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Advances in the numerical modeling of field-reversed configurations

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Abstract The field-reversed configuration (FRC) is a compact torus with little or no toroidal magnetic field. A theoretical understanding of the observed FRC equilibrium and stability properties presents significant challenges due to the high plasma beta, plasma flows, large ion gyroradius, and the stochasticity of the particle orbits. Advanced numerical simulations are generally required to describe and understand the detailed behavior of FRC plasmas. Results of such simulations are presented in this paper. It is shown that 3D nonlinear hybrid simulations using the HYM code [E. V. Belova *et al.*, Phys. Plasmas **7**, 4996 (2000)] reproduce all major experimentally observed stability properties of elongated (theta-pinch-formed) FRCs. Namely, the scaling of the growth rate of the $n = 1$ tilt mode with the S^*/E parameter (S^* is the FRC kinetic parameter, E is elongation, and n is toroidal mode number), the nonlinear saturation of the tilt mode, ion toroidal spin-up, and the growth of the $n = 2$ rotational mode have been demonstrated and studied in detail. The HYM code has also been used to study stability properties of FRCs formed by the counter-helicity spheromak merging method. A new stability regime has been found for FRCs with elongation $E \sim 1$, which requires a close-fitting conducting shell and energetic beam ion stabilization.

I. INTRODUCTION

The field-reversed configuration (FRC) is a compact torus with zero toroidal magnetic field and high plasma beta. The FRC is an example of a self-organized plasma state, in which no external coils are used to confine the plasma, and confinement is achieved by means of the self-induced poloidal magnetic fields. Although magnetohydrodynamic (MHD) theory predicts strong instabilities, experimentally it is found that FRC configurations are surprisingly robust, and have been shown to survive, for example, during translation from one chamber to another and multiple reflections from magnetic mirrors [1].

The traditional theta-pinch formation method usually produces highly kinetic FRCs with relatively low magnetic flux, small S^* (the FRC kinetic parameter S^* is the ratio of the separatrix radius to the ion skin depth) and large elongation E. Experimentally, these FRCs are observed to suffer the $n = 2$ rotational instability, which can be suppressed by the application of weak multipole magnetic fields after FRC formation [1,2]. The $n = 2$ rotational mode is the only experimentally observed global mode, which often prematurely terminates the these FRC configurations [1,3]. The $n = 1$ tilt mode, which has been theoretically shown to be strongly unstable within an MHD model, and which is considered to be the most dangerous global mode in prolate FRCs, has not been observed at all, or it has been seen to “decay away” without destroying the configuration [3–5].

The intriguing stability properties of prolate (theta-pinch-formed) FRCs have been the subject of numerous theoretical studies [6–12]. A theoretical understanding of the observed FRC behavior presents significant challenges due to the high plasma beta, plasma flows, large ion gyroradius, and the stochasticity of the particle orbits. Advanced numerical simulations are generally required to describe the self-consistent stability properties of kinetic FRCs [10–13]. The results of such simulations are presented in this paper. It is shown that reasonably good agreement between kinetic simulations using the HYM code and experimentally observed FRC stability properties are been obtained. In particular, thermal ion finite-Larmor-radius (FLR) effects have been shown to play a major role in reducing the growth rates of unstable modes in prolate kinetic FRCs, and the nonlinear saturation of the $n = 1$ tilt instability has been demonstrated numerically [12].

For large-scale, reactor-relevant configurations, new FRC formation methods are being investigated, including the counter-helicity spheromak merging method [14–17], and rotating magnetic field (RMF) current drive methods [18]. Since FLR stabilization does not scale to large FRC configurations, additional methods will be needed in order to stabilize the low-n MHD modes. These methods may include, for example, stabilization by a close-fitting conducting shell, and by RMF effects [19]. Injection of energetic ion beams may also provide an additional stabilizing mechanism, as well as plasma heating and current drive in future planned experiments [20,21]. This paper investigates the stability properties of oblate FRCs formed by counter-helicity spheromak merging, including the effects of a conducting shell and neutral beam injection (NBI) stabilization.

This paper is organized as follows. The numerical model is described in Section II. The results of nonlinear hybrid simulations of prolate FRCs are summarized in Section III. Nonlinear simulations of oblate FRCs including the effects of a conducting shell and NBI stabilization are described in Section IV, and the conclusions are summarized in Section V.

II. NUMERICAL MODEL

The stability properties of FRC configurations have been investigated numerically using the hybrid version of the 3D nonlinear simulation code HYM [10]. A full-orbit kinetic description is used for both the thermal ions and the beam ions, and the electrons are treated as a cold fluid. The numerical scheme implemented in HYM code has been described elsewhere [10]. The basic equations for the magnetic field $\mathbf{B}$, electric field $\mathbf{E}$, total plasma current $\mathbf{J}$, and electron flow velocity $\mathbf{v}_e$ are given by

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}, \tag{1}$$

$$\mathbf{E} = -\mathbf{v}_e \times \mathbf{B}/c + \eta \mathbf{J}, \tag{2}$$

$$\mathbf{J} = (c/4\pi)\nabla \times \mathbf{B}, \tag{3}$$

$$\mathbf{v}_e = -(\mathbf{J} - \mathbf{J}_i - \mathbf{J}_b)/en_e. \tag{4}$$

Here $\mathbf{J}_i + \mathbf{J}_b$ is the total ion current density (thermal ion plus beam), electron inertia effects and the displacement current are neglected, and quasineutrality $n_e = n_i + n_b$ is assumed for singly-charged ions. The ion density and current density are calculated from the distribution of the simulation particles, which follow the ion trajectories using the Lorentz-force equations. The nonlinear delta-f particle simulation method [22,12] is employed in order to reduce the numerical noise level. The nonlinear HYM simulations have been performed with a cylindrical grid size of $50 \times 32 \times 60$ in (r, ϕ, z) using 2×10^6 simulation particles. The region outside the separatrix (open field lines) is modeled as a low-density, high-resistivity plasma.

In the simulations, the thermal ion equilibrium distribution function is taken to be a local Maxwellian with $f_i \sim \exp(-\varepsilon/T_0)$, where $\varepsilon = m_i\mathbf{v}^2/2 + e\phi_0$ is the ion energy, ϕ_0 is the equilibrium electrostatic potential, and T_0 =const is the thermal ion temperature. The equilibrium thermal ion toroidal flow is neglected.

In Sec. IV, an exponential rigid-rotor distribution function is assumed for the beam ions, corresponding to $f_b(\varepsilon, p_\phi) = A\exp[-(\varepsilon - \Omega_0 p_\phi)/T_b)]$, where A is a normalization constant, $p_\phi = m_iRv_\phi - e\psi$ is the canonical toroidal angular momentum, ε is the beam ion energy, T_b is the beam ion temperature (assumed constant), and Ω_0 =const. The beam ion distribution function corresponds to a local, shifted Maxwellian distribution with constant toroidal angular rotation frequency Ω_0.

Self-consistent equilibria have been calculated including the contribution of the beam ions for simulations described in Section IV. The generalized Grad-Shafranov equation for poloidal flux ψ can be expressed as [23]

$$\Delta^* \psi = -R^2 p_i' + R J_{b,\phi}, \tag{5}$$

where Δ^* is the Grad-Shafranov operator, $p_i = n_i T_0$ is the thermal ion pressure, $J_{b,\phi}$ is the toroidal component of the beam current density, and the prime ($'$) denotes differentiation with respect to the poloidal flux. The equilibrium can be calculated provided the magnetic surface function $p_i(\psi)$ is specified, and the beam ion current density is calculated using the fast-ion distribution function f_b. The solution to Eq. (5) has been obtained iteratively with the total toroidal current used as an integral constraint during the iterations.

III. NONLINEAR STABILITY PROPERTIES OF PROLATE FRCs

Numerical studies of the nonlinear evolution of MHD modes with toroidal mode numbers $n \geq 1$ in kinetic, prolate FRCs (with $E \geq 4$) have been performed using the 3D nonlinear hybrid and MHD simulation code HYM [10]. It has been demonstrated that due to the strong FLR stabilization of the higher-n modes, the $n = 1$ tilt mode is the most unstable mode for $S^* \lesssim 50 - 80$, i.e., for nearly all experimentally-relevant non-rotating FRC equilibria [12]. An empirical FLR scaling of the tilt-mode linear growth rate in elongated elliptical FRCs has been obtained from a fit to the numerical results (Fig. 1) according to $\gamma = \gamma_{mhd} \exp(-3E\rho_i/R_s)$, where $\gamma_{mhd} = CV_A/R_sE$ is the MHD growth rate, and $C \approx 2$ is a constant. Other FRC parameters are defined as follows, R_s is the separatrix radius, ρ_i is ion thermal Larmor radius, V_A is a characteristic Alfvén velocity, and the elongation E is defined as the ratio of the separatrix half-length to its radius, $E = Z_s/R_s$.

A significant reduction of the growth rate occurs in the kinetic regime with small S^* and large E, and this reduction depends on the parameter S^*/E, because $E\rho_i/R_s \sim (S^*/E)^{-1}$. Moreover, nonlinear hybrid simulations have shown that the tilt instability saturates nonlinearly without destroying the configuration, when its linear growth rate is sufficiently small. In particular, the saturation of the $n = 1$ tilt mode is found for FRC parameters $S^* < 20$ and $E \sim 6$ (Fig. 4 of Ref. [12]). These numerical results explain the experimental data for prolate FRCs, which show that stability with respect to global MHD modes depends on the S^*/E parameter [5], and that the stable FRCs are observed for $S^*/E \lesssim 3 - 4$.

Figure 1 shows that the deviation from S^*/E scaling occurs for $S^*/E < 2$ for configurations with elongations $E \leq 6$ (i.e., for smaller values of $S^* < 12$). Linearized hybrid simulations have shown that this deviation is due to resonant-particle destabilization of FLR-stabilized MHD modes [11]. The growth rate of the resonant instability depends, in particular, on the stochasticity of the ion orbits. Namely, the wave-particle resonances are shown to occur only in the regular regions of the phase-space. Therefore, the stochasticity of the equilibrium particle orbits in prolate FRCs, which occurs due to the large curvature of the magnetic field lines near the FRC ends, has an important effect on the FRC stability properties. Analysis of the particle orbits in long configurations, for different values of S^*, have produced rather unexpected results [11]. Contrary to the usual assumption of a large degree of stochasticity of the ion orbits in FRCs, we have found that a significant fraction of the orbits (up to 60% of all confined orbits) is regular in configurations with $S^* \sim 10$. Furthermore, the number of regular orbits has been shown to scale approximately linearly with $1/S^*$, independent of E (see Fig. 9 of Ref. [11]). Although the magnetic moment is not conserved for most of the ion orbits, another adiabatic invariant, based on the smallness of the $1/E$ parameter, is conserved in low-S^* FRCs. A larger number of regular ion orbits in low-S^* configurations results in a stronger resonant drive for the $n = 1$ mode, which explains the deviation from S^*/E scaling for the $E \sim 6$ case in Fig. 1. For a given value of S^*/E, a larger value of E implies a larger S^* value, and therefore a larger number of ergodic orbits. For these reasons, the FLR-fluid-like behavior is evident for the $E = 12$ case (Fig. 1).

Nonlinear kinetic simulations performed for a set of FRC equilibria with $E = 4 - 6$ and $S^* = 10 - 80$ show that the $n = 1$ tilt mode saturates nonlinearly without destroying the configuration, provided the FRC kinetic parameter is sufficiently small, $S^* \lesssim 20$. In addition to the saturation of the tilt mode, the simulations show that the ions spin-up toroidally in the ion diamagnetic direction, and the $n = 2$ rotational mode grows in the nonlinear phase of the simulation (Fig. 4 of Ref. [12]). Initial conditions for the simulations are set at $t = 0$ so that the ions have a non-rotating Maxwellian distribution, which is consistent with experimental observations just after FRC formation. However, as the simulation proceeds, the ions gradually begin to rotate, and near the end of the simulation run the ion toroidal flow velocity is comparable to the ion diamagnetic velocity. The saturation of the tilt instability occurs in the presence of a significant ion toroidal rotation, and in the nonlinear phase, the ion rotation rate is comparable to the linear growth rate of the tilt mode. Therefore, the ion toroidal spin-up, in addition to driving the rotational instability, is likely to contribute significantly to the saturation of the tilt instability [24].

A separate set of 2D (axisymmetric) nonlinear hybrid simulations has been performed in order to investigate the mechanism of the ion spin-up [25]. The simulations show that there is a significant particle loss associated with the resistive decay of the poloidal flux. Namely, the magnetic field decay results in a slow change in the particle trajectories, and the initially weakly-confined ion trajectories eventually change into open-field-line trajectories. It is also found that most of the lost particles have negative toroidal velocity, so that there is a net flux of negative momentum away from the separatrix region, and therefore a net positive ion rotation inside the separatrix (where the positive direction corresponds to the direction of the toroidal current). The simulations show that the ion toroidal spin-up is related to the resistive decay of the internal flux, and the resulting loss of weakly-confined particles, and the details of the ion toroidal spin-up determine the nonlinear evolution of rotational instabilities.

Both 2D and 3D simulations with zero initial ion rotation demonstrate the formation of an approximate rigid-rotor velocity profile inside the separatrix in about 40-60 Alfvén times, depending on the plasma resistivity. Radial profiles of the ion toroidal flow velocity are shown in Fig. 2. The linear velocity profile ($V_\phi \sim R$) suggests that the distribution function f_i of the ions inside the separatrix evolves toward a shifted local Maxwellian distribution, which may indicate that the stochasticity of the ion orbits plays a significant role in FRC relaxation.

IV. STABILIZATION OF OBLATE FRCs BY A COMBINATION OF CONDUCTING SHELL AND BEAM ION EFFECTS

The counter-helicity spheromak merging method typically produces FRC configurations with relatively small elongation with $E \sim 1$. It is known that stability properties of oblate FRCs with $E \lesssim 1$ are different from prolate FRCs ($E \gg 1$). In particular, earlier studies have shown that the $n = 1$ tilt mode is an external mode in oblate FRCs, and that this mode can be completely stabilized by a close-fitting conducting shell even in the MHD regime [26]. With conducting-shell stabilization, the $n > 1$ internal co-interchange (kink) modes become the most unstable MHD modes [26]. In contrast to prolate FRCs, the thermal ion FLR stabilization of the low-n modes in oblate FRCs is weak even for low values of the S^* parameter. However, the localization of the low-n kink modes near the magnetic null suggests that neutral beam injection (NBI) may be a very effective stabilizing mechanism for oblate FRCs.

The equilibrium solutions of the generalized Grad-Shafranov equation have been calculated for thermal plasma and beam ion parameters consistent with the proposed MRX-FRC experiment [20], i.e., for $E = 1.1$, $S^* = 18$, and for the following beam ion parameters: beam toroidal velocity (at the magnetic null $V_0 = R_0\Omega_0$) $V_0 \approx 6V_A$, normalized peak density n_b/n_e =3%, and normalized temperature $\hat{T}_b = T_b/(m_i V_A^2/2) = 10$. It is found that the beam ions tend to coalesce between the magnetic null and the separatrix near the FRC midplane, in agreement with earlier calculations of FRC-beam equilibria [27,28]. Due to localization, the peak beam current density can be comparable to the local plasma current density, even when the fraction of the total current carried by the beam ions is small.

The calculated FRC-beam equilibria have been used as initial conditions for the linearized and nonlinear simulations using the hybrid version of the HYM code. Stability properties of the MHD modes with toroidal mode numbers $n = 1-4$ have been studied with and without the effects of a close-fitting conducting shell and the NBI ions. The growth rates of the most unstable mode for each toroidal mode number for three different sets of simulations are shown in Fig. 3a (where the growth rates are normalized to $\gamma_0 = V_A/Z_s$).

For an oblate FRC configuration with thermal ion parameter $S^* = 18$ and elongation $E \sim 1$, the stability parameter $S^*/E \sim 18$ is well above the empirical stability boundary [1] at $S^*/E = 3-4$. The $n = 1$ tilt mode and other $n > 1$ MHD modes are expected to be strongly unstable in this regime. Indeed, Fig. 3a shows that in the absence of a conducting shell and beam ion effects, the $n = 1$ mode is the most unstable mode, with growth rate $\gamma = \gamma_0 = 0.83\gamma_{mhd}$, where $\gamma_{mhd} = 1.2\gamma_0$ is the growth rate of the $n = 1$ mode in the MHD regime. The growth rate of the $n = 2$ mode is comparable to that of the tilt mode, whereas the growth rates of the $n = 3$ and $n = 4$ modes are reduced, probably due to stronger thermal ion FLR stabilization at higher values of n.

Analysis of the linear mode structure shows that the $n = 1$ tilt mode is an external mode, which has a large perturbation amplitude at the separatrix, whereas the higher-n modes, $n > 1$, are more localized and have smaller radial extent. The growth rate of the $n = 1$ tilt mode is reduced almost by an order-of-magnitude when a close-fitting conducting shell is used for stabilization (Fig. 3a, green curve). The growth rates of the $n > 1$ modes are also reduced due to conducting-shell effects. Conducting-shell stabilization for the above FRC parameters is stronger than had been previously found in MHD plasmas [26] due to the change in the linear mode structure caused by thermal ion kinetic effects. With conducting-shell stabilization, the $n = 2$ axially polarized kink mode becomes the most unstable mode, and all unstable low-n modes are localized near the magnetic null.

Figure 3a also shows the simulation results with the combined effects of the conducting shell and energetic beam ion stabilization (blue curve). It is evident that the beam ions have a strong stabilizing effect on the $n = 1$ and $n = 2$ modes, which are stabilized completely. For the same set of beam ion parameters, the growth rates of the $n = 3$ and $n = 4$ modes remain approximately the same, with the $n = 3$ mode being more unstable than the $n = 4$ mode. Numerical simulations with larger beam ion density, $n_b/n_e = 0.05$, show a reduction of the growth rates of these modes by a factor 1.5, but not complete stabilization.

A set of 3D nonlinear hybrid simulations has been performed in order to study the nonlinear evolution of the FRC configuration in the presence of a conducting shell and energetic beam ions with n_b/n_e =3% (Fig. 3b). These simulations show that the residual $n = 3$ (and $n = 4$) instability saturates nonlinearly at low amplitude (Fig. 3b), and therefore it is not a dangerous mode. Simulation runs which model a "sustained FRC" (i.e., without decay of the equilibrium current) show that after the $n = 3$ mode saturates, the resulting configuration remains stable with respect to all global MHD modes. Note that this is a new stability regime (i.e., a combination of small elongation, conducting shell, and beam ion effects), which has not yet been studied experimentally. The stabilizing effects of the neutral-beam-induced bulk ion rotation have not been included in the present numerical study, but these effects will likely contribute to the stabilization of the low-n MHD modes.

V. CONCLUSIONS

The hybrid simulations presented here demonstrate that ion FLR effects determine the linear stability properties of non-rotating prolate FRCs. It has been shown that the inclusion of nonlinear and ion-toroidal-flow effects is necessary for a satisfactory description of plasma behavior in low-S^* FRC experiments. In particular, nonlinear hybrid simulations have shown that the ion toroidal spin-up plays an important role in FRC nonlinear evolution, including that of the $n = 1$ tilt mode.

The 3D hybrid simulations have been able to reproduce all major experimentally observed stability properties of kinetic (theta-pinch-formed) FRCs. Namely, the scaling of the linear growth rate of the $n = 1$ tilt instability with the S^*/E parameter has been obtained for a class of elongated elliptical FRCs [12]; and ion toroidal spin-up, the nonlinear saturation of the tilt mode, and the growth of the $n = 2$ rotational mode have been demonstrated. It has been shown that the loss of ions with a preferential sign of toroidal velocity due to the resistive decay of the poloidal flux results in the ion toroidal spin-up, which reproduces very well the experimentally-observed ion rotation.

Stability properties of FRCs with smaller elongation $E \sim 1$, which are typically formed using counter-helicity spheromak merging methods, have also been studied. A new stability regime has been discovered which requires a close-fitting conducting shell and energetic beam ions for stabilization. It has been shown that in this regime, the $n = 1$ tilt mode and the $n = 2$ mode are linearly stable, whereas the residual weak instabilities of the $n = 3$ and $n = 4$ modes saturate nonlinearly at small amplitude. The resulting FRC configuration remains stable with respect to all global MHD modes, provided that the FRC current is sustained. Future theoretical studies will be performed in order to optimize the beam and bulk plasma parameters with respect to stabilization of global MHD modes in oblate FRCs.

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FIGURE CAPTIONS

FIG.1. Normalized growth rates obtained from linearized hybrid simulations of the $n = 1$ tilt instability for three different elliptical equilibria with $E = 4$, 6.25, and 11.6. The dashed curve corresponds to the scaling $\gamma = \gamma_{mhd} \exp(-3E\rho_i/R_s)$.

Fig.2. Radial profiles of the ion toroidal flow velocity at the FRC midplane at $t = 20$, 40, and $80t_A$ obtained from 2D hybrid simulations with $S^* = 20$ and $E = 4$. The separatrix radius is $R_s/R_c \approx 0.6$.

Fig.3. (a) Normalized growth rates of the $n = 1 - 4$ modes obtained from linearized hybrid simulations including thermal ion kinetic effects (red), the effects a of conducting shell (green), and the combined effects of a conducting shell and NBI stabilization (blue) for an FRC with $E = 1.1$. (b) Plots of the time evolution of the $n = 0 - 4$ Fourier harmonics of the ion kinetic energy obtained from 3D nonlinear hybrid simulations including the effects of the energetic beam ions and the close-fitting conducting shell.

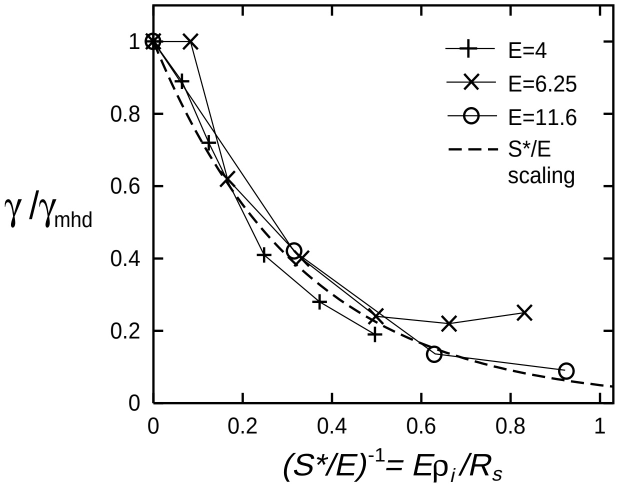

Figure 1.

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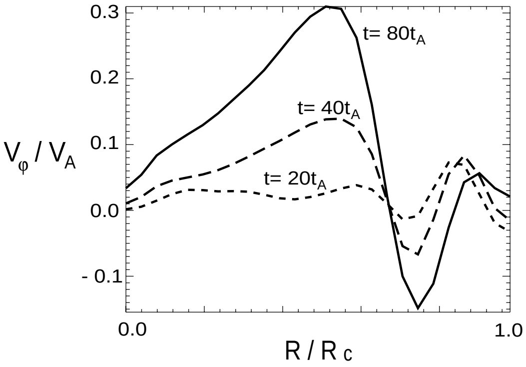

Figure 2.

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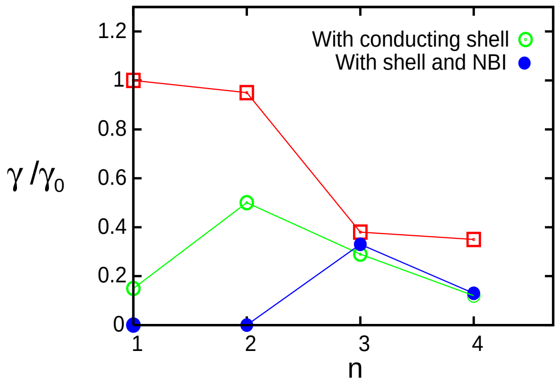
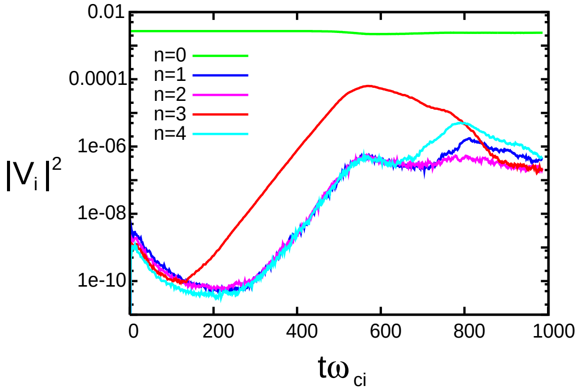


With conducting shell
With shell and NBI
γ/γ_0
0
0.2
0.4
0.6
0.8
1
1.2
1
2
3
4
n



n=0
n=1
n=2
n=3
n=4
$|V_i|^2$
0.01
0.0001
1e-06
1e-08
1e-10
0
200
400
600
800
1000
$t\omega_{ci}$

External Distribution

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