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by

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Abstract

Recently reported numerical results for axisymmetric devices with low aspect ratio A found radial transport enhanced over the expected neoclassical value by a factor of 2 to 3. In this paper, we provide an explanation for this enhancement. Transport theory in toroidal devices usually assumes large A, and that the ratio B_p/B_t of the poloidal to the toroidal magnetic field is small. These assumptions result in transport which in the low collision limit is dominated by banana orbits, giving the largest collisionless excursion of a particle from an initial flux surface. However in a small aspect ratio device one may have $B_p/B_t \sim 1$, and the gyroradius may be larger than the banana excursion. Here, we develop an approximate analytic transport theory valid for devices with arbitrary A. For low A, we find that the enhanced transport, referred to as omniclassical, is a combination of neoclassical and properly generalized classical effects, which become dominant in the low-A, $B_p/B_t \sim 1$ regime. Good agreement of the analytic theory with numerical simulations is obtained.

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A recent numerical study¹ of collisional transport in tokamaks with low aspect ratio ("spherical tori") found that the transport rate predicted by a Lorentz (*i.e.*, full-orbit) code GYROXY² is substantially larger than the rate computed using guiding-center (GC) codes such as ORBIT.³ The drift-kinetic equations used by GC codes are also the basis of standard neoclassical theory,^{4,5} so the disparity between the GC and Lorentz results indicates that something is missing in neoclassical theory, related to the effects of finite gyroradius for devices with low aspect ratio $A \equiv R/a$ (R and a are the device major and minor radii), and larger ratios B_p/B_t of the poloidal to toroidal magnetic field than those found in larger-A tokamaks. The total transport including these new effects was termed "omniclassical transport."

In this work, we provide an explanation for the omniclassical enhancement over the neoclassical rates from the ORBIT code. The explanation is a generalization of the classical transport one expects in a 1-dimensional (1D) or large-A(weakly 2D) system, where the diffusion coefficient $D_{cl} \simeq \nu \rho_g^2$ is small compared with the neoclassical coefficient $D_{nc} \simeq \nu (q\rho_g)^2 / \epsilon^{3/2}$ (with ν the collision frequency, ρ_g the particle gyroradius, and $\epsilon \equiv A^{-1}$). While some form of classical transport is one natural candidate to explain the numerical results in Ref. 1, how this mechanism generalizes to the strongly 2D case, and whether this in fact accounts for the observed disparity, need to be determined. Here, we present an approximate analytic calculation of the radial transport, from which emerges the appropriately generalized expression for the classical transport in this strongly 2D context, and we provide the demonstration that this expression accounts for the numerical findings.

In the GC code, a collision operator $C = C_{\parallel}$ is used which scatters only in pitch $\lambda \equiv v_{\parallel}/v_{\perp} \equiv v \cos \theta_v$, where v is the particle speed, and v_{\parallel} and v_{\perp} are its components parallel and perpendicular to the magnetic field **B**. This permits the particle "banana center" (bounce-averaged flux surface label) \bar{x} to wander diffusively. In GYROXY, as well as in the analytic calculation, C scatters not only in λ , but also in gyrophase ϕ_v , $C = C_{\parallel} + C_{\perp}$, providing a second statistically independent process by which \bar{x} can wander, enhancing the overall transport. As in the 1D case, the analytic theory finds that the classical transport has two contributions, one from scattering in λ , and a dominant contribution from scattering in ϕ_v . When the collision operator in GYROXY is constrained to scatter only in λ , we find that the transport rate drops toward that from ORBIT by an amount consistent with the theory.

The method used to compute D with ORBIT and GYROXY is as described in Ref. 1. A monoenergetic distribution of particles is loaded on a surface ψ_0 (ψ is the poloidal flux/ 2π) uniformly in poloidal angle θ and pitch variable λ . Integrating the particle orbits, D is computed from the Fokker-Planck expression $D = (1/2)d/dt \langle (\delta \psi)^2 \rangle (t)$. In Fig. 1 we plot $\langle (\delta \psi)^2 \rangle$ (arbitrary units) versus time from GYROXY and ORBIT, for a high beta equilibrium in the National Spherical Torus Experiment⁶ (NSTX) spherical torus with a 100 eV monoenergetic distribution launched on a surface ψ with maximum major radius $X(\psi) = 140$ cm. The collision frequency used is $10^{-4}\omega_0$, with ω_0 the cyclotron frequency, which is well within the banana regime. The top curve (a) is from GYROXY with full collision operator $C = C_{\parallel} + C_{\perp}$, the bottom curve (c) is from ORBIT, which has C_{\parallel} only, and the middle curve (b) is from GYROXY with C_{\parallel} only. One notes that the slope of this curve has dropped most of the way from that of full-C GYROXY curve to the ORBIT curve, as indicated above. The rapid displacement from the flux surface occuring initially reflects the mean square banana width in the case of ORBIT, and the much larger banana plus gyro width for the upper two curves. In Fig. 2 we plot D/D_{nc} versus $X(\psi)$, where D_{nc} is the GC result from ORBIT. The top curve (a) gives results using the full C, and the bottom curve (b) with Cconstrained to C_{\parallel} only. Doing so removes about 3/4 of the disparity between the GYROXY and ORBIT results.

To gain an analytic understanding of these results, we consider the diffusion of particles in the space of the three constants of motion J, and in particular,

diffusion in the banana-center flux surface \bar{x} . (A typical choice for **J**, employed in the "action-angle formalism",⁷ are the magnetic moment J_g , the bounce action J_b , and the angular momentum p_{ζ} . For our limited purposes here, we shall only need to make use of a small part of the machinery of this formalism.) One may define \bar{x} using the conservation of $p_{\zeta} \equiv \mathbf{e}_{\zeta} \cdot \mathbf{p} \equiv \frac{e}{c}A_{\zeta} + Mv_{\zeta}$ for collisionless orbits. Here, $\mathbf{e}_{\zeta} = R\hat{\zeta}$ is the contravariant basis vector for toroidal azimuth ζ . We use a flux coordinate system (x, θ, ζ) , with x the flux-surface label. The vector potential is given by $\mathbf{A} = \Phi \nabla \theta - \psi \nabla \zeta$, with $A_{\theta}(x) = \Phi$ the toroidal flux/ 2π , and $A_{\zeta}(x) = -\psi$. Thus, $p_{\zeta} = -\frac{e}{c}\psi + Mv_{\zeta}$, and for this reason, expressions are somewhat simpler if one adopts the choice $x \to \psi$, which we now do. Since p_{ζ} is a constant of the motion, it equals its orbit-average $\bar{p}_{\zeta} = \frac{e}{c} \bar{\psi} + M\bar{v}_{\zeta}$, and thus

$$\bar{\psi} = -\frac{c}{e}(p_{\zeta} - M\bar{v}_{\zeta}) = \psi - \frac{c}{e}M(v_{\zeta} - \bar{v}_{\zeta}).$$
(1)

After a little geometry, one can write v_{ζ} in terms of the more conventional components v_{\parallel}, v_{\perp} as

$$v_{\zeta} = R(b_t v_{\parallel} + b_p v_{\perp} \cos \phi_v) = Rv(b_t \lambda + b_p \sqrt{1 - \lambda^2} \cos \phi_v) \equiv v_{\zeta \parallel} + v_{\zeta \perp}, \quad (2)$$

where $b_{t,p} \equiv B_{t,p}/B$, the ratio of the toroidal or poloidal to the total magnetic field.

For trapped particles (trapping-state index $\tau = 0$), one has $\bar{v}_{\zeta} = 0$. For passing particles ($\tau = 1$), $\bar{v}_{\zeta \perp}$ again vanishes under the gyro-average, while $\bar{v}_{\zeta \parallel} \simeq v_{\zeta \parallel}$, an approximation improving for more deeply-passing particles. Thus, in Eq.(1), one has

$$(v_{\zeta} - \bar{v_{\zeta}}) \simeq (1 - \tau) v_{\zeta \parallel} + v_{\zeta \perp}.$$
(3)

As in the GYROXY simulations, we adopt a collision operator which scatters in λ and ϕ_v , but not in energy:

$$C \equiv C_{\parallel} + C_{\perp} \equiv (\nu v^2/2) \nabla_{\mathbf{v}} \cdot (\mathbf{I} - \hat{\mathbf{v}}\hat{\mathbf{v}}) \cdot \nabla_{\mathbf{v}} = \nu/2 [\partial_{\lambda}(1 - \lambda^2)\partial_{\lambda} + (1 - \lambda^2)^{-1/2}\partial_{\phi_v}^2].$$
(4)

 C_{\parallel} is the usual Lorentz collison operator used in the drift-kinetic framework. Both C_{\parallel}, C_{\perp} are self-adjoint in (λ, ϕ_v) space: for any functions F, G, $\int d^2 \mathbf{v} F C_{\parallel,\perp} G = \int d^2 \mathbf{v} G C_{\parallel,\perp} F$.

We compute the radial flux of banana centers due to the action of C: $d_t \langle \bar{\psi} \rangle \equiv \langle C \bar{\psi} \rangle$, where d_t is the collisionally-induced time derivative, and for any function F(z), $\langle F \rangle \equiv V^{-1} \int_V d^6 z f F$ is a phase-space integral ($d^6 z = d^3 \mathbf{x} d^3 \mathbf{p}$) of F weighted by distribution function f, over a thin toroidal shell of volume V centered at flux surface ψ . From Eq.(4), one has $d_t = d_t^{\parallel} + d_t^{\perp}$, yielding contributions to the flux induced by C_{\parallel} and C_{\perp} , respectively. From Eqs.(1)-(4), one thus has

$$d_t \langle \bar{\psi} \rangle = \left(-\frac{c}{e} M\right) d_t \langle (v_{\zeta} - \bar{v}_{\zeta}) \rangle = \left(-\frac{c}{e} M\right) \langle (C_{\parallel} + C_{\perp}) [(1 - \tau) v_{\zeta \parallel} + v_{\zeta \perp}] \rangle, \quad (5)$$

which one sees gives four terms, of which the term $\langle C_{\perp}(1-\tau)v_{\zeta\parallel}\rangle$ vanishes. The first term is the neoclassical flux, $\Gamma_{nc} \equiv (-\frac{c}{e}M)\langle (C_{\parallel}(1-\tau)v_{\zeta\parallel}\rangle)$, while the remaining two yield the generalized classical flux, $\Gamma_{cl} \equiv (-\frac{c}{e}M)\langle (C_{\parallel}+C_{\perp})v_{\zeta\perp}]\rangle$.

To complete the calculation, we need an expression for the particle distribution f. Any function $f = f_0(\mathbf{J})$ of \mathbf{J} satisfies the Vlasov equation, so for low ν , a good collisionless approximation is the local Maxwellian form $f_0(\bar{\psi}, E) = n_0/[2\pi TM]^{3/2} \exp(-E/T)$, with n_0 and T functions of $\bar{\psi}$, and $E \equiv Mv^2/2$ the particle energy. Using $\bar{\psi} = \psi - \delta\psi$, one has $f_0(\bar{\psi}) \simeq f_0(\psi) - \delta\psi\partial_{\psi}f_0$, where $\partial_{\psi}f_0 = -\kappa f_0$, with $\kappa = \kappa_n + \kappa_T (v^2/v_T^2 - 3)/2$, $\kappa_n \equiv -\partial_{\psi} \ln n_0$ and $\kappa_T \equiv -\partial_{\psi} \ln T$. From Eqs.(1) and (3), $\delta\psi = (\frac{c}{e}M)((1-\tau)v_{\zeta\parallel}+v_{\zeta\perp}) \equiv \delta\psi_b+\delta\psi_g$, with $\delta\psi_g$ the gyro-orbit radial excursion, and $\delta\psi_b$ the (bounce-related) radial drift excursion. Using these in (5), one finds

$$d_t \langle \bar{\psi} \rangle = -V^{-1} \int_V d^6 z \partial_{\psi} f_0 \frac{\nu}{2} (\frac{c}{e} M R v)^2 \times \{(1-\tau) b_t^2 (1-\lambda^2) + b_p^2 [(1-2\lambda^2) + \sqrt{1-\lambda^2}] \cos^2 \phi_v \}$$
(6)

The integrations over λ and ϕ_v here are elementary, and those over θ and v may be done for specific models. Setting $\kappa_T = 0$ to extract only the diagonal term D^{ψ} of the transport matrix, we find from Eq.(6)

$$d_t \langle \bar{\psi} \rangle = -\partial_{\psi} n_0 [\bar{D}_{nc}^{\psi}(\psi) + \bar{D}_{cl}^{\psi}(\psi)] = \kappa_n V^{-1} \int_V d^6 z f_0 [D_{nc}^{\psi}(v, \mathbf{x}) + D_{cl}^{\psi}(v, \mathbf{x})],$$
(7)

where $D_{nc}^{\psi}(v, \mathbf{x}) = \frac{1}{2} \nu \rho_g^2 (B_t R)^2 (1 - B/B_{\pi})^{1/2}$, and

 $D_{cl}^{\psi}(v, \mathbf{x}) = \frac{1}{4}(\frac{1}{3} + \frac{\pi}{2})\nu\rho_g^2(B_pR)^2$ are the neoclassical and generalized classical diffusion coefficients, which must be integrated over θ and v to obtain the averaged coefficients. Here, $B_{\pi} \equiv B(\psi, \theta = \pi)$ is the maximum value of B on surface ψ . The factor $\frac{1}{3}$ in D_{cl}^{ψ} is the contribution from the term $C_{\parallel}v_{\zeta\perp}$ in (5), while the dominant factor $\frac{\pi}{2}$ is from the term $C_{\perp}v_{\zeta\perp}$. Thus, removing C_{\perp} from the GYROXY simulation should remove most of the enhancement of D over D_{nc} , as seen in Fig. 2. The dominant contribution from this approximate theory is about $\frac{\pi}{2}/(\frac{1}{3} + \frac{\pi}{2}) \simeq 82\%$ of D_{cl} , while the factor in Fig. 2 is about 70%. Note that ratios of these quantities are independent of particle energy, being due to equilibrium field geometry, and thus these results apply to heat transport as well as particle transport.

We put these diffusion coefficients in a more familiar form by transforming from ψ to a flux function $r(\psi)$ having units of length, which approximates an average minor radius. One has $d_t \langle \bar{r} \rangle = -\partial_r n_0 \bar{D}^r$, with $\bar{D}^r = (\partial_{\psi} r)^2 \bar{D}^{\psi}$, and similarly for D^r . Taking $r \equiv \sqrt{2\Phi/B_0}$, with B_0 the toroidal field strength on the magnetic axis, one has $(\partial_{\psi} r) = (q/B_0 r)$, and thus $D_{nc}^r(v, \mathbf{x}) = \frac{1}{2}\nu(\rho_g q)^2(B_t R/B_0 r)^2(1 - B/B_\pi)^{1/2}$, and $D_{cl}^r(v, \mathbf{x}) = \frac{1}{4}(\frac{1}{3} + \frac{\pi}{2})\nu\rho_a^2(qB_p R/B_0 r)^2$.

The θ -dependences in these expressions lie in ρ_g , $(1 - B/B_\pi)$, and B_pR . For a small- ϵ device, $\rho_g \simeq \text{const}$, $\oint \frac{d\theta}{2\pi}(1 - B/B_\pi)^{1/2} \simeq \frac{2}{\pi}\sqrt{2\epsilon}$, $(B_tR/B_0r)^2 \simeq 1/\epsilon^2$, and $(qB_pR/B_0r)^2 \simeq 1$, resulting in the familiar dependences $D_{nc}^r(v, \mathbf{x}) \simeq \sigma_{nc}\nu(\rho_g q)^2/\epsilon^{3/2}$ and $D_{cl}^r(v, \mathbf{x}) \simeq \sigma_{cl}\nu\rho_q^2$, with numerical coefficients

 $D'_{nc}(v, \mathbf{x}) \simeq \sigma_{nc} \nu (\rho_g q)^2 / \epsilon^{5/2}$ and $D'_{cl}(v, \mathbf{x}) \simeq \sigma_{cl} \nu \rho_g^2$, with numerical coefficients $\sigma_{nc} = \frac{\sqrt{2}}{\pi}$, and $\sigma_{cl} = \frac{1}{4}(\frac{1}{3} + \frac{\pi}{2})$.

In Fig. 3 we average these expressions using the NSTX geometry to compute the analytic counterparts of the numerical results in Fig. 2. One notes the approx-

imate agreement of the numerical and analytic results.

Summarizing, we have provided an explanation for the omniclassical enhancement of the total transport over neoclassical rates observed in Ref. 1, and developed an approximate transport theory which predicts both the enhancement over neoclassical from the full-orbit GYROXY code with full collision operator, and the residual finite-gyroradius enhancement in GYROXY present when its collision operator is constrained to scatter in pitch only. The enhancement comes from a generalization to strongly 2D geometries of classical transport, which for low-*A* configurations like NSTX can dominate over the neoclassical contribution.

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Figures

- FIG. 1. Plots of $\langle (\delta \psi)^2 \rangle$ versus time from the full-orbit code GYROXY and guidingcenter code ORBIT, from whose slopes (straight lines) the diffusion coefficients are computed. See text for details.
- FIG. 2. The ratio D/D_{nc} of the diffusion coefficient from GYROXY to that from ORBIT, versus radial variable X. See text for details.
- FIG. 3. The same ratio D/D_{nc} as in Fig. 2 from the analytic expressions derived in the text.



Figure 1:



Figure 2:



Figure 3:

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