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The transition to collisionless ion-temperature-gradient-driven plasma turbulence: A dynamical systems approach

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The transition to collisionless ion-temperature-gradient-driven plasma turbulence is considered by applying dynamical systems theory to a model with ten degrees of freedom. Study of a four-dimensional center manifold predicts a "Dimits shift" of the threshold for turbulence due to the excitation of zonal flows and establishes the exact value of that shift in terms of physical parameters. For insight into fundamental physical mechanisms, the method provides a viable alternative to large simulations.

Understanding the regulatory mechanisms of turbulent transport is an important problem in the magnetic confinement of plasmas. It has been recognized that nonlinearly generated $\boldsymbol{E} \times \boldsymbol{B}$ poloidal (zonal) flows (ZF's) play a central role in that process [1]. It is frequently said that ZF's shear apart eddies associated with the underlying turbulence, thus reducing the radial transport. They are also important in geophysical contexts [2].

An extreme example demonstrating the importance of ZF's is the so-called *Dimits shift*, which is a nonlinear upshift of the critical temperature gradient for the onset of ion-temperature-gradient-driven (ITG) turbulence. Let that gradient be measured by a dimensionless parameter ϵ, and let the threshold for linear instability be ϵ_c. According to large collisonless gyrokinetic [3] and gyrofluid [4] simulations of ITG systems near marginal stability, there is a regime $\epsilon_c < \epsilon < \epsilon_*$ for some ϵ_* in which only ZF but no drift wave (DW) activity (and hence no radial transport) is observed. The Dimits shift is defined to be $\Delta\epsilon \doteq \epsilon_* - \epsilon_c$ ($\doteq$ is used for definitions); ϵ_* is identified with the onset of (weak) DW turbulence. Rogers *et al.* [5] identified ϵ_* with the onset of tertiary modes that grow to nonlinear amplitudes and damp the ZF's. They discussed three stages as ϵ is increased: (i) primary instability of the DW's D; (ii) secondary instability of zonal modes Z (driven by D), which then (totally) suppress D; (iii) tertiary instability: Z is destabilized. Such nomenclature might suggest that one search for a sequence of three bifurcations occurring at $\epsilon_1 \equiv \epsilon_c$, ϵ_2, and $\epsilon_3 \equiv \epsilon_*$. However, there is a fundamental difficulty with any steady-state scenario that relies on the nonlinear interaction $D + D \to Z$: It is impossible to close a steady-state loop $D \to Z \to D$ with $D = 0$ but $Z \neq 0$. (Such a loop with nonzero values of both D and Z is considered in the statistical theory of fully developed DW–ZF turbulence [6].) Dastgeer *et al.* [4] seem to suggest that certain resonances enhance the ZF response, but even then Z cannot be driven if $D \equiv 0$. No distinct ϵ_2 is observed in the simulations. Instead, as Rogers *et al.* noted, ZF's are excited by a burst of DW's [through a Kelvin–Helmholtz (KH) instability of radial streamers], which then die away leaving only the ZF's in steady state.

Although large simulations have proven to be invaluable for the detailed modeling of complex behavior in modern tokamaks, they are cumbersome, expensive, and frequently ill suited for the identification and detailed understanding of basic conceptual issues. Here we consider the opposite extreme and perform a dynamical systems analysis [7–9] of the "simplest" model of an electrostatic, collisionless (undamped ZF's), curvature-driven ITG system near marginal stability. Although the physics does involve ZF's in a fundamental way, we will show that there are just two bifurcation points of interest. Also, for the first time in any model, we are able to predict the Dimits shift exactly as a function of physical parameters.

A bifurcation is a change "in the qualitative structure of the solutions" [7] of a system of nonlinear equations as a parameter such as ϵ passes through certain values. For many physicists, intuition about bifurcation phenomenology has been strongly influenced by the simplest normal forms exhibiting bifurcations [7]. For systems with linear waves, the two-dimensional (2D) Hopf bifurcation is especially relevant. If that pertained to the collisionless ITG problem and if it were supercritical [10], then slightly above linear threshold the DW's would saturate at a small amplitude $\propto (\epsilon - \epsilon_c)^{1/2}$; for an example, see the calculations of collisionally damped ZF's in Refs. 11 and 12. However, such behavior is not observed. If the bifurcation were subcritical [10], then the DW's would jump to a finite level as ϵ is increased beyond ϵ_c; that behavior is not observed either.

In fact, the standard Hopf bifurcation does not apply to the strictly collisionless problem. A systematic way of proceeding is to exploit the Center Manifold Theorem [7, 8], which states that at linear threshold the system dynamics are (in the absence of positive linear eigenvalues) attracted to a smooth n_0-dimensional invariant subspace [the *center manifold* (CM)] as $t \to \infty$. At the point of bifurcation, the CM is tangent to the linear eigenspace corresponding to the n_0 modes whose eigenvalues λ ($\partial_t \to e^{\lambda t}$) have zero real part. The theorem is easily extended (by suspension) to systems parameterized by a bifurcation parameter ϵ. Since an ITG model [13] must involve at least two coupled fields (usually potential vorticity ω and pressure P) in order that the system self-consistently produces a linear growth rate (the Hasegawa–Wakatani paradigm [10] is similar in this regard), the dimensionality of the CM is the sum of at least 2 (for the complex DW amplitude) plus 2 (for the two real undamped zonal fields); thus the CM is at least 4D. This feature has not been recognized previously; it is responsible for the unusual behavior that underlies the Dimits shift. We will demonstrate that explicitly for a simple model with ten real degrees of freedom, both by perturbative construction of the CM and qualitative analysis of the dynamics on the CM, and by exact calculation of the relevant fixed point of the full nonlinearity.

We consider a simplified gyrofluid ITG system [14] for $\boldsymbol{u} = (\omega, P)^{\mathrm{T}}$, driven by magnetic curvature [the unit vectors $\widehat{\boldsymbol{z}}$, $\widehat{\boldsymbol{x}}$, and $\widehat{\boldsymbol{y}}$ are associated with the magnetic field, radial, and (essentially) poloidal directions, respectively]. In vector form, the system (considered as 2D in the plane perpendicular to $\widehat{\boldsymbol{z}}$) is $\partial_t \boldsymbol{u}(\boldsymbol{x}, t) = \widehat{\mathsf{M}} \cdot \boldsymbol{u} + \widehat{\boldsymbol{N}}(\boldsymbol{u}, \boldsymbol{u})$, with the hat denoting a differential operator and the nonlinear term (bilinear in the field vector) describing simple $\boldsymbol{E}\times\boldsymbol{B}$ advection: $\widehat{\boldsymbol{N}}(\boldsymbol{u}, \boldsymbol{u}) = -\widehat{\boldsymbol{z}} \times \boldsymbol{\nabla}\varphi \cdot \boldsymbol{\nabla}\boldsymbol{u}$. The electrostatic potential φ follows from $\varphi = \widehat{\mathcal{D}}^{-1}\omega$, where $\widehat{\mathcal{D}} \doteq \widehat{\alpha} + \widehat{\nabla}^2$, $\widehat{\alpha}$ being zero for convective cells ($k_\parallel = 0$) and the identity operator otherwise [6]. The linear matrix is assumed to have the form

$$\widehat{\mathsf{M}} = \begin{pmatrix} -i(\widehat{\Omega} - i\widehat{\eta}) & -i\widehat{b} \\ i\widehat{c} & -\widehat{d} \end{pmatrix}. \tag{1}$$

Here $\widehat{\Omega} \doteq 2(\widehat{\mathcal{D}}^{-1} + \tau)\widehat{\partial}_y$, with τ being the ratio of ion and electron temperatures, is associated with the linear frequency of DW's; $\widehat{\eta} \doteq \mu\widehat{\nabla}^2$ describes weak collisional damping on the DW's (only); $\widehat{b} \doteq 2\widehat{\partial}_y$ provides the linear coupling between ω and P; $\widehat{c} \doteq \tau\widehat{\mathcal{D}}^{-1}(\epsilon - \widehat{\epsilon}_c)\widehat{\partial}_y$ scales with the distance from marginality of the considered Fourier mode, where $\epsilon \doteq L_n/L_T$ is the ratio of background density and temperature gradients and $\widehat{\epsilon}_c$ is that ratio calculated at marginality of the considered mode; and $\widehat{d} \doteq \widehat{\nu} + \widehat{\eta}$, where $\widehat{\nu} = -i\nu\widehat{\partial}_y$ represents the Landau damping effect in the gyrofluid closure [14].

We consider energetically self-consistent Galerkin truncations [9] in which the fields are represented as $\sum_{\boldsymbol{k}} \boldsymbol{u}_{\boldsymbol{k}}(t)\sin(k_x x)e^{ik_y y}$. In choosing a standing wave in x, we subscribe to an argument from Ref. 11, which asserts this as a crude representation of the localizing effect of magnetic shear. The lowest truncation retains $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, and $\boldsymbol{u}_3$, where $1 \equiv (k_x, k_y)$, $2 \equiv (2k_x, 0)$, and $3 \equiv (3k_x, k_y)$. $\boldsymbol{u}_1$ represents both the first (as a function of increasing ϵ) bifurcating linearly unstable D as well as a damped eigenmode; $\boldsymbol{u}_3$ is a DW sideband S; and $\boldsymbol{u}_2$ represents zonal variation Z, present in the system as a result of the nonlinear interaction between D and S. This model does not retain streamers ($k_x = 0$), so does not capture the KH mechanism of Ref. 5; however, it does permit ZF's to be generated from a DW transient.

In the Fourier representation, the $\widehat{\mathsf{M}}$ operator involves various $\boldsymbol{k}$-dependent coefficients Ω_i, η_i, b_i, c_i, and d_i with $i = 1, 2, 3$. Signs and phases have been chosen so that all coefficients are real and positive, except that $c_1 < 0$ below marginality and $c_3 < 0$ when just mode $\boldsymbol{u}_1$ or no mode is unstable. By definition of the collisionless problem (no linear zonal damping), η_2 is taken to vanish. η_1 and η_3 are unfolding parameters in the sense of Ref. 7. When $\eta_1 = 0$, the DW threshold is $\epsilon_c = 0$; at that threshold, the DW eigenvalues are $\lambda_+ = -i\Omega_1$ and $\lambda_- = -\bar{\nu}$, with eigenvectors $\boldsymbol{e}_+ = (1, 0)^{\mathrm{T}}$ and $\boldsymbol{e}_- = (0, 1)^{\mathrm{T}}$.

The equations are

$$\dot{\omega}_1 = -i(\Omega_1 - i\eta_1)\omega_1 - ib_1P_1 + \frac{1}{2i}\left[\left(\frac{1}{\mathcal{D}_1} - \frac{1}{\mathcal{D}_2}\right)\omega_1\omega_2 - \left(\frac{1}{\mathcal{D}_3} - \frac{1}{\mathcal{D}_2}\right)\omega_3\omega_2\right], \tag{2a}$$

$$\dot{P}_1 = ic_1\omega_1 - d_1P_1 + \frac{1}{2i}\Big[\Big(\overbrace{\frac{\omega_1}{\mathcal{D}_1}P_2}^{(1)} - \overbrace{\frac{\omega_2}{\mathcal{D}_2}P_1}^{(0)}\Big) - \Big(\overbrace{\frac{\omega_3}{\mathcal{D}_3}P_2}^{(2)} - \overbrace{\frac{\omega_2}{\mathcal{D}_2}P_3}^{(3)}\Big)\Big], \tag{2b}$$

$$\dot{\omega}_3 = -i(\Omega_3 - i\eta_3)\omega_3 - ib_3P_3 - \frac{1}{2i}\left(\frac{1}{\mathcal{D}_1} - \frac{1}{\mathcal{D}_2}\right)\omega_1\omega_2, \tag{2c}$$

$$\dot{P}_3 = ic_3\omega_3 - d_3P_3 - \frac{1}{2i}\Big(\overbrace{\frac{\omega_1}{\mathcal{D}_1}P_2}^{(4)} - \overbrace{\frac{\omega_2}{\mathcal{D}_2}P_1}^{(3)}\Big), \tag{2d}$$

$$\dot{\omega}_2 = \left(\frac{1}{\mathcal{D}_1} - \frac{1}{\mathcal{D}_3}\right)\mathrm{Im}(\omega_1\omega_3^*), \tag{2e}$$

$$\dot{P}_2 = \mathrm{Im}\Big(\overbrace{\frac{\omega_1}{\mathcal{D}_1}P_3^*}^{(4)} + \overbrace{\frac{\omega_3}{\mathcal{D}_3}P_1^*}^{(2)}\Big) - \mathrm{Im}\Big(\overbrace{\frac{\omega_1}{\mathcal{D}_1}P_1^*}^{(1)}\Big). \tag{2f}$$

Here $\omega_2 \equiv z_\omega$ and $P_2 \equiv z_P$ are real. A consistency check can be obtained by noting that the quantities $\mathcal{W} \doteq |\omega_1|^2 + |\omega_3|^2 + \omega_2^2$ and $\mathcal{P} \doteq |P_1|^2 + |P_3|^2 + P_2^2$ are conserved by the nonlinearities. The cancellations of terms under $\dot{\mathcal{P}}$ are shown by the numbering; term 0 vanishes separately.

Numerically, it is observed that for $\epsilon_c < \epsilon < \epsilon_*$ and almost all initial conditions (IC's), a burst of fluctuations occurs that eventually dies away leaving only ZF's. For many IC's, the final state is unique; that is, many trajectories are attracted to a stable, nontrivial fixed point (at $\boldsymbol{z} = \boldsymbol{z}_0$, with all other fields vanishing). However, other IC's lead to final states dependent on the IC's. For $\epsilon > \epsilon_*$, the model does not saturate in general. That is of little concern for a qualitative discussion of the Dimits shift; higher-order truncations [15] do saturate. (The value of ϵ_* depends on the order of truncation.)

All of this behavior can be explained, and ϵ_* can be predicted, by a bifurcation analysis that involves the construction of the CM. As we noted, it is critical to realize that when the zonal components are undamped [$\eta_2 = 0$; see Eqs. (2e) and (2f)], the CM is 4D. To construct the CM, we write $\boldsymbol{u}_1 = D\boldsymbol{q} + \boldsymbol{y}_1$, $\boldsymbol{z}_2 = \boldsymbol{z}$, and $\boldsymbol{u}_3 = \boldsymbol{y}_3$, where $\{D, \boldsymbol{z}\}$ provides four real coordinates on the center eigenspace and the $\boldsymbol{y}$'s describe the nonlinear curvature of the CM with respect to that space. Symmetry considerations [9] dictate that $\boldsymbol{y}_i = \mathsf{W}_i \cdot \boldsymbol{z} D + \cdots$, where D and $\boldsymbol{z}$ are treated as small and the W_i are constant matrices to be determined. That may be accomplished [7, 8] by equating the time derivative of the power-series expansion of $\boldsymbol{y}_i$ with the evolution equation that follows from the restriction of Eqs. (2) to the CM. In detail, we follow the projection method advocated by Kuznetsov [8], which does not require a preliminary linear diagonalization. To lowest order, we are led to the 4D system

$$\dot{D} = \Gamma(\boldsymbol{z})D, \quad \dot{\boldsymbol{z}} = (-\boldsymbol{a}\epsilon + \mathsf{A} \cdot \boldsymbol{z})I, \tag{3a,b}$$

where $I \doteq |D|^2$. This can immediately be reduced to a 3D system for I and $\boldsymbol{z}$ by writing $D = \rho e^{i\theta}$ and noting that the θ dependence entirely decouples; Eq. (3a) is then replaced by $\dot{I} = 2\Gamma(\boldsymbol{z})I$. The two-vector $\boldsymbol{a}$ and the 2×2 matrix A are known, and $\Gamma(\boldsymbol{z})$ is known through $O(z^2)$ [$\Gamma(\mathbf{0}) > 0$ is the linear DW growth rate]. All elements of A are positive; both of its eigenvalues are negative. The supporting algebra and explicit formulas for these quantities and others in the subsequent analysis will be displayed in a longer paper to be published elsewhere.

For this reduced dynamics, not only is the origin $I = 0$, $\boldsymbol{z} = \mathbf{0}$ a fixed point (linearly unstable for $\epsilon > \epsilon_c$), the entire $I = 0$ plane is invariant. This unusual behavior is the first indication that for the collisionless problem the origin does not have the same preferred status as in other, more conventional problems [11, 12]. Indeed, the system also admits a nontrivial fixed point F at $I_0 = 0$ and $\boldsymbol{z}_0 = \epsilon \mathsf{A}^{-1} \cdot \boldsymbol{a}$. We will show that the stability of F determines the Dimits shift.

One may perform a phase-plane analysis of Eqs. (3b) by noting that I cancels out under $\dot{z}_P/\dot{z}_\omega = dz_P/dz_\omega \equiv v(\boldsymbol{z})/u(\boldsymbol{z})$. A sketch of the phase trajectories is shown in Fig. 1. All qualitative properties of this figure can be determined analytically. F is attracting in the $\boldsymbol{z}$ plane (for all ϵ); it passes through the origin as ϵ passes through 0. In the submarginal region $\epsilon < \epsilon_c$, $\Gamma(\boldsymbol{z}) < 0$ for all $\boldsymbol{z}$'s. Then all trajectories starting in the vicinity of the $I = 0$ plane are attracted to that plane and end up close to the initial starting point (the final $\boldsymbol{z}$ may be either F or may depend on IC's).

In the supermarginal region $\epsilon_c < \epsilon < \epsilon_*$, $\Gamma(\boldsymbol{z}) > 0$ in the vicinity of $\boldsymbol{z} = \mathbf{0}$ but is negative in the vicinity of F. Then most trajectories starting close to the origin initially move away from it; they end up either at F [for sufficiently large $I(t = 0)$] or sometimes on the $I = 0$ plane at positions depending on initial conditions. Such dynamics are consistent with the observed behavior above marginality: an initial burst of DW's generates ZF's, which then annihilate the DW's leaving only a steady ZF component as $t \to \infty$. This generation (secondary instability)/annihilation process is *transient*, so does not involve a distinct bifurcation point ϵ_2.

As ϵ is increased further through some ϵ_*, $\Gamma(\boldsymbol{z}_0)$ becomes positive, many IC's are repelled from the $I = 0$ plane, and the system cannot saturate. (Simulations verify that higher truncations do saturate with nonzero levels of DW activity and characteristic chaotic behavior.)

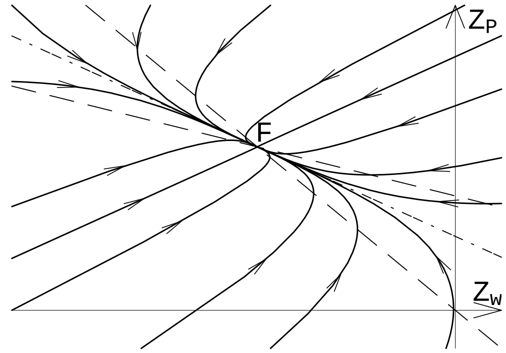

FIG. 1: Representative phase trajectories in the $\boldsymbol{z}$ plane, showing the nontrivial fixed point F (stable in the $\boldsymbol{z}$ plane) the overall stability of which determines the Dimits shift. Upper dashed line: $v(\boldsymbol{z}) = 0$ (slope $dz_P/dz_\omega = 0$); lower dashed line: $u(\boldsymbol{z}) = 0$ (slope $= \infty$); dash-dotted lines: eigenvector directions (one such line is obscured by the trajectory at approximately 45°).

Perturbative CM calculations provide only approximations to F and ϵ_*, and they cannot address the global structure of the phase space. Fortunately, the present model is simple enough that certain quantities can be calculated exactly. Rigorous equations for $\boldsymbol{z}_0$ are motivated by the observation that dynamics ought to relax rapidly to the CM. Since ω_1 has a component in the CM, we define the normalized variables $P_1' \doteq P_1/\omega_1$, ω_3', and P_3'; $\boldsymbol{z}$ is not normalized. Although for $\epsilon < \epsilon_*$ all original variables (except for $\boldsymbol{z}$) are dynamically driven to zero, the normalized variables remain nonzero as $t \to \infty$. This expedites tracking the fixed point $\boldsymbol{z}_0(\epsilon)$. Upon deriving evolution equations for the normalized variables from Eqs. (2), passing to an amplitude–phase representation, and requiring that the primed amplitudes and phases be steady, we are led after tedious algebra to tractable equations for the position of F. Further nontrivial algebra shows that to lowest order in ϵ the prediction agrees with that found from the perturbative CM construction. For any ϵ, numerical solution of the fixed-point equations demonstrates agreement with the numerically observed $\boldsymbol{z}_0$ through six decimal places.

Although this nonperturbative calculation captures all (possibly global) fixed points of the original system (the perturbative CM calculation is local), we have found only the $\boldsymbol{z}_0$ described previously. We have no categorical proof that no other fixed points exist, although no other stable ones emerge from an admittedly very incomplete numerical search of the phase space. We believe that if they do exist they are all saddles, which would not modify the qualitative asymptotics we have described.

With nonperturbative results in hand, we can formulate an exact equation for the Dimits shift. We write

$\omega_1 = \rho_1 e^{i\theta_1}$, divide Eq. (2a) by ω_1, and take the real part, obtaining (at $\boldsymbol{z}_0$) $\dot{\rho}_1/\rho_1 = -\eta_1 + b_1 Y_1'$, where $Y_1' \doteq \operatorname{Im} P_1'$. [Nonlinear terms do not contribute due to the steady-state condition $\operatorname{Im}\omega_3' = 0$, which follows from Eq. (2e).] F is therefore destabilized in the I direction when $Y_1'(\epsilon_*) = \eta_1/b_1$. An exact formula for $Y_1'(\epsilon)$ is available, and numerical work demonstrates agreement with our simulation value of ϵ_*. The destabilization process is not a KH instability but rather an ITG instability modified by stabilizing ZF shear.

An important observation emerges by considering the limit of small DW collisional dissipation $\eta_{1,3} \to 0$, for which it can be shown that $\epsilon_* \propto \eta_1/(\eta_3 - \eta_1)$. This ratio remains nonzero in the limit. Thus a nonzero Dimits shift arises even in the absence of collisional dissipation, which substantially enhances the relevance of our model to the large-scale collisionless simulations.

Now consider the addition of very weak zonal damping: $\dot{\boldsymbol{z}} = -\beta \boldsymbol{z} + \cdots$. ($\beta$ can also be considered to be an unfolding parameter.) This introduces a new, very long time scale and slightly perturbs the position of F. For $\epsilon < \epsilon_*$, arbitrary initial conditions typically move rapidly to the vicinity of the original fixed point, then slowly relax to the final steady state. (That state involves a small, nonturbulent component of D.) This disparity of time scales underlies the bursting behavior observed in Ref. 16 for weakly collisional runs. That does not occur in the lowest-order truncation studied here, but does occur in higher-order ones [15], whose additional degrees of freedom allow F to be destabilized in other directions and, thus, the trajectories to be ejected from its vicinity after the slow relaxation. Preliminary long-time (many-burst) integrations of such truncations show relaxation to a quasiregular state; limited computational resources precluded the authors of Ref. 16 from integrating more than a few bursts. Further analysis of the regime immediately above ϵ_* is very desirable and is in progress.

For sufficiently large β, the zonal modes are strongly stable and should no longer be used as coordinates on the CM, which is now 2D. For this case, a standard Hopf bifurcation occurs; the (straightforward) details will be presented elsewhere, as will a discussion of the modulational instability described by the associated Ginzburg–Landau equation. The radical differences in behavior between the undamped and strongly damped limits arises because of the interchange of the limits $t \to \infty$ and $\beta \to 0$ [17]. The signature of that interchange is the differing dimensionality of the CM's for the two cases. The Dimits shift occurs when the limit $\beta \to 0$ is taken first.

In summary, we have considered a very simple yet instructive model for the transition to collisionless ion-temperature-gradient-driven plasma turbulence. The excitation of zonal flows, important in a variety of physics contexts, plays a crucial role in the dynamics of that transition. That has been known previously, but only in rather qualitative terms. Here, by using tools from dynamical systems theory, we have shown how the nonlinear upshift of the critical temperature gradient for the onset of turbulence (known as the Dimits shift) is related to a certain fixed point of the nonlinear system (which arises as a particular expression of the balance between forcing and dissipation) and how that shift can be calculated in terms of the physical parameters of the model. Further work remains: higher-order truncations exhibit characteristic signatures of chaos, and it would be interesting to elucidate the details. All in all, dynamical systems analysis of model nonlinear systems possessing relatively small numbers of degrees of freedom is a viable and very instructive alternative to large-scale, brute-force simulations. Both have their places in the quest to understand the "anomalous" transport properties of magnetically confined plasmas and related nonlinear systems.

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