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Four Lectures on Fusion Power

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Four Lectures on Fusion Power

I The Interest in Thermonuclear Power Production

The study of the problems of controlled thermonuclear reactions is of considerably more than academic importance. The practical economic interest in a successful outcome can be divided into two parts: the long-range viewpoint in which we concern ourselves with the eventual depletion of our currently used fuels and the short range viewpoint in which we recognize that even a rather small reduction in the cost of energy is of vast importance to the economy. In the power industry a great deal of capital investment has gone into the development of better coal fired systems in recent years to raise the thermal efficiency from 35 to 41 percent, and every gain of one percent has been a noteworthy achievement. If fusion power plants should make a few percent reduction in true power costs, the current human effort would be justified many times over. On the other hand, the long-range viewpoint seems to stimulate more imagination and interest for the average person than the prospect of reducing the cost of power by a fraction of a mill per kilowatthour.

Let us examine the history and speculate on the future of the energy needs of mankind. Such a vast subject requires vast units for a quantitative discussion, and such a unit is the Q. The unit is defined as 10^{18} BTU, a number probably meaningless in itself to all of us. Someone has calculated that this is the amount of energy required to get Lake Ontario up to the boiling point, a Herculean task of greater magnitude, but of considerably less merit than the cleaning of the Augean stables, but even this gives little feeling for the magnitude of energy defined by one Q.

I like to think of the Q as the amount of energy that will be required to operate the world for one year when industrial civilization as we know it now has spread over a substantial fraction of the world. The world now consumes between 0.1 and 0.2 Q per year, and if the United States' standard of living were to spread to one-third or one-half of the earth's present population, we should require about one Q per year. Our past history of consumption has been estimated by Putnam[1] as follows: prior to the industrial revolution mankind consumed between 6 and 9 Q largely in the form of human and animal muscle power. During the first century of industrial civilization (1850-1950), 4 Q were consumed, and during our present half century (1950-2000), some 15 Q will be released. It is clear that an annual rate of 1 Q is being rapidly approached and, in fact, Putnam estimates that the first half of the 21st century will require 100 Q, with the annual rate exceeding 2 Q at the end of that period. Of course it is always dangerous to extrapolate, and in particular one cannot say that since an annual growth rate of electrical power production of greater than 6% has been experienced in recent years, it will probably continue and then use this exponential growth curve to predict demand several decades hence.

There are certain physical problems related to climate associated with large increases of power released on earth. Let us examine the heat balance of the earth. The surface of the earth receives from the sun 4 Q daily, in comparison with which the amount reaching the surface from the hotter inner regions and that liberated by man in combustion seems negligible. However our climate is quite sensitive to surface temperature. An equilibrium value occurs such that we radiate to space at the same rate at which we receive energy. This rate of radiation is closely proportional to the fourth power of our surface temperature. Thus if mankind's energy release ever reaches 15 Q per year (1% of the sun's input), our surface temperature will rise $3/4^{\circ}C$, a significant fraction of the temperature excursion ($5^{\circ}C$) responsible for periodic glaciation. Although not intolerable, this would produce noticeable changes. In fact, in our present era the ice caps are slowly melting and such a rise in average temperature would accelerate the process. Geologists have estimated that the final melting of the ice caps will raise mean sea level by one hundred feet. Although this would give Princeton refreshing sea breezes on hot summer evenings, it is of some concern to Plasma Physics Laboratory personnel since the 100' contour passes directly through our laboratory, and the surf may be lapping at the foundation of Model C before we have solved the thermonuclear problem.

Is there any reason for serious consideration of climatic problems induced by increased liberation of energy? If there is, when will the problem develop? If one assumes that power production will remain proportional to our economic activity and then makes the more questionable assumption that the economy of the world will grow 4% per year, the 15 Q annual requirement will be reached in 125 years, which is not completely remote. Furthermore, local problems will arise in perhaps half that time when the United States requires about 1 Q a year. Waste heat from our power plants is normally dumped into rivers, and their capacity for absorbing heat is limited by the temperature rise of the water one is willing to allow. Humidity and temperature will rise significantly near power centers within a few decades.

One might be tempted to assert that these potential troubles with the climate can neatly be averted by utilizing solar power. By this method useful energy can be produced at no increase in surface temperature. But check the assumption quantitatively. If one assumes that future technology succeeds in producing a 40% conversion efficiency, and if one assumes 5% of the earth's surface to be covered with converters (probably the wildest assumption of all in this highly speculative set of lectures) with a utility of 50%, one could realize only 15 Q per year, a quantity more easily liberated from fuel with a tolerable climatic problem. The capital costs of solar power would be so great that this method will not be able to compete to supply industrial power.

Barring completely revolutionary new discoveries, it appears that we must fuel our economy. Do we have the fuel?

Putnam[1] has also considered this question. His extensive report considers all manner of fuel resources and can be summarized in the following way. He defines economic reserves to be those known at present plus an allowance for as yet undiscovered reserves of sufficiently high quality to be recoverable at no more than twice current costs. On this basis he estimates that we have left 6 Q of oil and gas, 32Q of coal, and 600 Q of the fissionable fuels, Uranium and Thorium. Of course there is far more available, but only at increased cost. A few conclusions to be drawn from this are: 1) Immediately and inevitably, the real cost of power is on its way up; 2) Within a few decades fission must take over the prime role of energy supply; 3) Serious investigation is needed to determine if we really face an exhaustion problem in the more remote future. There are subsidiary questions also such as what to do with the radioactive wastes that would result from large scale power production by fission.

The above conclusions ignore the possibility of fusion power. If we assume that a practical competitive power producing process fueled by light elements is developed, all three of the above conclusions may be invalidated. Figure 1 summarizes the situation. There is a ten thousand million Q potential reserve in the deuterium present in water. For reasons to be explored later in these lectures, a reactor using pure deuterium as a fuel gas may be neither developable nor desirable within the century. A fuel mixture of deuterium and tritium is much more attractive, and in this case, the primary fuels are deuterium and lithium. Lithium is abundant and cheap, with reserves of many hundreds of Q's.

If the fusion power development program should not succeed, the future is not entirely bleak if fission breeder reactors become economically competitive. As Dr. Weinberg of Oak Ridge has explained[2], it is conceivable that industrial civilization could be maintained indefinitely into the future on the basis of recovery of Uranium from granite. The mining operation would be similar in magnitude to present coal recovery. This is based, however, on a rather modest asymptotic energy requirement - less than 2 Q per year, and it is very likely that thirty years from now this will seem as foolish as the doctrine of the "mature economy" prevalent thirty years ago appears now.

Speculation about the remote future is a fascinating pastime related to the popularity of science fiction, but is not justification for large scale attacks on technical problems. The needs of future generations should not be ignored, but the strongest reason for concentrated effort on the fusion power program now is the fact that a thermonuclear power plant would be useful today, may be available in the seventies, and when available will probably be the cheapest source of power.

In view of this there is an extensive effort throughout the earth on this problem with dozens of laboratories and hundreds of scientists pursuing this valuable goal. Long recognized as a possible mechanism for energy release, it has only been since 1951 that quantitatively supportable proposals have been made for systems that may eventually be reactors and for experimental programs of investigation. Many questions remain for which answers must be available before it can be established whether or not such machines are attainable and if so, whether they are potentially practical. The past

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CONSUMPTION

THRU 1850	6-9 Q
1850-1950	4
1950-2000	15
2000-2050	100

PUTMAN, 1953

ECONOMIC RESERVES

COAL:	32 Q
OIL AND GAS:	6
URANIUM AND THORIUM:	600

PUTMAN, 1953

ENERGY RESOURCE IN OCEANS' DEUTERIUM: 10^{10} Q

Figure 1 World Energy

decade has produced an extraordinary amount of knowledge in this corner of the new science of plasma physics, but much, including some of the most fundamental questions, remains to be established.

One such question is whether or not it is possible to confine a plasma of sufficiently high temperature and density for a sufficiently long period to release a usefully large amount of fusion power. (The actual magnitude of these numbers will be discussed in detail in what is to follow.) Closely related to this is whether energy can be added to a confined plasma below reacting temperature to heat it above the fusion ignition point without destroying the confinement.

In the United States, effort on this problem, sponsored by the Atomic Energy Commission under the name Project Sherwood, began principally at three laboratories, at Princeton University, Los Alamos Scientific Laboratory, and the University of California. The early history may be read in the book by Bishop[3]. In the beginning these laboratories represented the stellarator, pinch, and mirror concepts respectively, but as time passed the interest in each laboratory broadened, and new laboratory devices were proposed and constructed while other laboratories began contributing their efforts also.

A functioning thermonuclear reactor would be a very copious source of neutrons, a valuable and perishable commodity for numerous uses, including weapons material production. Consequently, in the early fifties before one appreciated fully the complexity of the task, Project Sherwood was classified due to the possibility of rapid progress and early application to materials production. Within a few years however, as the hazy problem of stability congealed into a recognizable shape of alarming proportions, it became apparent that a lengthy program lay ahead. Declassification occurred in 1958 at the Second International Conference on the Peaceful Uses of Atomic Energy. Since that time all nations are exchanging information through normal scientific channels, resulting in a much larger group of informed researchers able to contribute to knowledge in the field.

Final solutions are still remote. Although in theory diffusion of a plasma across a confining magnetic field should be extremely slow, much slower than is necessary for a reactor, few plasma devices have been able to demonstrate this. The diffusion experiments with cesium plasma, reported to you in companion lectures, seem consistent with classical diffusion, but these experiments take pains to eliminate all disturbances to the plasma. When a confined plasma experiences an electric current flowing parallel to the field, inhomogeneous fields or spatial assymmetries in the velocity distribution, diffusion is enhanced by very large factors as you have heard in other lectures of this course.

It appears, in theory, that any particular magnetic field geometry possesses a critical β above which the plasma becomes unstable. Since fusion reactions are binary collisions, the rate of reaction will vary as the square of β, thus the maximum value of β attainable in a given machine must be known to provide the necessary design data. To date no comparisons between theory and experiment have been carried out on this critical question.

In view of all these unknowns, isn't it premature to consider in any detail the problem of the characteristics of future power plants based on confined plasmas? It is premature in the sense that no firm designs or final conclusions can be drawn. Nevertheless it is important to look ahead in the light of present knowledge. For example, it would be very discouraging if, after constructing an elaborate framework of highly optimistic assumptions reaching far above the firm foundation of present knowledge, we were to make predictions of vast power plants of no possible commercial interest today and with but slight hopes for future utility. In contrast, we intend to show in these lectures that reasonable assumptions lead to the prospect of immediate application of fusion plants when available. The key assumptions are that successful confinement and heating will be attained.

One of the best ways to discover areas requiring design study or even research efforts for missing factual data is to attempt a design of a system even when everyone recognizes that it is too early to do it correctly and completely. In what follows a number of problems subsidiary to the principal one of confinement and heating will be identified and briefly discussed.

We shall begin with general considerations applying to any thermonuclear reactor and work toward more detailed examination of two particular proposed geometries, the mirror and the stellarator.

II General Considerations

A. Properties of the Nuclear Reactions

There are four nuclear fusion reactions with cross-sections greater than a millibarn at energies of 50 kev. These reactions are

	Reaction	E	E^*
1)	$D + T \rightarrow He^4\,(3.52\ \text{Mev}) + n\,(14.06\ \text{Mev})$	17.58	3.52
2)	$D + D \rightarrow He^3\,(0.82\ \text{Mev}) + n\,(2.45\ \text{Mev})$ $D + D \rightarrow T\,(1.01\ \text{Mev}) + p\,(3.03\ \text{Mev})$	3.6	2.4
3)	$D + He^3 \rightarrow He^4\,(3.67\ \text{Mev}) + p\,(14.67\ \text{Mev})$	18.34	18.34
4)	$T + T \rightarrow He^4 + n + n$	11.32	

E is the total energy release, and E^* is the energy release to charged particles.

Other reactions have probabilities far less than these and need not be considered. We assume a plasma in kinetic equilibrium at a temperature T kev with a negligible amount of impurities. The reaction rate R in a plasma of two species of nuclei of densities n_1 and n_2 will be given by

$$R = n_1 n_2 (\overline{\sigma v}) \tag{1}$$

where $(\overline{\sigma v})$ is the average of the cross-section times the relative velocity over the velocity distribution of the system. In the case of a single species, the rate is given by

$$R = 1/2\, n^2 (\overline{\sigma v}) \quad . \tag{2}$$

If a Maxwellian distribution is assumed, the averages can be calculated and can be found in several references. They monotonically increase with temperature below 100 kev. This does not mean, however, that the higher the temperature, the better (at least up to 100 kev) because there is another restriction frequently overlooked in the literature. Any containment scheme for a controlled thermonuclear reactor will be pressure limited, that is to say there will be some maximum pressure above which, for one reason or another, it will be impossible to operate. The particle density n and the operating temperature T are not independent variables, but are limited in their permissible product. Therefore in seeking the best temperature at which to operate a reactor, to maximize R, we do not maximize $(\overline{\sigma v})$ but note that

$$R = n_1 n_2 (\overline{\sigma v})$$

$$R = \frac{P_{max}^2}{k^2 T^2} (\overline{\sigma v})$$

$$n_{max} = \frac{P_{max}}{k T} . \qquad (3)$$

Therefore it is the quantity $\left(\frac{\overline{\sigma v}}{T^2} \right)$ which is indicative of the fusion power level of a given machine as a function of temperature. This quantity is plotted in Figure 2 for the four reactions we are interested in.

Anyone who attempts to describe all possible reactors from the most general point of view will encounter an extraordinary number of variables:

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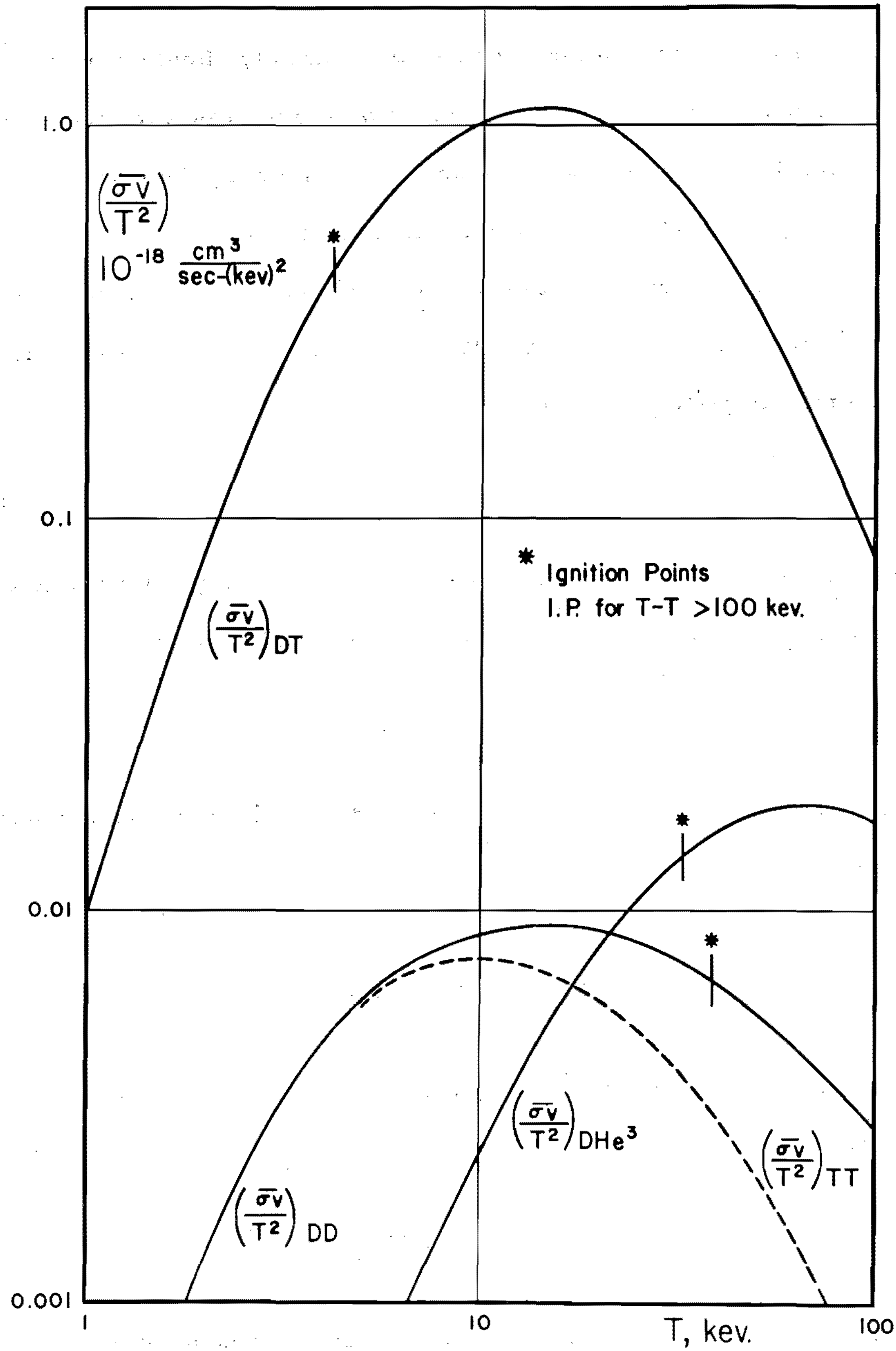

Figure 2 Reaction Rate as a Function of Temperature

type of machine, type of fuel, temperature, density, field strength, β, size, pulsed, dc, or cyclic operation, etc. If we are to draw any conclusions in a reasonable length of time, we must start restricting the variables we wish to investigate. Let us begin with the fuel selection.

We shall have much to say about radiation in what follows, but for the moment suffice it to say that the power density of bremsstrahlung radiation for our gas will be given by

$$P_B \sim n^2 T^{1/2} \quad . \qquad (4)$$

We should like to be able to supply this from the energy delivered to charged particles in the plasma by the fusion reaction, which is given by

$$P_F \sim n^2 (\overline{\sigma v}) E^* \quad ; \qquad (5)$$

where E^* is the energy released to charged particles in a single fusion event, the ratio is

$$P_F/P_B \sim \frac{(\overline{\sigma v}) E^*}{T^{1/2}} \qquad (6)$$

or in terms of the factor $(\overline{\sigma v}/T^2)$ which we have computed and plotted in Figure 2

$$P_F/P_B \sim \left(\frac{\sigma v}{T^2} \right) E^* T^{3/2} \quad . \qquad (7)$$

This ratio is very much less than one at temperatures below a kilovolt for all reactions. As the temperature rises, the ratio reaches one at a temperature called the <u>ideal ignition temperature</u>, a temperature at which in the absence of all other losses, the plasma temperature will remain constant. These temperatures for the different reactions are: DT, 4.2 kev; DD, 39 kev; DHe^3, 33 kev; TT, > 100 kev.

We are now able to make our first decision. Note that only in the case of the DT reactor is the ignition point below the optimum. We should intend to operate a DT reactor at 14 kev, well above its ignition point, but reactors based on the others should be operated at the lowest possible temperature at which they will go. How much is the advantage of the DT reactor quantitatively? Let us compare the relative total energy release for two identical systems, one fueled by deuterium gas and the other by a DT mixture, both operating at their best temperature. At the same total pressure the ratio of fusion power released by the two fuels is given by:

$$\frac{P_{DT}}{P_{DD}} = \frac{\frac{1}{4}\left(\frac{\overline{\sigma v}}{T^2}\right)_{DT\,opt} E_{DT\,total}}{\frac{1}{2}\left(\frac{\overline{\sigma v}}{T^2}\right)_{DD\,opt} E_{DD\,total}} \quad . \tag{8}$$

Inserting the numbers we find a ratio of more than 100. This comes not only from the large difference in usable $(\overline{\sigma v}/T^2)$ but also from the greater release of energy from the DT reaction. This calculation also assumes that all bremsstrahlung losses are recovered in heat eventually. If some of this heat is assumed lost, the ratio grows. Similar considerations of the other reactions give similar results. In view of the inherent difficulty of the fusion problem, it would be foolish to discard this advantage of at least two orders of magnitude. Henceforth we shall assume the DT reactor.

Before abandoning the others completely, we should consider briefly the possibility of direct conversion to electricity of energy released by fusion reactions. In principle this can be done by cycling the confining magnetic field and delivering electrical energy to an external secondary coil winding at the expense of the energetic charged products in the plasma. This will recover a fraction of the charged particle energy, but, of course, none of the energy carried off by the neutrons, which in the case of the DT reactor have most of the energy. It might be of interest in the DD reactor or the DHe^3 reactor where substantial fractions of the energy appear in the charged particles, if these reactors ever become possible.

Pulsed reactors have occasionally been considered, but do not seem promising. In pulses short compared to instability times, pressures of a million atmospheres would be required as shown by Lawson[4]. A big disadvantage with this concept of machine is that energy must be supplied to heat an entire fuel charge to reaction temperature during each cycle, rather than merely replacement fuel. We shall discuss the problem of the minimum confinement time required later in this lecture.

B. Need for Neutron Thermalizing Blanket

In the case of the DT reactor, 80% of the energy released in the reaction is carried by the neutron which leaves the plasma. In order to capture this energy it is necessary to stop the neutron to convert its kinetic energy into heat. This requires a blanket surrounding the vacuum tube of a thickness approaching one meter. This element of the machine seems to be essential even if it were found possible to use a cyclic energy recovery system of the type alluded to above because of the shielding problem due to the ever present neutron flux. One should not be misled into thinking that use of the DHe^3 reaction would eliminate the neutron problem. We do not have Maxwell demons to ride on the deuterons and steer them only into He^3 collisions. DD reactions will also take place to an appreciable extent. Figure 3 plots the rate ratio for the two reactions as a function of temperature. One would be faced with about one neutron in every twenty fusion events. This leads to an intense flux and would require a blanket with sufficient cooling to carry off the delivered energy. This line of argument has been developed to show that the blanket problem is not an argument to seek alternatives to the DT reaction and to justify the assumption that a thermal cycle with heat generation in a blanket will be used.

Although the future may bring efficient direct conversion of heat to electricity, there is no efficient system near application now, whereas the steam cycle has been carried to a relatively high state of perfection. Large supercritical plants can achieve a 41% thermal efficiency, and small units exceed 30%. We shall assume a 33% steam plant efficiency in what follows. It is reasonable to expect as high as 36% from present designs which can operate at steam temperatures within the operating limits of current blanket proposals, but surely some energy will be required for unforeseen auxiliaries or system inefficiency, and it is only prudent to remain conservative in the performance estimates for our conventional systems. If we are ever successful in the difficult and central task of heating and confining a thermonuclear plasma for usefully long periods, it would be disastrous to fail to produce power because we can't achieve the assumed performance from the conventional portion of the plant.

C. Considerations for Any Reactor

From the above arguments we shall now assume that our machine is a continuously operating, DT reactor, utilizing a blanket and associated steam cycle, with plasma confinement by a magnetic field[5]. This class includes the stellarator, the magnetic mirror, the astron, various forms of cusped geometries and perhaps other proposed machines.

As noted above, the fusion power released in the plasma varies in accordance with

$$P_F \sim n^2 (\overline{\sigma v}) \quad . \qquad (9)$$

It is very convenient to eliminate the particle density in calculations in this

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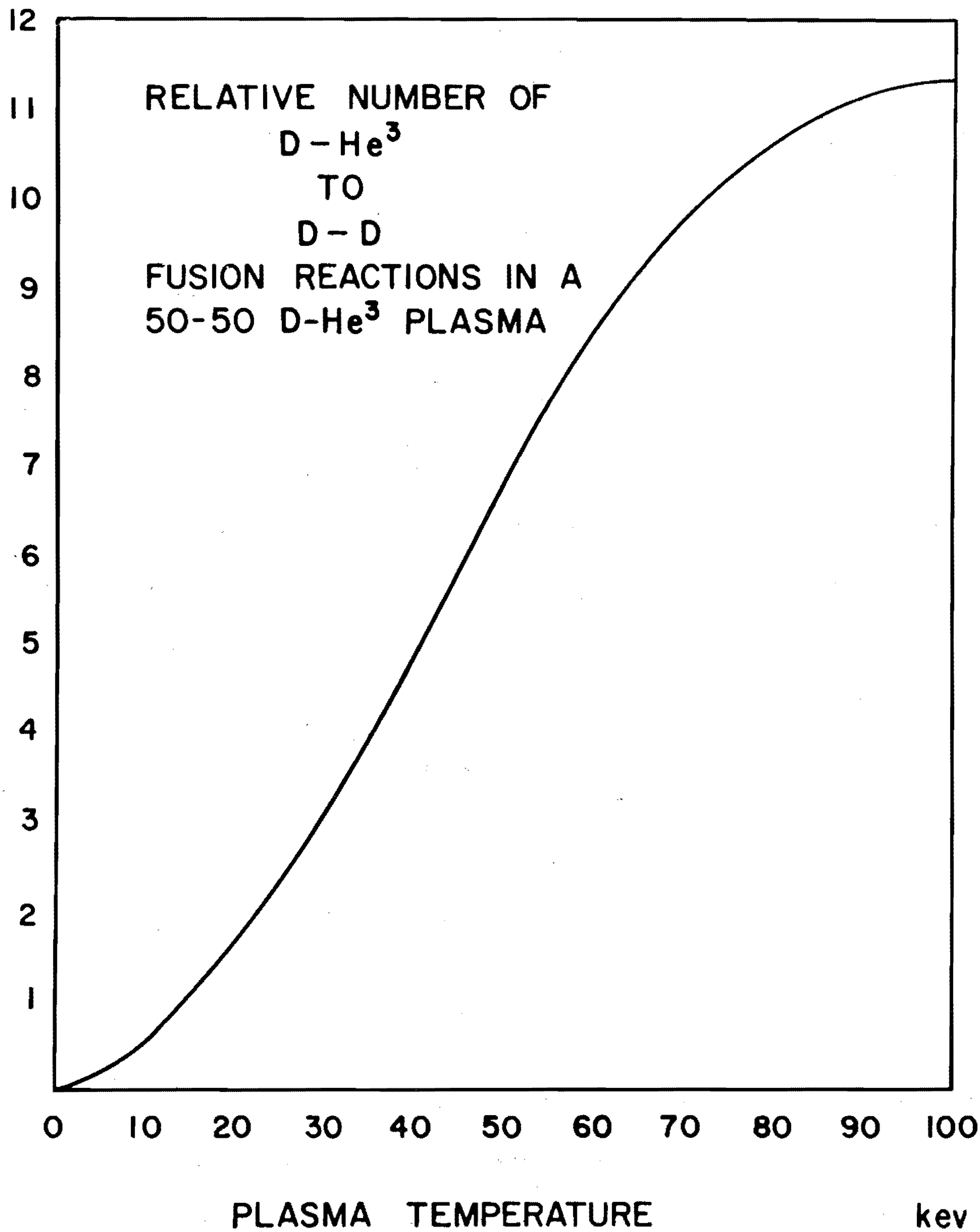

Figure 3 DHe^3 - DD Relative Reaction Rates subject by introducing the dimensionless parameter β defined by

$$\beta = \frac{8\pi n k T}{B^2} \tag{10}$$

By means of this we can replace the particle density n by

$$n = \frac{\beta B^2}{8\pi k T} \tag{11}$$

wherever it appears. In particular the fusion power density varies as

$$P_F \sim \left(\frac{\overline{\sigma v}}{T^2}\right)\beta^2 B^4 \quad . \tag{12}$$

Figure 2 shows $\left(\frac{\overline{\sigma v}}{T^2}\right)$ as a function of T from which it is clear that we should operate as close to 15 kev as we can. As we shall see later, there will be special reasons for departing from this ideal temperature. We assume a constant temperature operation. For a complete machine the total power release by fusion reaction will be given by

$$P_F = k_1 \beta^2 B^4 r^3 \quad . \tag{13}$$

In the absence of magnetic materials or superconductors, the power required to operate any magnet may be represented by

$$P_M = k' \rho B^2 r \tag{14}$$

where ρ is the resistivity of the conductor material, B is the magnetic field strength at any particular point in the system, r is a characteristic length of the device (such as the radius of the internal opening) and k' is a proportionality constant depending only on the geometric shape of the electric conductor system used to generate the field. If a particular conductor is selected, ρ will be established and combined with k' for a new constant k_2, giving

$$P_M = k_2 B^2 r \quad . \tag{15}$$

It follows that the power for sale from a fusion power plant is given by

$$P_S = P_F - P_M = k_1 \beta^2 B^4 r^3 - k_2 B^2 r \quad . \tag{16}$$

If nature were such that the second of these two terms tended to be very much smaller than the first, the future of fusion power plants would appear quite bright, once physical possibility were proven. Unfortunately this is not the case. Evaluations of k_1 and k_2, on the basis of various assumptions derived from technical considerations, show that these two terms are of the same order of magnitude[6].

If we are to achieve a practical power plant, P_F must be considerably larger than P_M. If it were only slightly larger, the plant would be circulating a very large amount of power within itself in order to sell a small quantity, and the resulting investment cost per output kilowatt would be impossibly high. In other words, the ratio

$$\frac{P_F}{P_M} = \frac{k_1 \beta^2 B^2 r^2}{k' \rho} \tag{17}$$

must be greater than 1 to have any power yield at all, and should be considerably higher (probably a minimum of 2) to be of practical interest.

From equation (17) there are four obvious ways to proceed to improve this ratio. The first is to select the best type of machine. This involves the k coefficients and β. Much laboratory experimentation must be done to determine the highest value of β attainable and to determine which form of machine can deliver the best k's.

The second method is to increase the field strength, but this has technical limitations.

The third method is simply to scale up the size of the reactor. The presence of r^2 in equation (17) implies that if fusion reactors are proved physically possible, they can certainly be made practical if built in large enough sizes. A serious difficulty with this approach is that the cost rapidly increases with increasing size and although the investment cost per kilowatt may be brought down within reason by a unit of large enough size, the output might be too large for any market in the foreseeable future.

The fourth method would be to reduce the resistivity ρ of the conductor material. As I explained in the lecture on coil design, this can be done by refrigeration of pure materials in the neighborhood of $20^{\circ}K$ or below, or by the use of superconductors, to be treated later.

There are several substances that might be useful in cryogenic, normally conducting magnets. Post has suggested sodium[7]. Laquer has operated cryogenic magnets of copper for several years[8]. Aluminum is another possibility. This approach may ultimately be of very great importance, particularly since it improves the relative yield P_F/P_M without simultaneously increasing the generated power P_F. Factors of improvement of the order of 10 may be achieved by this method. An unknown factor, however, is the cost required for the extremely large cryogenic refrigerators that would have to be developed for such an installation. In recent years, considerable progress has been made in this direction.

Another, less obvious, approach is cyclic field peaking. Division of the two terms in equation (16) to yield equation (17) implies time independence of all the variables. Suppose the confining field B is periodic in time. The mean value of B^4 over a cycle divided by the mean value of B^2 over the cycle will have different values depending on the waveshape. All other factors remaining constant, the former measures the fusion power liberated, and the latter, the magnet power consumed.

It is possible to select waveshapes which improve the ratio of P_F/P_M; Figure 4 presents several examples. To illustrate the potential advantages, in the figure is a tabulation of the mean value of B^4 divided by the mean value of B^2 and normalized to constant magnet power by dividing by the mean value of B^2, i.e., $\overline{B^4}/(\overline{B^2})^2$. For dc the value is, of course, 1, and the other curves should be compared with this. The waves shown are: simple sine wave, optimized biased sine wave, optimized first and third harmonic mixture, and an optimized biased first and third harmonic mixture. In principle the value of this ratio could be raised to arbitrarily high values by inclusion of higher and higher harmonics to give a more peaked wave. But skin-effect considerations limit the highest usable frequencies, and maximum allowable temperature excursions limit the lowest, so that improvement factors greater than three are unlikely. Since the problem is basically one of increasing a ratio (P_F/P_M) from some value between 1 and 2 to a value in excess of 2, such an improvement could be of great importance.

Clearly the type of machine exhibiting the highest intrinsic economic potential should be chosen for a fusion plant, but high fields, large size cryogenic cooling, and cyclic field peaking cannot all be combined to obtain an ultimate facility. In fact, almost all combinations of these approaches lead to technical difficulties which would require extensive development for full understanding. As examples, the following may be mentioned.

The combination of high fields with large sizes leads to the problem of structural strength, discussed in a previous lecture. If high fields are combined with cryogenic cooling, a phenomenon known as the magnetoresistive effect causes trouble. The resistance of a conductor rises as the magnetic field through it is increased. At room temperatures, this is a very unimportant effect, but at cryogenic temperatures it can be the major component of the resistance. This effect rules out copper. If peaking is to be combined with high fields and large volumes, one finds that thousands of volts per turn may be required in the magnet to cycle the large fluxes involved, and insulation becomes a serious problem. Finally, the

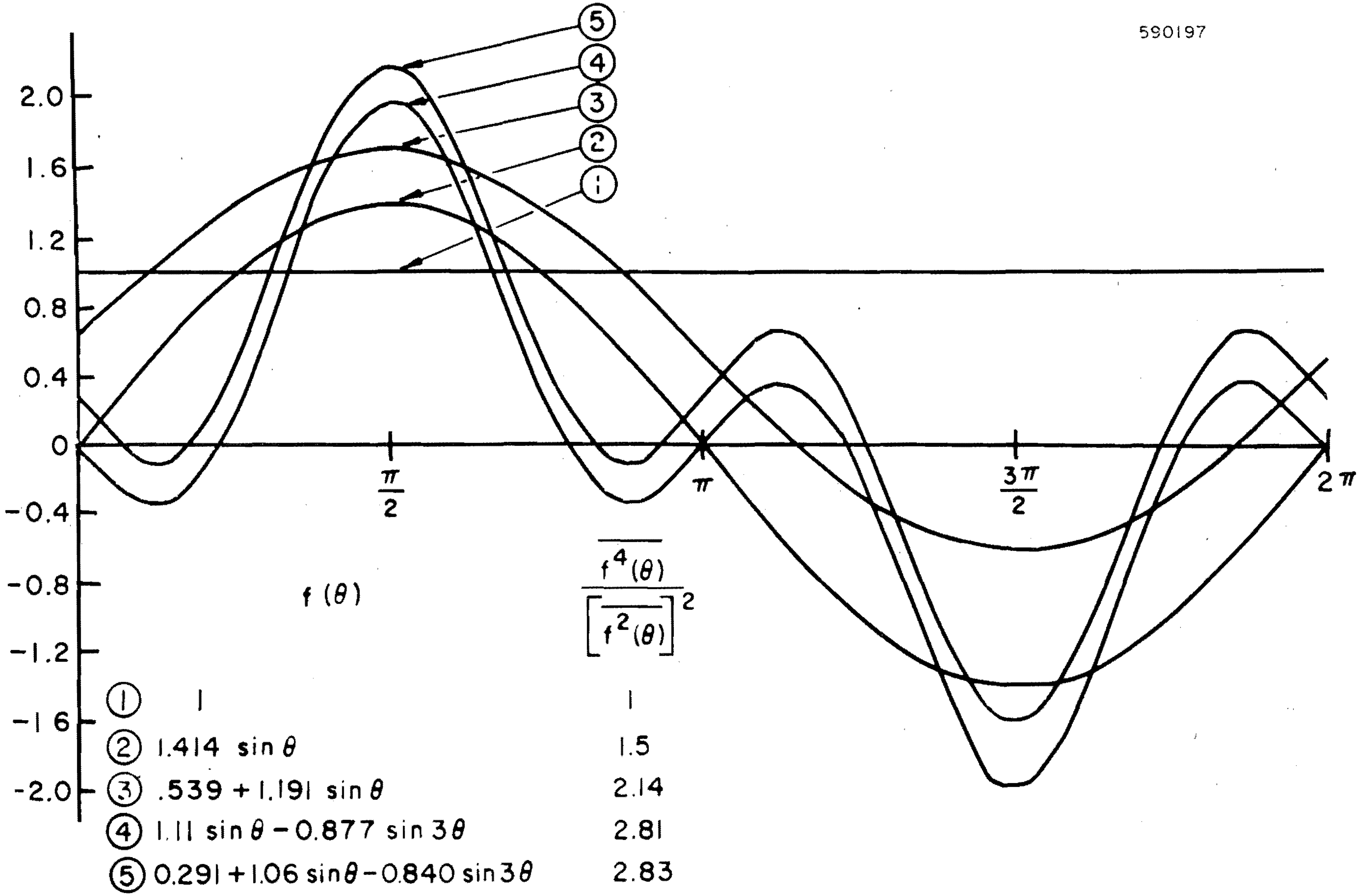

Figure 4 Field Peaking

combination of cryogenic cooling and cyclic field peaking seems out of the question because of the skin-effect problems. The skin depth decreases with the square root of the resistivity, and becomes so small at cryogenic temperatures (0.01 inch at 60 cycles) that one would have to wind large magnets with wire so fine that fabrication and insulation would be impossible. Of the above approaches the two most promising methods seem to be cryogenic dc magnets and ambient temperature peaked-field magnets.

If superconductors become practical, and there is every reason to believe they will, most of these considerations can be ignored, with certain qualifications to be developed later. In brief, superconductors will be the answer to economical steady dc field production, but they must be in dewars containing liquid helium in a region of low neutron flux, because it will not be economical to pump large amounts of heat (as would be delivered by neutrons) out of the cryostat. This means that superconducting windings must be located outside the blanket.

D. Bremsstrahlung

The power density of bremsstrahlung radiated from a Maxwellian plasma has been calculated by Spitzer[9]. His result can be written in the form

$$P_B = (0.535 \times 10^{-23}) n_e n_i Z^2 T_e^{1/2} \quad \frac{\text{ergs}}{\text{sec-cm}^3} \qquad (18)$$

where T is in kev, n_e and n_i are the electron and ion density, and Z is the number of electron charges on the ion. We have made use of this above to calculate the ideal ignition temperature by equating this to the fusion power released to charged particles. In practice the actual ignition temperature will be higher due to synchrotron radiation (to be discussed below), to impurity radiation and other losses. Let us consider the impurity radiation. The presence of the Z^2 term makes high Z impurities very costly in radiation. At an electron temperature of 15 kev, a pure DT plasma would radiate the following fraction of the energy released to charged particles

$$\frac{P_B}{P_F} = \frac{(0.535 \times 10^{-23})\, n_i^2 (15)^{1/2}}{\frac{1}{4}\, n_i^2 \left(\frac{\overline{\sigma v}}{T^2}\right)_{15} (15)^2 E^*}$$

$$\frac{P_B}{P_F} = 5.7\% \quad . \tag{19}$$

Thus we can allow no more than a factor of seventeen rise in the radiation losses. Unavoidably we will have He^4 nuclei present in the plasma as the ash of the reaction and to about one half percent concentration. Since the Z of He^4 is 2, this will produce an increase of about 2% in the total radiation, a negligible amount. On the other hand, should we have a bad impurity, say 5% oxygen nuclei, the radiation due to this impurity would be more than three times that due to the hydrogen species. Clearly a large number of impurity ions cannot be tolerated. On the other hand, the percentage of impurities allowable - of the order of tenth or hundredth of a percent - is high compared to that which will be present due to base pressure in the vacuum system. The unsolved problem is whether emission from the wall can be held to tolerable levels.

E. Synchrotron Radiation

We shall assume a continuously operating flat density profile DT reactor which is in kinetic equilibrium and quiescent. This assumption is important in order to rule out any possibility of synchronized motion of groups of electrons which might result in coherent emission and greatly augmented losses. The first calculation will neglect absorption in order to provide an upper limit to the loss by synchrotron radiation. The classical total energy loss rate by an electron accelerated a cm/sec^2 is given by

$$- \frac{dW}{dt} = \frac{2}{3} \frac{e^2}{c^3} a^2 \quad . \tag{20}$$

An electron moving in a magnetic field experiences an acceleration, $\bar{a}$, given by

$$\bar{a} = \frac{e}{mc} (\bar{V} \times \bar{B}) \quad , \tag{21}$$

therefore

$$- \frac{dW}{dt} = \frac{2}{3} \frac{e^4 V_{\perp}^2 B^2}{m^2 c^5} \quad . \tag{22}$$

In a plasma in kinetic equilibrium each degree of freedom has on the average $kT/2$ of kinetic energy. Therefore in perpendicular motion (two degrees of freedom), an average particle possesses

$$1/2\, m V_{\perp}^{2} = kT \qquad \text{or}$$

$$V_{\perp}^{2} = \frac{2kT}{m} \quad . \tag{23}$$

Thus the average power loss per electron in the plasma is

$$-\frac{dW}{dt} = \frac{4}{3} \frac{e^{4} B^{2}}{m^{3} c^{5}} (kT) \quad . \tag{24}$$

By definition

$$\frac{B^{2}}{8\pi} + nkT = \frac{B_{o}^{2}}{8\pi} \tag{25}$$

$$\beta = \frac{8\pi nkT}{B_{o}^{2}} \quad , \qquad \text{or}$$

$$B = B_{o}(1 - \beta)^{1/2} \tag{26}$$

and

$$n = \frac{\beta B_{o}^{2}}{8\pi kT} \quad . \tag{27}$$

The particle density n is made up of three components, n_D, n_T, and n_E, of deuterons, tritons, and electrons. We assume that the machine is fueled with a 50-50 mixture of deuterons and tritons, or

$$n_D = n_T = \frac{n_E}{2}$$

and

$$n = 2 n_E \quad . \tag{28}$$

Thus

$$n_E = \frac{\beta B_o^{\ 2}}{16\pi k T} \quad . \tag{29}$$

Since each electron radiates independently, the total power density loss for the plasma is given by multiplying (24) by n_E

$$- \frac{dw}{dt} = \frac{4}{3} \frac{e^4 B^2}{m^3 c^5} (n_E k T) \quad . \tag{30}$$

Inserting (26) and (29) in (30), we get the final result for the upper limit to the power density radiated by synchrotron radiation, p_s

$$p_s = - \frac{dw}{dt} = \frac{1}{12\pi} \frac{e^4 B_o^{\ 4}}{m^3 c^5} \beta(1 - \beta) \quad . \tag{31}$$

If we plan to supply these losses by the energy released to charged particles by the fusion reaction (rather than by energetic particle injection or by external heating), we may calculate this power density input, p_f, for comparison. This is given by

$$p_f = n_D n_T (\overline{\sigma v}) E^*$$

where E^* is the energy released to the He^4 product nucleus (3. 5 Mev.). Using (28) and (29) this becomes

$$p_f = \frac{B_o^{\ 4} \beta^2 E^*}{1024 \pi^2 k^2} \left(\frac{\overline{\sigma v}}{T^2} \right) \quad . \tag{32}$$

Taking the ratio of these we get

$$\frac{P_f}{P_s} = \frac{3}{256\pi} \frac{E^* m^3 c^5}{e^4 k^2} \left(\frac{\overline{\sigma v}}{T^2}\right) \frac{\beta}{1-\beta} \quad , \tag{33}$$

and by setting this equal to one we find the critical β above which a DT reactor will function without continuous supply of external energy (which, if not excessive, could, inprinciple, be done).

Evaluating all terms in expression (33) for T = 15 kev, one finds the numerical expression

$$\frac{P_f}{P_s} = (3.43) \frac{\beta}{1-\beta} \tag{34}$$

from which one finds the critical β to be

$$\beta_c = 23\% \quad . \tag{35}$$

Absorption of Radiation

If absorption is considered, the critical β will be reduced. To consider this question we first approximate the spectral distribution of the emitted radiation. To do this we rewrite (24) relativistically corrected:

$$-\frac{dw}{dt} = \frac{2}{3} \frac{e^4 V^2 B^2}{m^2 c^5 (1 - V^2/c^2)} \quad . \tag{36}$$

Landau and Lifschitz[10] have calculated the total radiation power density in the nth harmonic to be

$$- \frac{dw_n}{dt} = \frac{2e^4B^2(1 - V^2/c^2)}{m^2c^2V} \left[\frac{nV^2}{c^2} J'_{2n} \left(\frac{2nV}{c}\right) - n^2(1 - V^2/c^2) \int_0^{V/c} J_{2n}(2nx)\, dx \right] . \qquad (37)$$

By dividing (37) by (36) we get coefficients c_n which give the relative power radiated in each harmonic:

$$c_n = 3 \left[\frac{n(1 - V^2/c^2)^2}{(\frac{V}{c})} J'_{2n} \left(\frac{2nV}{c}\right) - n^2 \frac{(1 - V^2/c^2)^3}{(\frac{V}{c})^3} \int_0^{V/c} J_{2n}(2nx)\, dx \right] \qquad (38)$$

from which it follows

$$\sum_{n=1}^{\infty} c_n = 1 \quad . \qquad (39)$$

The first few c_n for our conditions are plotted in Figure 5. The effect of absorption on the critical β can be calculated as follows. If a fraction η of the total synchrotron radiation is reabsorbed before loss from the plasma, the minimum condition for sustaining plasma temperature is

$$\frac{p_f}{(1 - \eta)\, p_s} = 1 \quad , \qquad (40)$$

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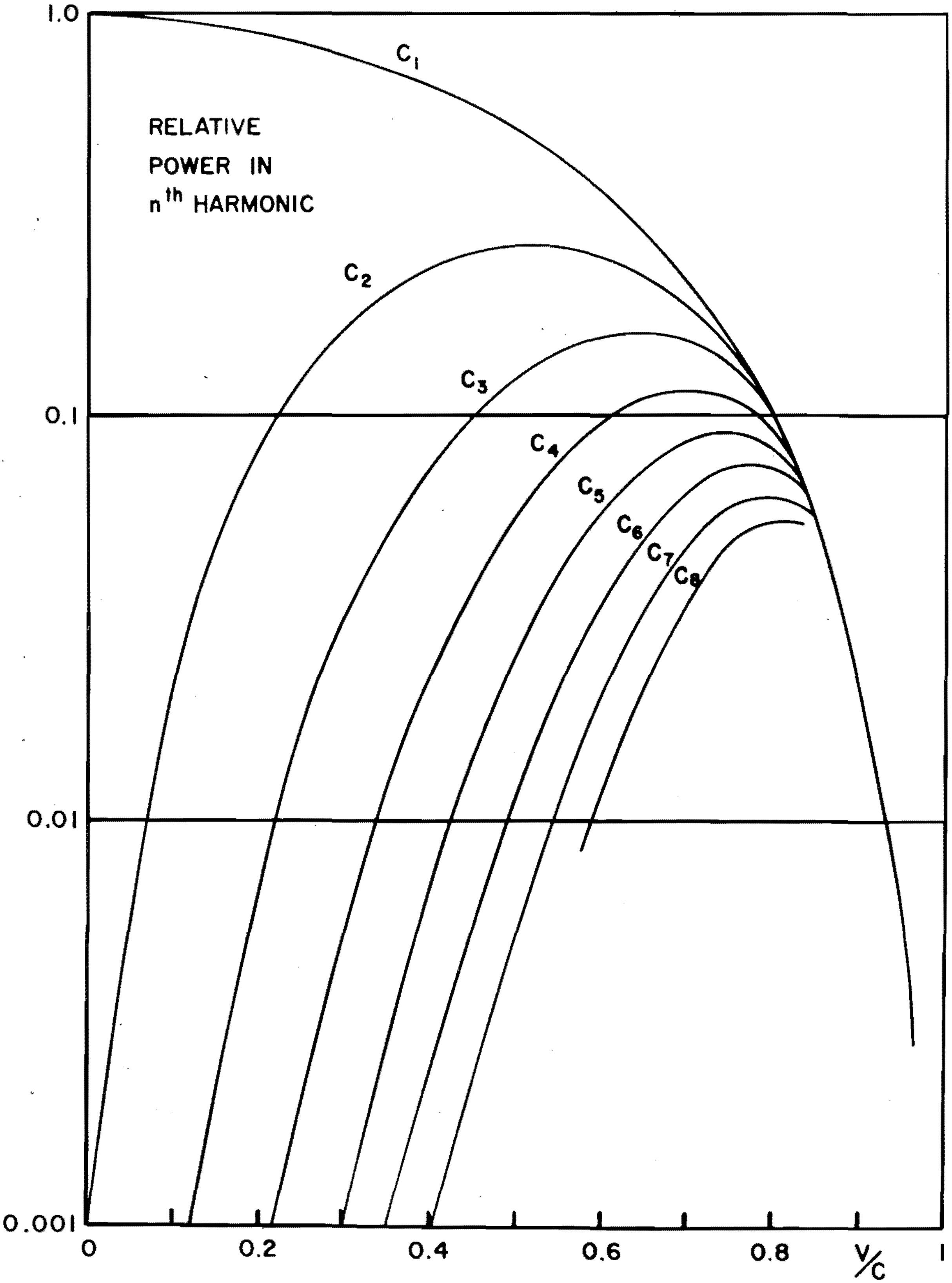

Figure 5 Synchrotron Radiation as a Function of Electron Velocity

and from (34) we may write

$$\frac{3.43}{(1 - \eta)} \frac{\beta_c}{1 - \beta_c} = 1 \quad . \tag{41}$$

Solving this we find

$$\beta_c = \frac{1 - \eta}{4.43 - \eta} \tag{42}$$

which is plotted in Figure 6.

The greater the density of the plasma, the greater will be its absorption, but it is an intricate calculation to determine this function. As a very rough estimate, we shall simply assume that all of the energy in the fundamental (the electron's gyromagnetic frequency) will be absorbed when the plasma density reaches the point where this frequency equals the plasma frequency and extrapolate linearly through this point. This condition is represented by the relation

$$1 = \frac{\beta}{1 - \beta} \left(\frac{mc^2}{4kT}\right) \tag{43}$$

which results in a β of 11% being sufficient to absorb c_1 or 87% of the total synchrotron radiation. The light line in Figure 6 represents this assumption which leads to the conclusion that β must exceed 8% for a DT reactor to maintain its temperature without external sources.

F. Confinement Time Needed

1. Classical Diffusion

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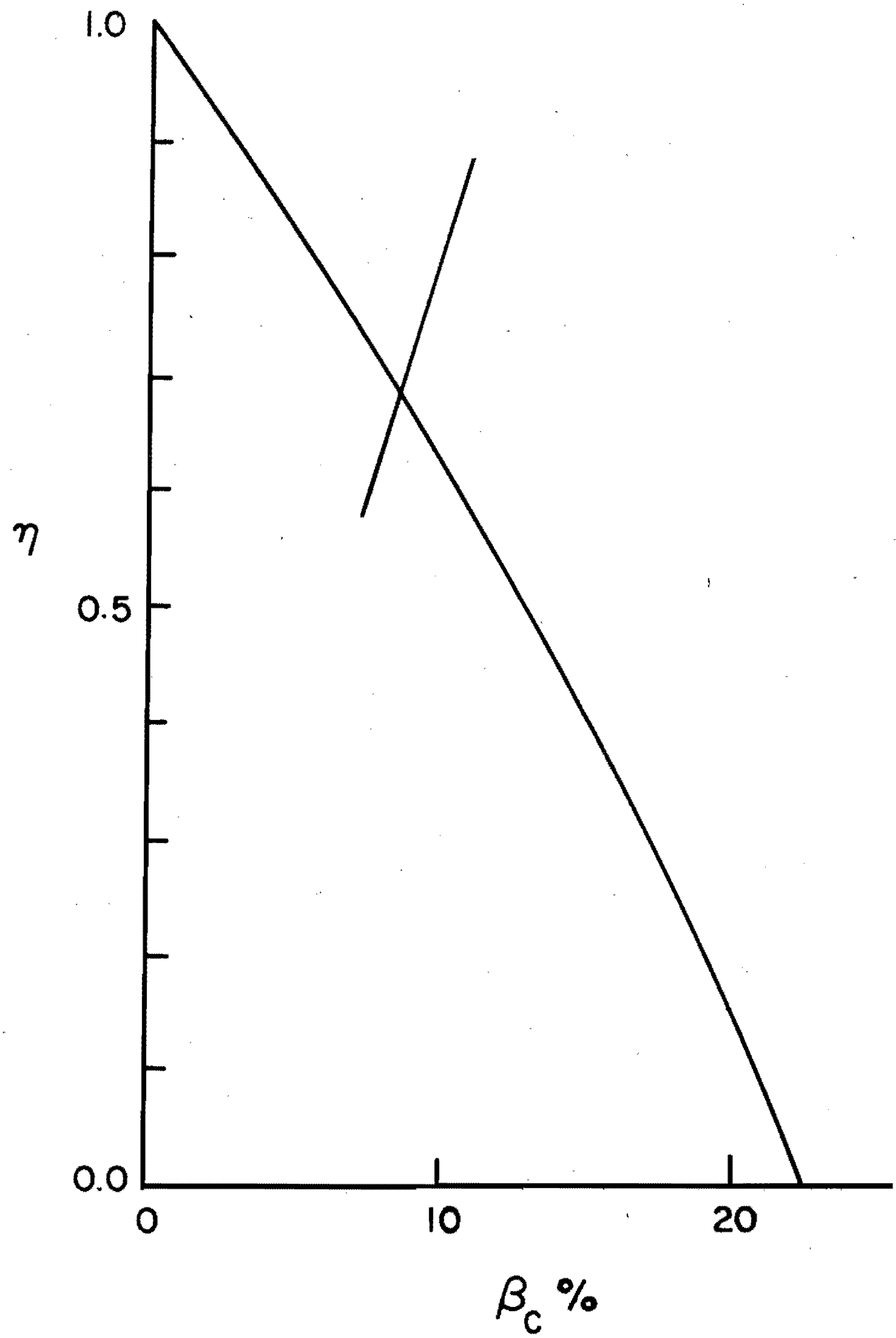

Figure 6 Effect of Absorption on Critical Beta

Classical diffusion theory gives the confinement time for a plasma cylinder of radius r centimeters as

$$\tau = \frac{2 r^2 T^{3/2}}{\beta} \times 10^{-3} \quad \text{seconds} \qquad (44)$$

where T is in kev. This predicts an extraordinarily long period of confinement - several minutes at reactor conditions. Plasma diffusion in experimental apparatus, with the exception of the cesium plasma devices, is much faster than this. Although improvement in confinement times in various machines under investigation is anticipated, it is generally not expected that the slow, classical rates will be accomplished. Nor is it necessary that such long confinement times be achieved. Let us examine the question of what the minimum time required is.

2. Minimum Regenerative Heating Time

A continuously operating steady state reactor is analyzed in which cold fuel gas is continuously injected in equilibrium with the natural losses.

If a cold gas of hydrogen isotopes is injected uniformly at a rate of m particles per cubic centimeter per second, and if an unfed plasma decays with a confinement time τ, the density will vary in time by

$$\frac{dn}{dt} = m - \frac{n}{\tau} \quad . \qquad (45)$$

For steady state conditions

$$m = \frac{n}{\tau} \quad . \qquad (46)$$

The incoming particles are to be heated to reacting temperature T. Therefore they require an input power density of

$$p_i = \frac{n}{\tau}\left(\frac{3}{2} k T\right) \qquad \text{ergs/sec-cm}^3 \quad . \qquad (47)$$

If this power is to be supplied by the release of fusion energy, this power density must be equaled or exceeded by

$$p_F = \frac{1}{16} n^2 (\overline{\sigma v}) (E^* - \bar{E}_B) \qquad (48)$$

where it is assumed that there are n/4 deuterons and an equal number of tritons per cm^3. $(\overline{\sigma v})$ is the reactivity at the temperature T, E^* is the energy released to charged particles, and $\bar{E}_B$ is the average energy lost to radiation per fusion event. The minimum confinement time for the maintenance of temperature will be given by Equations (47) and (48) to get

$$\tau_{min} = \frac{24 k T}{n(\overline{\sigma v})[E^* - \bar{E}_B]} \quad . \qquad (49)$$

By making use of

$$n = \frac{\beta B^2}{8 \pi k T} \qquad (50)$$

we get the final result

$$\tau_{min} = \frac{192 \pi k^2 T^2}{\beta B^2 (\overline{\sigma v}) [E^* - \bar{E}_B]} \quad . \qquad (51)$$

By inserting the proper constants for 15 kev, one finds

$$\tau_{min} = \frac{2 \times 10^8}{\beta B^2} \text{ seconds} , \qquad (52)$$

which is the result shown in Figure 7.

3. Fractional Burnup

It is interesting to compare this with the mean reaction time for a confined nucleus. For n/4 deuterons and n/4 tritons, the reaction rate is $1/16\, n^2 (\overline{\sigma v})$. Therefore the probability per second of a reaction for either a deuteron or a triton is this quantity divided by n/4. It follows that the mean reaction time is the reciprocal of this or

$$\tau_R = \frac{4}{n(\overline{\sigma v})} . \qquad (53)$$

From (49) we have

$$\tau_{min} = \frac{24\, k\, T}{n(\overline{\sigma v})[\, E^* - \overline{E}_B\,]} . \qquad (54)$$

Therefore,

$$\frac{\tau_{min}}{\tau_R} = \frac{6\, k\, T}{[\, E^* - \overline{E}_B\,]} = \frac{4(\frac{3}{2} k\, T)}{[\, E^* - \overline{E}_B\,]} . \qquad (55)$$

Since we are assuming a mean ion energy of 15 kev whereas E^* is 3. 5 Mev, this ratio is 0. 017. In other words, at least 1. 7% of the fuel must react before leaving the reaction tube if the temperature of a reacting DT plasma is not to fall when cold fuel is being added to make up for the particle losses.

This calculation has been done on the same basis as that for the ideal

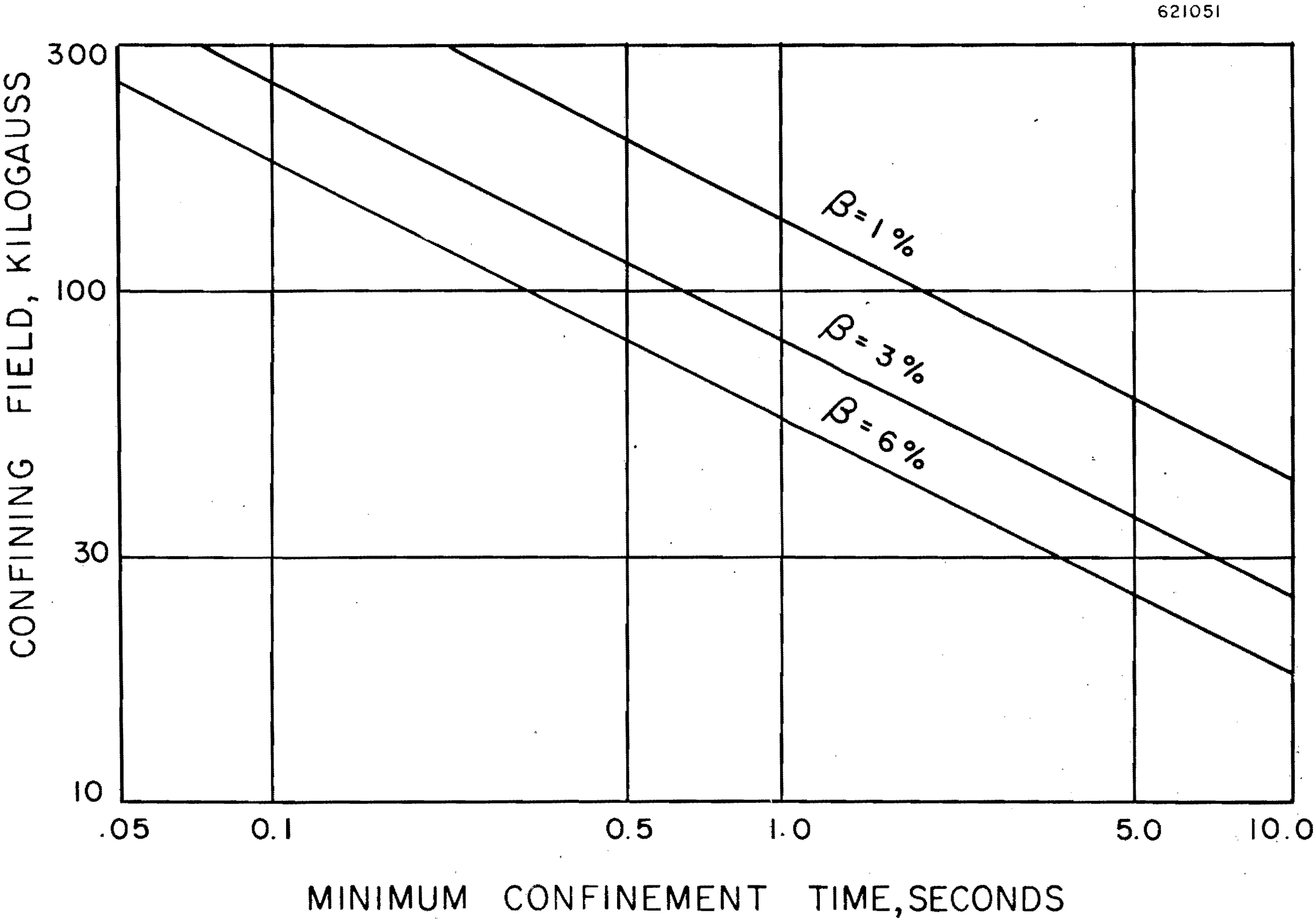

Figure 7 Continuous DT Thermonuclear Reactors

ignition temperature. Bremsstrahlung has been accounted for, but synchrotron radiation has not. Inclusion of this radiation will further increase the minimum regenerative heating time required.

If these confinement times should be proven not attainable, it could still be possible to produce a successful reactor by heating the incoming gas prior to injection. As we shall see below, some machines under investigation require this as they are known to be incapable of the minimum regenerative heating time.

4. Ash Pressure

We are assuming a plasma of equal densities of deuterium and tritium, $n_D = n_T = \frac{n_E}{2}$ with a total particle density $n = 4n_D$. As the reaction

$$D + T \rightarrow He^4 + n$$

proceeds, the He^4 product nuclei will accumulate, leading to a density n_α of reaction products. (The neutrons, of course, leave immediately.) The question arises of whether this constituent of the plasma will contribute significantly to the total pressure. This is important because if n_α should rise too high, the consequent reduction in initial pressure of deuterium and tritium might lower the power generation below the minimum amount needed to maintain the reaction. Since the He^4 are born at high energy, it would not take a large particle density to produce a high pressure.

We equate production rate to loss rate:

$$\frac{1}{16} n^2 (\overline{\sigma v}) = \frac{n_\alpha}{\tau} \quad . \qquad (56)$$

One might speculate that due to the large gyration radius of the energetic He^4, their confinement time might be shorter than the plasma confinement time. To be conservative we shall not adopt this, but take τ to be the regenerative heating time calculated in (49) above. Inserting this, we find ϵ, the relative density of He^4, to be:

$$\epsilon = \frac{n_\alpha}{n} = \frac{3/2\, kT}{[E^* - \bar{E}_B]} . \qquad (57)$$

For the conditions we have assumed this is about 0.4%, which seems low. But let us calculate the relative β

$$\frac{\beta_\alpha}{\beta} = \frac{n_\alpha \frac{2}{3} \bar{E}_\alpha}{nkT}$$

or

$$\frac{\beta_\alpha}{\beta} = \frac{\bar{E}_\alpha}{[E^* - \bar{E}_B]} . \qquad (58)$$

Since the He^4 are born with E^*, the number could be quite high, even exceeding one if the He^4 retained their full energy throughout their time of confinement.

We must now calculate the mean He^4 energy. To do this we apply (5-29) of scripture[11] from which we can find the time constant of the energy decay of the He^4, τ^*.

$$\tau^* = \frac{A\,T^{3/2}}{n_e \ln \Lambda} \times 10^{13} \quad \text{seconds} \qquad (59)$$

where Λ is the ratio of the Debye shielding distance to the impact parameter for 90° deflection (see reference 11 for further discussion) with T in kev. Since

$$\bar{E}_{\alpha} = \frac{\int_0^{\tau} E(t)\,dt}{\int_0^{\tau} dt} \qquad (60)$$

and

$$E(t) = E^{*} e^{-t/\tau^{*}} \quad , \qquad (61)$$

we get

$$\bar{E}_{\alpha} = \frac{\tau^{*}}{\tau} \left[1 - e^{-\frac{\tau}{\tau^{*}}} \right] E^{*} \quad . \qquad (62)$$

Neglecting E_B with respect to E^{*}, and $e^{\frac{-\tau}{\tau^{*}}}$ with respect to one, we get

$$\frac{\beta_{\alpha}}{\beta} \approx \frac{\tau^{*}}{\tau} \quad . \qquad (63)$$

As an example, for A = 4, T = 15, $n_e = 1.65 \times 10^{15}$ and $\Lambda = 17$, Equation (59) gives a τ^{*} of 83 milliseconds. If the plasma confinement time is a half second, β_{α}/β will be 0.166 or the reactants' partial pressure will be 86%. This reduction is enough to require consideration.

5. Rise in Electron Temperature

In the last section we discussed the decay of the energy of the He^4 products in the plasma. In order to supply the energy to the incoming cold fuel, this energy must eventually be transferred to it, and another difficulty arises in this process. The transfer of energy from the He^4 to the plasma takes place predominantly to the electrons. Thus we must pass energy from He^4 to electrons and from electrons to the cold ions. This means that the electron temperature rises above the ion temperature. As pointed out by Spitzer[12] this sets an upper limit to the temperature at which we can operate because as the electron temperature rises, the rate of transfer decreases. Let us examine this question.

The rate of energy release to charged particles is given by

$$p = \frac{n^2}{16} (\overline{\sigma v}) E^* \quad . \tag{64}$$

We assume that this goes to the electrons, elevating their temperature. The incoming cold ions are being heated with a power (the ions have a density of n/2) of

$$p_i = \frac{3nk}{4} \frac{dT}{dt} \quad . \tag{65}$$

As shown by Spitzer[11], (Equation 5-30)

$$\frac{dT}{dt} = \frac{\Delta T}{t_{eq}} \tag{66}$$

where ΔT is the difference in temperature between the electrons and ions, and t_{eq} is the equipartition time.

In equilibrium, (64) and (65) are equal, or

$$\frac{3nk}{4} \frac{\Delta T}{t_{eq}} = \frac{n^2}{16} (\overline{\sigma v}) E^* \quad . \tag{67}$$

It can be shown that the equipartition time for our conditions (a 50-50 DT mixture) is given by

$$t_{eq} = \frac{2.92 \times 10^{12} T_e^{3/2}}{n} . \qquad (68)$$

Substituting,

$$\frac{3n^2 k \Delta T}{4(2.92 \times 10^{12}) T_e^{3/2}} = \frac{n^2}{16} (\overline{\sigma v}) E^*$$

$$\frac{3k(T_e - T_i)}{T_e^{3/2}} = \frac{(2.92 \times 10^{12})}{4} (\overline{\sigma v}) E^* \qquad (69)$$

$T_e > T_i$, let $T_e = p T_i$, $p > 1$

$$\frac{p - 1}{p^{3/2}} = \frac{2.92 \times 10^{12}}{12k} \left(\frac{\overline{\sigma v}}{T^2}\right) T^{5/2} E^* . \qquad (70)$$

Numerically, for our conditions, and defining f by

$$f = \frac{p - 1}{p^{3/2}} = (0.857 \times 10^{15}) T^{5/2} \left(\frac{\sigma v}{T^2}\right) , \qquad (71)$$

we can plot f as a function of p or a function of T. These appear in Figure 8. From this pair of curves the ratio of T_e to T_i can be found for any operating temperature. The curve for f vs. p is not continued beyond its maximum at p = 4. At temperatures above 11 kev, the electrons' temperature runs away since energy is entering them faster than they can pass it along to the ions.

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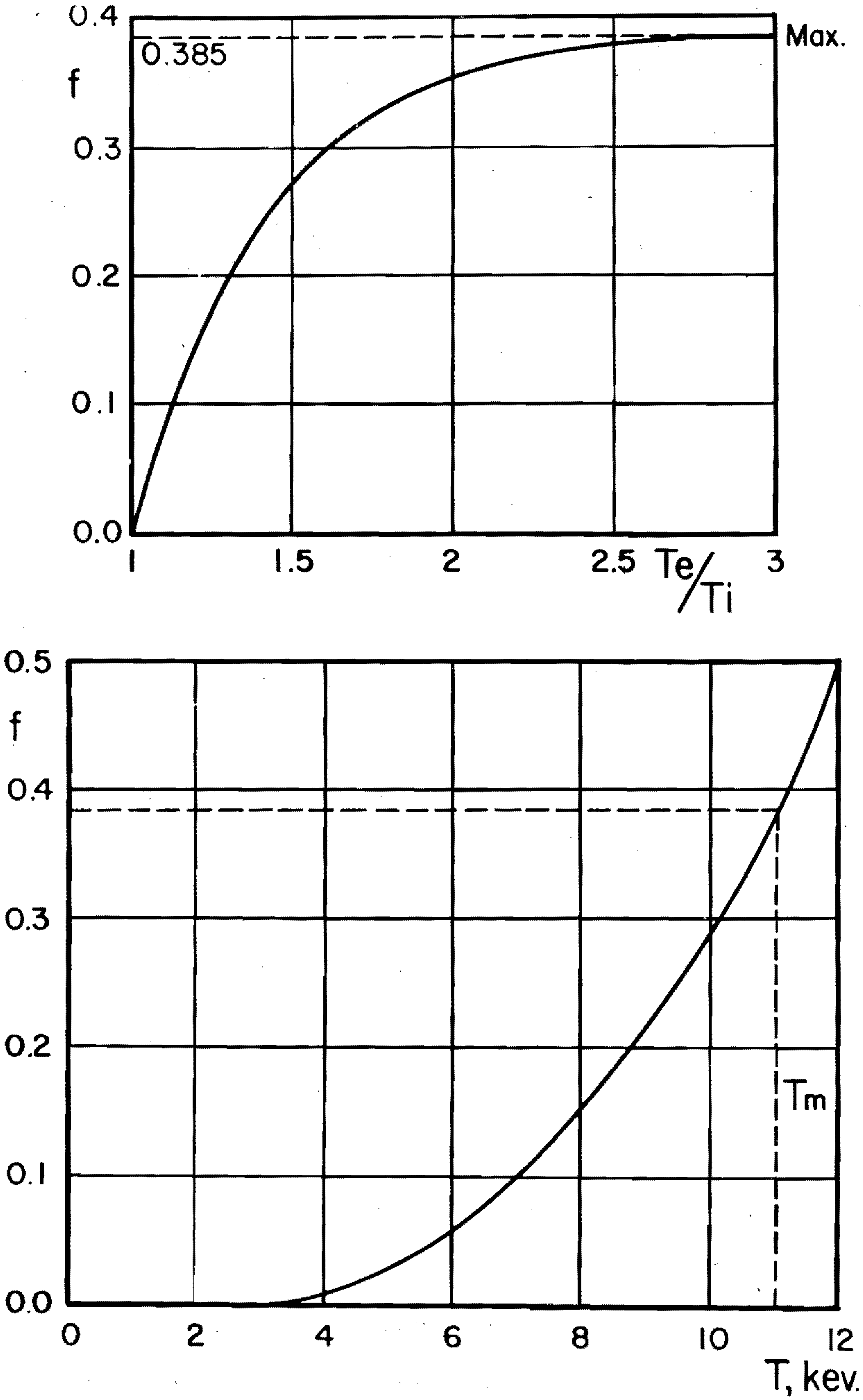

Figure 8 T_e/T_i as a Function of T

They radiate the excess. In an operating reactor, if the temperature fluctuated above 11 kev while cold fuel injection continued, the plasma temperature would fall until it reached equilibrium at 11 kev. Thus our optimistic plans to operate at the 15 kev optimum point on the $\left(\frac{\overline{\sigma v}}{T^2}\right)$ curve must be abandoned. From Figure 2, however, it is seen that the peak is fairly broad, and this is not a severe disadvantage.

G. Proposed Machines

Various machines have been proposed as potential thermonuclear reactors. Machines utilizing the pinch effect would have to be toroidal (ZETA type) to avoid contamination by electrode material. Since the pinched plasma would be a single turn secondary of a transformer, it would necessarily be a cyclic or pulsed machine. Since the stability problem today seems to be too severe a problem, prospects are not bright for this type of machine, and it will not be considered here.

The magnetic mirror machine is in a much more hopeful state. A difficult problem with the mirror is the end loss. The losses are severe and require energetic injection of fuel. The question of stability is still an open one which current research is investigating. Since the mirror is a promising approach, we shall consider it in more detail below.

Cusped machines will not be considered. The loss problem is bad enough in a mirror and will be even worse for a cusp. Nevertheless, if hydromagnetic instability turns out to be the dominant one, a cusp machine may turn out to be the only successful reactor since it should, in principle, be capable of the highest β confinement (approaching one). We skip treatment of the cusp not because it is unpromising, but because its analysis would be very similar to that of a mirror with higher losses but higher β.

The stellarator should be capable of long time confinement since, in theory, stable equilibrium confinement exists. The major problem of the stellarator is to produce reasonably high β's. The inhomogeneous field in the U bends will be a source of trouble unless the plasma conductivity is high enough to short out transverse electric fields in these regions. If the diamagnetic currents which flow parallel to the field to remove this electric field produce oscillations or turbulence which raises the effective resistivity of the plasma, classical confinement, or even sufficiently high confinement for a reactor, may be difficult to achieve. At the present time plasma temperatures are too low for adequate tests of theory. These questions are under active investigation now. Many residents of Princeton believe the stellarator to be the most promising approach to fusion power, and we shall consider it in more detail below.

The astron, if it works, should also be capable of equilibrium confinement, but investigation of this type of machine has not progressed to the point where a final machine can be visualized with any degree of accuracy.

Finally, in extremis, should magnetic field confinement fail completely, there are the bomb-in-a-hole people ready to step in with H-bomb technology, periodically detonating one in a cavern, recovering the heat by various processes from the wall. Those of us in CTR laboratories view these proposals as frightfully crude and see no reason yet to lay down our instruments to pick up the sledge hammer.

III Power Plant Estimates

The first careful study of the possibility of practical power production by controlled thermonuclear reactions was carried out by Spitzer, et al. in 1954[6]. This early study was completed before the modern work on hydromagnetic stability was undertaken and consequently is far too optimistic in selection of operating β. Levels as high as 75% were assumed in this report, whereas today it is not anticipated that these levels can ever be approached except, perhaps, in cusped machines. Later studies by Post[13,14] and Mills[15,16] consider lower β, but their work is open to criticism on a number of points. Post has assumed in some cases unreasonably high efficiencies for certain auxiliaries in the plant; both authors have ignored the He^4 ash problem and synchrotron radiation; and Mills seems to have been unaware of the problem of transferring energy from the He^4 products to the incoming fuel via the electrons as discussed above. He mentions, but does not discuss, the difficulties of designing a stabilizing winding within the blanket in an intense neutron flux. The excellent book by Rose and Clark[17] also discusses matters of interest in the power production problem.

The above referenced reports attempted a broad study examining the effects of changing many variables. Time does not permit any attempt to redo the work with all appropriate changes. Let us take the results of references 14 and 16, change them to meet the objections raised above, and see if we can evaluate the constants k_1 and k' in equations (13) and (14). We shall attempt to do this in as consistent a manner as is possible wherever the machines are similar. There is a considerable similarity since both machines are essentially long solenoids containing a lithium bearing blanket associated with a steam-electric plant, and a vacuum tube to contain the plasma. We shall assume both solenoidal magnets to be superconductors, both machines capable of confining a total β of 12%, and both electric plants capable of 33% efficiency.

A. The Mirror

According to equation (13) the gross electric plant output for a reactor will be

$$P_f = k_1 \beta^2 B^4 r^3 \quad . \qquad (13)$$

We shall use Post's paper to evaluate k_1 for the mirror. We make the optimistic assumption that all of the energy of radiation and mirror-loss particles is recovered as heat and delivered to the steam.

By combining several equations in reference 14 we get an expression equivalent to his equation (43) for the recovered electric power per cubic centimeter of reacting plasma

$$P_n = 5.5 \times 10^2 \frac{1}{Q} \left((Q+1)\eta_t - \frac{1}{\eta_s} \right) \left(\frac{\overline{\sigma v}}{T^2} \right) \beta_i^2 B^4 \qquad (72)$$

where Q is the ratio of nuclear power released to particle energy escape rate, η_s is the efficiency of the injector, η_t is the efficiency of the electric plant, and β_i is the partial β due to ions. Post considers mirror ratios of 2, 3.3, and 10. We pick 3.3 and use his result for

$$Q = 6.1 \times 10^{14} (\overline{\sigma v}) T_i^{1/2} \quad . \qquad (73)$$

It is this factor which represents the mirror loss problem. It requires the ion temperature in a mirror to be higher than that for optimum $\left(\frac{\overline{\sigma v}}{T^2}\right)$. The optimum seems to lie near 40 kev, and adopting this as our operating temperature, we can compute Q to be 3.5. Taking $\eta_t = 0.33$ and accepting Post's admittedly highly optimistic estimate of 0.9 for η_s, we get

$$p_n = 0.6 \times 10^2 \left(\frac{\overline{\sigma v}}{T^2}\right) \beta_i^{\,2} B^4 \quad \text{watts/cm}^2 \quad . (74)$$

Since $\overline{\sigma v}/T^2$ is 5.5×10^{-19} at 40 kev, we get

$$p_n = 3.3 \times 10^{-17} \beta_i^{\,2} B^4 \quad \text{watts/cm}^3 \quad . \quad (75)$$

Next we must determine what fraction of the total β is represented by β_i. We write

$$\beta = \beta_i + \beta_e + \beta_\alpha \quad . \quad (76)$$

In a reactor in complete kinetic equilibrium, β_e will be the same as β_i, but Post assumes his electrons are at 1/4 the ion temperature (see reference 14 for an explanation). We have indicated in the section on ash pressure how to compute β_α. Post's quantity ϕ, defined by

$$\phi = 4.1 \times 10^{10} (\overline{\sigma v}) T^{3/2} \quad , \quad (77)$$

gives the fractional density of He^4 and is 0.92%. The mean confinement time for our conditions will be about 65 milliseconds, whereas the slowing down time is 124 milliseconds. For these values the average energy of the

He^4 is 78% of its birth value, or

$$\frac{\beta_\alpha}{\beta_i} = \frac{n_\alpha}{n_i} \cdot \frac{\overline{E}_\alpha}{\overline{E}_i} = (0.0092)\,\frac{0.78(3.52)}{0.060} = 0.42 \quad . \qquad (78)$$

Rewriting (76) we get

$$\beta = \beta_i + 0.25\beta_i + 0.42\beta_i \qquad \text{or}$$

$$\beta = 1.67\beta_i$$

$$\beta_i^{\,2} = \frac{\beta^2}{2.8}$$

which gives us the nuclear power density as

$$p_n = 1.22 \times 10^{-17}\beta^2 B^4 \quad . \qquad (79)$$

To scale to the radius of the vacuum vessel, we note that Post adopts models where $\ell = 100\,r$ giving a volume of $\pi(100\,r^3)$, or

$$P_f = 3.85 \times 10^{-15}\beta^2 B^4 r^3 \quad . \qquad (80)$$

Thus we have evaluated k_1.

$$k_1 = 3.85 \times 10^{-15} \quad \text{watts/cm}^3\text{-gauss}^4 \ . \qquad (81)$$

The power loss to the magnet scales in accordance with equation (14)

$$P_M = k'\rho B^2 r \quad . \qquad (14)$$

We have agreed to superconductors for the main solenoid, but the mirrors generate a field strength 3.3 times that of the solenoid. We assume that these will be above the critical field of the superconductor and will therefore have to be normal conductors. Nevertheless they can be cryogenic coils, and we assume them (including their refrigerators) to require only 1/10 the power copper coils would. Post publishes curves for a "standard mirror coil" of copper which are equivalent to the relation

$$P_M = 6 \times 10^{-5} B_M^{\,2} r' \quad . \tag{82}$$

We correct this as follows: We need two mirrors (X2); we shall use a cryogenic system ($\div$ 10); B_M = 3.3B (X10.9); r' (the inner radius of the mirror) will be assumed to be 3r (X3). Therefore our magnet power expression becomes

$$P_M = 3.92 \times 10^{-4} B^2 r \tag{83}$$

or

$$k'\rho = 3.92 \times 10^{-4} \quad \text{watts/cm - gauss}^2 \tag{84}$$

and

$$\frac{P_f}{P_M} = 9.8 \times 10^{-12} \beta^2 B^2 r^2 \quad . \tag{85}$$

The plant will not be economic unless this ratio is at least 2. If we assume a ratio of 2, a β of 12%, and a magnetic field of 120 kG, we find the economic minimum radius of 31.4 cm and net output power for the plant of 170 MW.

B. The Stellarator

According to reference 16, k_1 for a stellarator is 2.5×10^{-14}. We must reduce this to meet the objections mentioned above. In the first place Mills has assumed the operating temperature to be 15 kev, whereas we have shown above that the electron temperature will run away at this temperature, and that 11 kev is the absolute maximum. We adopt 10 kev as a conservative operating ion temperature. At this level $(\overline{\sigma v}/T^2)$ is reduced from its optimum value of 11.5 to 11×10^{-19} $cm^3/sec\text{-}kev^2$. Furthermore the electron temperature (from Figure 8) will be 1.55 times the ion temperature which is involved in the calculation of β, or

$$\beta_e = 1.55\,\beta_i \quad . \tag{86}$$

We must correct for β by including β_α (the ash pressure) as described earlier. Now

$$\beta = \beta_i + \beta_e + \beta_\alpha \quad , \tag{76}$$

and we have shown in (63) et seq. above that

$$\frac{\beta_\alpha}{\beta_e + \beta_i} = 0.166 \quad . \tag{87}$$

Inserting (86) we get

$$\frac{\beta_\alpha}{2.55\,\beta_i} = 0.166 \quad .$$

or

$$\beta_\alpha = 0.42\,\beta_i \quad . \tag{88}$$

Thus (76) becomes

$$\beta = \beta_i + 1.55\,\beta_i + 0.42\,\beta_i$$

$$\beta = 2.97\,\beta_i$$

rather than

$$\beta = 2\,\beta_i \tag{89}$$

as assumed in reference 16. Thus β^2 must be corrected by

$$\beta^2_{new} = \left(\frac{2}{2.97}\right)^2 \beta^2_{old} \quad . \tag{90}$$

Reducing the old k_1, giving the new as

$$k_1 = 1.1 \times 10^{-14} \quad \text{watts/cm}^3\text{-gauss}^4 \quad . \tag{91}$$

The stellarator, as well as the mirror, requires normally conducting magnets. We assume, as we did for the mirror, that the solenoid is generated by superconductors. Where the mirror machine has mirrors to power, the stellarator has stabilizing windings. These must lie on the vacuum tube in a neutron flux, and therefore cannot be cryogenically cooled. We assume copper of $\rho = 2 \times 10^{-6}$ ohm-cm. We assume an $\ell = 2$ (4 conductor) stabilizing winding of 30° angle carrying a current sufficient to generate a field B/2 at a distance within the vacuum tube of 0.08 r. The current flows through rectangular conductors $r/2 \times \pi r/4$. The axial length of the machine is 160 r. This gives a value for k_2 of

$$k_2 = 6.5 \times 10^{-4} \quad \text{watts/cm - gauss}^2 \tag{92}$$

and

$$\frac{P_f}{P_M} = 1.7 \times 10^{-11} \beta^2 B^2 r^2 \quad . \tag{93}$$

Assuming again a ratio of 2, a β of 12%, and a magnetic field of 120 kG, we find an economic minimum radius of 23.8 cm and a net output power for the plant of 270 MW.

C. Discussion

It is rather remarkable how similar these two hypothetical machines appear. A popular misconception is that a fusion plant will be at least unwieldy if not of vast proportions. Actually they should be more compact than present steam plants. Figure 9 shows a modern 650 MW steam plant. It consists of two independent 325 MW units. The boiler for each is ten stories high.

In view of the large number of unknowns in this subject, both the mirror and the stellarator will probably change significantly in concept as time goes on. The process of evaluating the constants k_1 and k_2 points out that in addition to the basic plasma physical problem of heating and confinement, there are severe engineering problems with which to struggle. The mirror has its injector efficiency problems; the stellarator its stabilizing winding design. There are many others, probably less critical.

One is the blanket design which we have not discussed in these lectures due to lack of time. This problem is discussed in reference 6, and more modern work on different aspects of the problem is being done by E. F. Johnson in the Department of Chemical Engineering at Princeton, by David Rose and his group at M.I.T. and by a group at Oak Ridge. At the present time it does not appear that the blanket can be made much less than one meter thick. The problems are complex and involve the neutron budget, tritium recovery efficiency, and the problem of tritium inventory, an expensive commodity. In reading the literature you will find many references to the reaction

$$Li^6 + n \rightarrow T + He^4$$

which is usually treated as the only source of T. Since Li^6 is present to only about 7.5% in natural lithium, it is interesting that blanket designsbased on this breeding reaction are apparently feasible. Recently the cross-section for the following reaction has been published[18]

$$Li^7 + n \rightarrow n + T + He^4 \quad .$$

The cross-section is large and not only produces T from the much more abundant Li^7, but leaves an extra neutron available. This will be very helpful in blanket design.

D. Costs

Time does not permit a discussion of this topic. It is covered from the capital cost standpoint in reference 16. It appears that if fusion reactors become possible without expensive departure from the equipment visualized

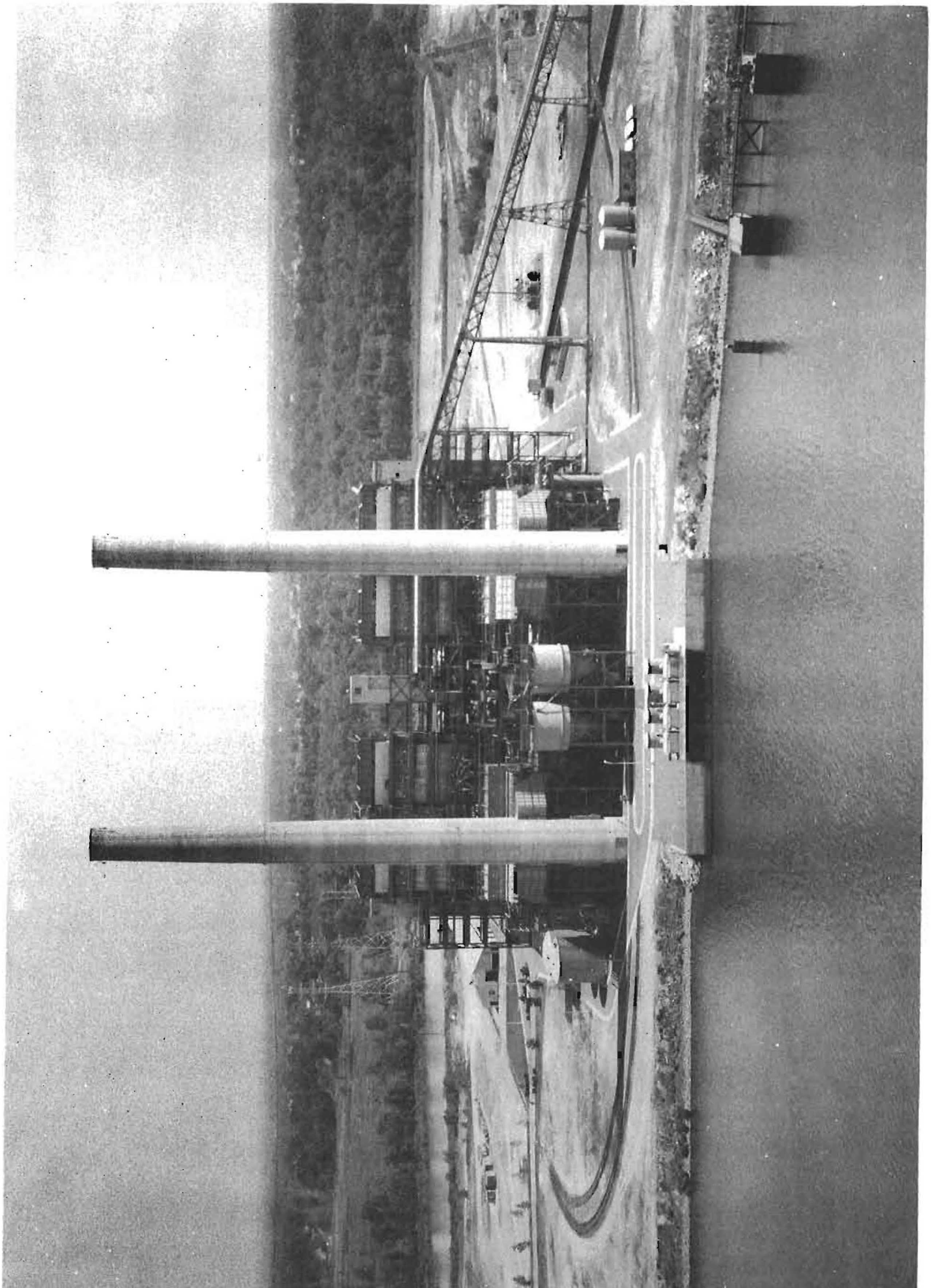

FIGURE 9 MODERN GENERATING STATION

as necessary now, they should be highly competitive commercially, producing power at lower than current costs.

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