C_k\, [\varepsilon(\omega,\mathbf{k})]^2 \approx 0, \tag{38}$$

where C_k is the appropriate modecoupling coefficient. It is then reasonable to argue that the experimentally observed ICE which is proportional to this nonlinear saturated level of wave energy $\varepsilon^\infty(\omega \approx m\omega_{c\alpha},\mathbf{k}) \approx 2\gamma(\omega \approx m\omega_{c\alpha},\mathbf{k})/C_k$ is proportional to the concentration of alpha particles and, hence, to the rate of neutron emission. That is, in such cases the nonlinear saturated level of ICE is proportional to their linear growth rates, in agreement with the recent experimental observations in TFTR.

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