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Alpha Channeling in Mirror Machines

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Charged particles trapped in a magnetic mirror can be cooled by shining radio frequency waves into the mirror trap. This cooling effect relies upon waves in different axial locations resonating with ions having specific axial velocities. The ions are then forced to diffuse along highly constrained orbits, such that they can only exit the magnetic trap at low energy. This cooling effect may have application to magnetic fusion mirror machines, where the free energy of the fusion byproducts, the $\alpha$ particles, might be channeled into the waves that effect the cooling, thereby both extracting the $\alpha$-particle energy and making that energy available in a convenient form for more useful purposes.

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Because of their engineering simplicity, high-$\beta$, and steady-state operation, mirror machines and related open-trap machines such as gas dynamic traps, are an attractive concept for achieving controlled nuclear fusion. In these open-trap machines, the confinement occurs by means of magnetic mirroring, without the magnetic field lines closing upon themselves within the region of particle confinement. Unfortunately, these concepts have not achieved to date very spectacular laboratory results, and their reactor prospects are dimmed by the prospect of a low Q-factor, the ratio of fusion power produced to auxiliary power [1, 2]. Nonetheless, because of its engineering promise, over the years numerous improvements have been proposed to enhance the reactor prospects of mirror fusion, such as tandem designs, end-plugging, and electric potential barriers.

I suggest that important improvements in open traps might be had through the use of rf fields interacting with mirror-confined ions, such that the ions diffuse in the rf fields along highly constrained orbits, losing energy as they are forced out of the trap. This effect is similar to the alpha channeling effect practiced in tokamaks on the $\alpha$-particles, which are the charged byproducts of DT fusion [3]. In a tokamak reactor, by channeling the $\alpha$-particle energy to fuel ions, the tokamak could be run in a hot-ion mode, with the fusion ash removed quickly, and with the effective fusion reactivity more than doubled [4]. The $\alpha$-particle energy could also be diverted to other useful purposes, like providing the current necessary for steady state operation [5]. If these mechanisms can be implemented, they would lower significantly the cost of electricity by tokamak fusion [6].

The $\alpha$-channeling effect in tokamaks is implemented by injecting waves that diffuse resonant particles along highly constrained diffusion paths in the phase space connecting high-energy $\alpha$ particles in the tokamak interior with low-energy $\alpha$ particles at the periphery, so that a population inversion occurs along that path [3]. Because of the population inversion, the waves cause hot $\alpha$ particles to diffuse to the periphery and cool at the same time. These same waves, operating along similar diffusion paths, can simultaneously diffuse fuel ions from the periphery and heat them as they are brought to the tokamak center. This useful fueling and heating effect occurs because the population inversion for the fuel ions is opposite to that of the $\alpha$ particles. There are no MeV fuel ions in the center, but there are many relatively cold fuel ions near the periphery. Thus, in a tokamak, the same wave that taps $\alpha$ particle energy, while rejecting the $\alpha$ particles to the periphery, also fuels the plasma by sucking in fresh fuel ions and heating them.

Similar possibilities might be expected in mirror machines. Both mirrors and tokamaks are devices with a symmetry direction, so that the diffusion paths can be written similarly. For interaction by means of the resonance $\omega - k_\| v_\| = n\Omega$, where $\omega$ is the wave frequency, $\Omega$ is the cyclotron frequency, $k_\|$ is the wave parallel wavenumber, $n$ is the harmonic of the resonance, and $v_\|$ is the wave parallel velocity, the diffusion paths obey

$$dP_\|/dW = n\phi/\omega,$$
$$d\mu/dW = qn/m\omega,$$

where $\mu = mv^2/2B$ is the ion magnetic moment, $q$ is the ion charge, and $W = \mu B + mv_\|^2$ is the kinetic energy. The canonical angular momentum, $P_\phi = R(mv_\phi - qA_\phi)$ and the radius $R$ are defined with respect to the symmetry direction, namely toroidal in the tokamak and azimuthal in the mirror. While the $\alpha$-channeling effects in either tokamaks or mirrors will employ diffusion paths defined by Eqs. (1) and (2), the most successful $\alpha$-channeling effects in a mirror machine will exploit the fact that the periphery of the mirror machine is defined very differently. Since the tokamak is a closed field device, its periphery lies past the last closed magnetic surface, so particles exit the device by diffusing across magnetic field lines. In mirror geometry, the open geometry defines a very different periphery in the phase space that includes both configuration space and velocity space. Particles can also leave a mirror machine across field lines through radial diffusion, but they are more likely to leave through the open field lines at the ends of the mirror through velocity-space
diffusion. Thus, because of the complex periphery of the mirror machine, the arrangement of waves that accomplish the channeling effect will be very different.

The advantages of this effect in open-trap machines, while similar to those in a tokamak, retain key differences as well. Although the open geometry of the mirror makes fueling and ash removal much easier, the heating of ions and the quick removal of ash may be more critical in improving the $Q$ of the mirror reactor. In a tokamak, $\alpha$-particles take up valuable plasma pressure; in a mirror machine, the fusion ash takes up valuable electric potential. The quick removal of the energetic ash, which would otherwise be magnetically confined longer than the fuel ions, makes room for fuel ions. Also, maintaining the temperature disparity economically may be more important, since the mirror runs in an extreme hot-ion mode.

I now show how, exploiting the open field line geometry, an efficient $\alpha$-channeling effect can be produced in mirror geometry using multiple regions of rf power. For simplicity, first suppose that mirror-trapped ions or $\alpha$ particles are wave-heated at just one axial position $z = z_{rf}$, where $z$ is the axial direction along the mirror. I will assume for simplicity the “simple mirror”, with vanishing plasma potential, but these ideas will pertain more generally to mirror-type, open-ended configurations as well. The trapped particles can be described by their perpendicular and parallel energies, $W_{\perp 0} \equiv W_{\perp}(z = 0)$ and $W_{|| 0} = W_{||}(z = 0)$, as they cross the mirror midplane at $z = 0$. The particles affected by the rf waves then lie between the rays shown in Fig. 1. The lower ray represents the trapped-untrapped boundary for mirror ratio $R_M \equiv B_{max}/B_0$, where $B_0$ is the magnetic field minimum at the midplane and $B_{max}$ is magnetic field maximum. The upper ray represents the boundary for particles reaching the region of rf power at $z = z_{rf}$, where the magnetic field is $B_{rf}$, with $B_0 < B_{rf} < B_{max}$. The upper ray is determined by the mirror ratio $R_f \equiv B_{rf}/B_0$.

Particles with higher perpendicular energy than the upper ray are mirror-reflected before reaching the region of rf, whereas particles with a lower perpendicular energy than the lower ray are not mirror-confined at all.

Upon interaction with the rf field at $z = z_{rf}$, the perpendicular energy $W_{\perp}(z_{rf})$ of the particle at $z = z_{rf}$ changes so that $W_{\perp}(z_{rf}) \rightarrow W_{\perp}(z_{rf}) + \Delta W_{\perp}$. The parallel energy similarly changes by $W_{||}(z_{rf}) \rightarrow W_{||}(z_{rf}) + \Delta W_{||}$. Since the wave-particle interaction is a stochastic process, the energy increments $\Delta W_{\perp}$ and $\Delta W_{||}$ can be positive or negative, but they are related to each other through the diffusion path. For interaction by means of the resonance $\omega - k_{||}v_{||} = n\Omega$, the diffusion path can be written from Eq. (2) as $\Delta W_{\perp} = \Delta W_{||}(\omega - n\Omega) = \Delta W_{||}n\Omega/(k_{||}v_{||})$. For example, for Landau damping, we have $n = 0$, with $\Delta W_{\perp} = 0$, so that the diffusion occurs in the parallel direction only. For cyclotron interactions, we have $n \neq 0$; in the limit then of $k_{||} \rightarrow 0$, we have $\Delta W_{||} = 0$, so that only perpendicular diffusion occurs.

Assuming the adiabatic invariance of the magnetic moment outside the resonance, the energy kicks $\Delta W_{\perp}$ and $\Delta W_{||}$ at $z = z_{rf}$ result in the midplane change

$$W_{\perp 0} \rightarrow W_{\perp 0} + \Delta W_{\perp}/R_{rf},$$

$$W_{|| 0} \rightarrow W_{|| 0} + \Delta W_{||} + \Delta W_{\perp} \left(1 - R_{rf}^{-1}\right).$$

Note that all of the parallel energy kick in the rf region is recovered in the parallel energy at the midplane, but, since $R_{rf} > 1$, not all of the perpendicular energy kick in the rf region is recovered in the perpendicular energy at the midplane, with some of that energy recovered as parallel energy at the midplane position.

Now note two necessary conditions for the channeling effect: First, the diffusion paths must connect high-energy particles in the interior with low-energy particles in the periphery of a confinement device, where the periphery is defined as a point of marginal confinement and the interior is defined as a point where particles are well-confined. In a mirror trap, one periphery is the trapped-passing boundary, or the lower ray in Fig. 1. Second, the diffusion to high energy must be limited. I will show that both of these conditions can be satisfied in a mirror geometry through the case of perpendicular diffusion only, or $\Delta W_{||} = 0$, which is realizable through $k_{||} \rightarrow 0$. The more general case of finite $k_{||}$ offers other flexibilities, but this important and simple limit is shown here to offer sufficient flexibility to realize the $\alpha$ channeling effect with high efficiency.

To construct the channeling effect, first note that from Eqs. (3) and (4), for $\Delta W_{||} = 0$, the slope of the energy change in midplane coordinates, $\Delta W_{\perp 0}/\Delta W_{|| 0} = (R_{rf} - 1)^{-1}$, which is the same slope as the rf interaction boundary. This means that due to the rf power particles diffuse in energy parallel to that boundary. However, only particles resonant with the wave are affected. The resonance condition, $\omega - k_{||}v_{||} = n\Omega$, selects a parallel energy $W_{|| res}$ at $z = z_{rf}$, which is a function of the local wave and magnetic field parameters. The resonant
region in midplane coordinates then obeys

\[ W_{\perp 0} = (W_{\parallel 0} - W_{\parallel \text{res}})/(R_{\text{rf}} - 1). \]  

(5)

Thus, particles kicked by the rf wave at \( z = z_{\text{rf}} \) diffuse along the trajectory indicated by Eq. (5). Since the slope of the diffusion path is the same as the slope of the resonance condition, particles remain in resonance as they are diffused, and Eq. (5) represents that diffusion path.

Along the diffusion path, particles that gain energy remain mirror-trapped, but particles that lose energy eventually encounter the trapped-passing boundary and are lost. The parallel and perpendicular energies upon exit can then be calculated to be

\[ W_{\perp \text{exit}} = W_{\parallel \text{res}}/(R_M - R_{\text{rf}}) \]  

(6)

\[ W_{\parallel \text{exit}} = W_{\parallel \text{res}} (R_M - 1)/(R_M - R_{\text{rf}}). \]  

(7)

Thus, by picking \( W_{\parallel \text{res}} \) small, the energy lost at the boundary can be made small.

Since particles diffusing to high energies are not lost, whereas the lost particles are at small energy, eventually all particles will be lost cold, thereby achieving the channeling effect. However, the particles affected are only those in resonance, namely only those diffusing along the trajectory indicated by Eq. (5). At first glance, it appears that a range of resonant parallel energies a large range of \( W_{\parallel \text{res}} \) will be required, which would mean that the amount of energy lost at the trapped-passing boundary becomes large. However, interestingly, it is possible to access the full range of trapped particles by arranging for several regions of rf as shown in Fig. 2, but still employing a limited range of small \( W_{\parallel \text{res}} \).

![FIG. 2: Mirror field with rf regions at axial positions \( z = z_{\text{rf}1} \) and \( z = z_{\text{rf}2} \). The magnetic field maximum is at \( z = z_M \).](image)

To see this, consider multiple regions of rf power, with the rf power in each region having its own prescribed wave frequency and wavenumber. For ions resonant in region \( i \), say at \( z = z_{\text{rf}i} \), diffusion takes place along the trajectory indicated by Eq. (5) for wave parameters and mirror ratios corresponding to the \( i \)th region, so that

\[ W_{\perp 0} = (W_{\parallel 0} - W_{\parallel \text{res}i})/(R_{\text{rf}i} - 1). \]  

(8)

Particles resonant in one region need not be resonant in a second region. The criteria for resonance in more than one region is that the diffusion paths cross within the trapping region. Fig. 3 shows two diffusion paths (dashed lines) due to two regions of rf, one at \( z = z_{\text{rf}1} \) corresponding to the mirror ratio \( R_{\text{rf}1} \), and one at \( z = z_{\text{rf}2} \) corresponding to the mirror ratio \( R_{\text{rf}2} \). For the case here, I choose the same rf resonant velocity for both regions. Since each diffusion path is parallel to its own corresponding trapped-passing boundary, it is clear that these paths can then meet only at \( W_{\perp 0} = 0 \), or clearly outside the trapped-passing boundary. More generally, for multiple rf regions, it can be seen that from Fig. 3 that the diffusion paths do not cross within the trapped particle region so long as the resonant regions at larger \( z_{\text{rf}} \) are arranged with parallel resonant energies \( W_{\parallel \text{res}} \) not too much smaller than the resonant energies at smaller \( z_{\text{rf}} \). If the diffusion paths do not cross, then there is a strong constraint on the energy at exit; resonant particles exiting at the trapped-passing boundary must exit with the exit energies given by Eqs. (6) and (7).

Although particles resonant at \( z = z_{\text{rf}1} \) diffuse into the rf region \( z = z_{\text{rf}2} \) before becoming untrapped, when they are in the rf region at \( z = z_{\text{rf}2} \), their parallel velocity is not resonant with the rf waves at \( z = z_{\text{rf}2} \). Thus, each set of resonant particles maintains the diffusion path set by one rf region. The resonance conditions can be arranged at each axial location to correspond to the same relatively low parallel energy \( W_{\parallel \text{res}} \). Fitting the wave into the rf region will in practice set a lower limit on the resonant energy \( W_{\parallel \text{res}} \). For \( W_{\parallel \text{res}} \) small, essentially all the energy is extractable as it can be seen from Fig. 3, or equivalently from Eqs. (6) and (7).

![FIG. 3: Diffusion paths in midplane energy coordinates for particles resonant in the rf regions shown in Fig. 2. The solid rays are the particle coordinates for particles mirroring at \( z = z_{\text{rf}1} \) (upper ray) or \( z = z_{\text{rf}2} \) (lower ray). The dashed rays are the diffusion paths for particles, resonant at instantaneous parallel energy \( W_{\parallel \text{res}} \), at \( z = z_{\text{rf}1} \) (upper ray) or \( z = z_{\text{rf}2} \) (lower ray). The dashed rays are parallel to the associated solid rays and offset by the resonant parallel energy.](image)
If the diffusion paths do cross within the trapped region of velocity space, then the strong condition on the exit energy may not obtain. However, the probability distribution of particles interacting with multiple waves along multiple paths might still highly favor exit at low energy. The crossing of diffusion paths using multiple waves was demonstrated in tokamak geometry as being potentially useful in accessing a larger part of phase space for channeling or in using to advantage waves which separately would not be as efficient [7, 8].

In the above, only velocity space diffusion is exploited, as described by Eq. (2). However, the channeling effect can be enhanced by exploiting also coupled diffusion in velocity and real space, as described by Eq. (1). For \( n_\phi \neq 0 \), diffusion in energy and radius are linked by \( \Delta R = \Delta W/(eB\omega/k_\phi) \), like channeling in closed geometry [3]. By choosing the poloidal phase velocity \( \omega/k_\phi \) such that an \( \alpha \) particle, loses its birth energy in a radial step of approximately the mirror radius, then the \( \alpha \) particle can exit cold at the radial periphery. Governed by both Eqs. (1 and 2) with \( n_\phi \neq 0 \), particles must then exit at low energy exit either at the trapped-passing boundary or at the radial periphery, whether the rf is situated near the midplane, or at off-midplane locations. In either manner of exit, the diffusion to high energy is bounded since it is tied to diffusion in real space to the mirror axis or plasma center, precisely in analogy with \( \alpha \)-channeling in tokamaks [3]. The bound at high energy means that, for a given rf power, energy is channeled faster into the resonant particles. Because it can be excited in inhomogeneous magnetic fields with azimuthal localization and large poloidal mode numbers, like in tokamaks [9, 10], an excellent wave candidate for \( \alpha \)-channeling would be the mode-converted ion Bernstein wave.

The cooling of the \( \alpha \)-particles is accompanied by the advantageous heating of the trapped fuel ions along the reverse diffusion path, like for the \( \alpha \)-channeling effect in a tokamak. In the mirror geometry implementation envisioned here, there is the opportunity also to choose the smallest \( W_{||_{res}} \) for the lowest \( z = z_{rfi} \) so that cold fuel ions will first encounter the diffusion paths, defined by Eq. (8), that carry fuel ions advantageously to the most highly trapped region of velocity space.

These effects are also particularly useful for mirror concepts in which the ions are much hotter than the electrons [11], or at least one species of ions is [12]. At ignition, the \( \alpha \)-particle heating \( P_\alpha = n_\alpha \epsilon_\alpha/\tau_\alpha \) must exceed the ion heat loss \( P_i = nT_i/\tau_i \), where \( \epsilon_\alpha \) is the \( \alpha \) birth energy, \( \tau_\alpha \) is the slowing down times of energetic \( \alpha \)-particles, and \( \tau_i \) is the ion energy containment time. Rewriting this as \( n_\alpha/\tau_i > (T_i/\epsilon_\alpha)/(\tau_\alpha/\tau_i) \), and assuming \( \tau_\alpha > \tau_i \), it can be seen that for ions of several hundred keV the \( \alpha \)-particle density is very significant [2], although for lower ion temperatures it will be smaller [13]. For \( \alpha \)-particle density 30\% of the ion density [2], the fuel ions are much diluted. Since the reactivity scales as the ion density squared, at constant confined positive charge, for this case the prompt loss of \( \alpha \)-particles alone increases the fusion reactivity by a factor of 2.8. The second important savings will come because if the \( \alpha \)-particle energy is captured in waves that are absorbed by ions, then not only is the \( \alpha \)-particle energy optimally converted, but higher disparities in ion and electron temperatures, or in temperatures of different ions, may be tolerated.

In summary, \( \alpha \)-particles born in mirror traps might be made to slow down collisionlessly on rf fields rather than on the electrons. The quick expulsion of the spent fuel reduces the mirror electric potential leading to better confinement of the fuel ions. The same rf field may be expected to heat the fuel ions, drawing their turning points closer to the midplane and their gyrocenters closer to the mirror axis. The rf wave predicted to accomplish these effects must be localized axially and azimuthally and must heat ions perpendicularly. A suitable wave might be a mode-converted ion Bernstein wave injected into the mirror across magnetic field lines, much like that suggested for tokamaks. Although for simplicity only the simplest mirror geometry is considered, the basic channeling mechanisms predicted here should serve also to enhance the reactor prospects of more complicated open-field traps. For mirror reactors utilizing advanced fuel reactions, and hence at higher ion and electron temperatures, the effect of channeling to the fuel ions the fusion byproduct energy may assume even greater importance.

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