

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,  
UNDER CONTRACT DE-AC02-76CH03073

PPPL-3847  
UC-70

PPPL-3847

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by

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July 2003



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# Zonal flow dynamics and size-scaling of anomalous transport

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(Dated: July 24, 2003)

## Abstract

Non-linear equations for the slow space-time evolution of the radial drift wave envelope and zonal flow amplitude have been self-consistently derived for a model nonuniform tokamak equilibrium within the coherent 4-wave drift wave-zonal flow modulation interaction model of Chen, Lin and White [Phys. Plasmas **7**, 3129, (2000)]. Solutions clearly demonstrate turbulence spreading due to non-linearly enhanced dispersiveness and, consequently, the device-size dependence of the saturated wave intensities and transport coefficients.

PACS numbers: 52.55.Dy, 52.55.Fa, 52.55.Tn

The dependence of plasma confinement on the device size is obviously a very crucial issue in fusion energy research. Assuming drift waves are responsible for the anomalous transport, size-scaling can be reduced, in the simplest model, to the dependence of drift-wave fluctuation intensity on  $\rho_*$ . Here  $\rho_* = \rho_i/L_p$  with  $\rho_i$  and  $L_p$  being, respectively, the ion Larmor radius and the plasma inhomogeneity scale length. The coherent 4-wave drift wave-zonal flow modulation interaction model of Chen, Lin and White [1] has captured the essential features observed in global gyrokinetic simulations in the  $\rho_i/L_p \rightarrow 0$  limit. We are thus motivated to adopt the same model as a theoretical paradigm, including finite  $L_p$  (i.e., finite  $\rho_*$ ) plasma inhomogeneities. In this finite- $\rho_*$  coherent 4-wave model, not only the drift wave (pump) radial envelope will be localized, leading to reduction in the modulational instability growth rate due to the finite interaction region; but more interestingly the damped pump and sidebands will disperse outward leading to radial spreading of the drift wave turbulence, qualitatively similar to that observed in recent simulations [2]. As we will show in the following, this turbulence characteristic behavior crucially depends on the wave dispersive properties of the radial envelope; which, in turn, depend intrinsically on the toroidal geometry of the considered system. As a consequence, the model we propose here predicts that numerical simulations of turbulent transport in cylindrical plasmas should be generally and profoundly different from those in a torus.

Following the theoretical formalism introduced in Refs. [1, 3], we assume that fluctuating fields are given by a single  $n \neq 0$  drift wave,  $\delta\phi_d$ , and a zonal ( $n = m = 0$ ) scalar field perturbation  $\delta\phi_z$ :

$$\begin{aligned}
\delta\phi_d &= \delta\phi_0 + \delta\phi_+ + \delta\phi_- \quad , \\
\delta\phi_0 &= e^{in\varphi} \sum_m A_{0,m} e^{-im\vartheta} \phi_0(nq - m, r) + c.c. \quad , \\
\delta\phi_{\pm} &= e^{\pm in\varphi} \sum_m A_{\pm,m} e^{\mp im\vartheta} \phi_{\pm}(nq - m, r) + c.c. \quad , \\
\delta\phi_z &= A_z(r) + c.c. \quad ,
\end{aligned} \tag{1}$$

where  $m$  and  $n$  are, respectively, poloidal and toroidal mode numbers. To simplify notations, time dependencies are suppressed, while  $(r, \vartheta, \varphi)$  denote a right handed toroidal coordinate system and  $q$  is the tokamak safety factor. Equations (1) explicitly indicate the existence of two characteristic spatial scales for high- $n$  drift waves [4]. The long scale reflects the characteristic radial variation of  $A_0, A_{\pm}, A_z$ , i.e. of mode envelopes and zonal flow, and is typically shorter than the equilibrium scale  $L_p$ . The short radial scale, instead, is associated

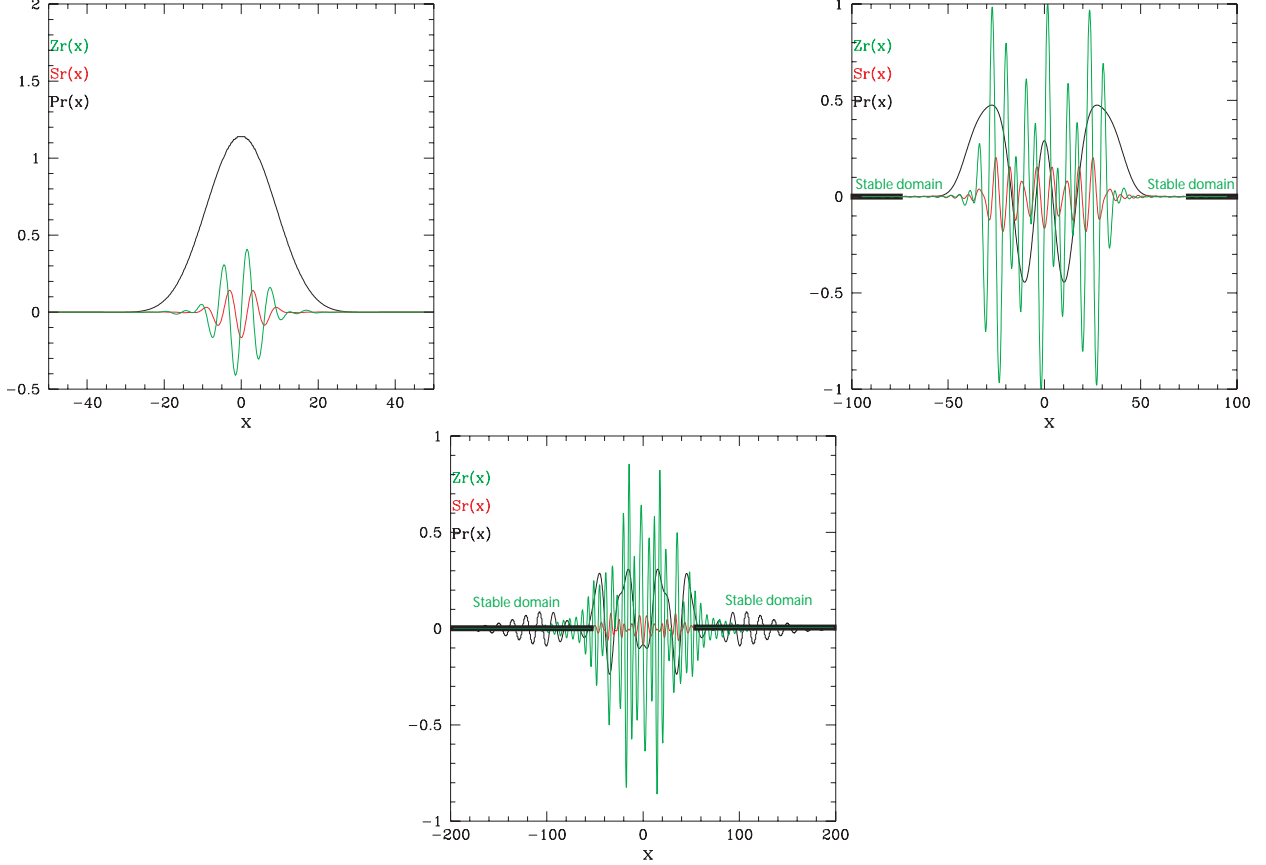


FIG. 1: Radial structure of the real part of the wave fields,  $\text{Re}P$  (black),  $\text{Re}S$  (red) and  $\text{Re}Z$  (green), at three subsequent times  $\tau = 20$  (left),  $\tau = 50$  (center) and  $\tau = 125$  (right), for  $A = 1.15$  and  $L = 200$ . The radial domain where the pump is linearly stable is also indicated.

with the parallel (to the ambient magnetic field) mode structure. It is  $\approx n^{-1}dr/dq$  and can be formally separated via the Fourier transform

$$\phi_0(nq - m, r) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i(nq-m)\theta} \psi_0(\theta, r) d\theta \quad , \quad (2)$$

where  $r$  dependencies reflect slow residual radial variations on the equilibrium scale. Similar equations can be written for  $\phi_{\pm}(nq - m, r)$ .

The typical time for setting up the parallel mode structure is of  $O(\omega^{-1})$ , the inverse mode frequency. Zonal flows, meanwhile, have characteristic times that are long compared with  $\omega^{-1}$  and typically of  $O(\gamma_L^{-1})$ , the inverse drift wave growth rate. Assuming formally proximity to marginal stability such that  $\gamma_L \ll |\omega|$ , non-linear dynamics, thus, will only affect radial envelope and zonal flow structures, leaving parallel mode structures essentially unchanged. That is, radial envelope and zonal flow structures could be completely different

from those predicted by linear theory and need to be determined consistently from non-linear equations for their slow space-time evolution. Detailed derivations of these equations will be given elsewhere. The basic approach for such derivations, however, can be found in Refs.[1, 3] in the  $k_{\perp}^2 \rho_i^2 \ll 1$  limit,  $k_{\perp}$  being the perpendicular (to the ambient magnetic field) drift wave vector. These equations are the quasi-neutrality conditions for the fluctuating fields of Eq. (1); explicitly written in a closed form via direct solution of the non-linear gyrokinetic equation [5]. As stated above, the possibility of reducing these non-linear system of Partial Differential Equations (PDE) (2-D in space plus time) to a non-linear system of Pseudo Differential Equations ( $\Psi$ DE) (1-D in space plus time) relies both on time and spatial scale separation of the mode structures; i.e.,  $\gamma_L \ll |\omega|$  and  $|n^{-1} dr/dq| \ll \Delta$ ,  $\Delta$  representing the characteristic width of the radial envelope and zonal flow structures[6]. Under these conditions, one can assume the parallel mode structures  $\psi_0(\theta, r)$ ,  $\psi_{\pm}(\theta, r)$  to be those predicted by linear theory and adopt an eikonal ansatz for radial envelope and mode structures

$$\begin{aligned}
A_{0,m} &= A_0(r) = e^{i \int n \theta_k dq} \ , \\
A_{-,m}^* &= A_{+,m} = A_+(r) = e^{i \int n \theta_k dq} \left( e^{i \int n \theta_z dq} + c.c. \right) \ , \\
A_z &= e^{i \int n \theta_{kz} dq} + c.c. \ .
\end{aligned} \tag{3}$$

Note, in Eqs. (3), we take  $k_z \approx k_r$ , and assume  $|\partial_r k_r / k_r^2| \ll 1$  and  $|\partial_r k_z / k_z^2| \ll 1$  for consistency. Here,  $k_z^{-1} = \theta_{kz}^{-1} n^{-1} dr/dq$  and  $k_r^{-1} = \theta_k^{-1} n^{-1} dr/dq$ ,  $2\pi k_z^{-1} (k_r^{-1})$  being the characteristic wavelength of zonal flow (drift wave) oscillations. Within this framework, it is possible to average the quasi-neutrality conditions for drift wave and zonal flow and reduce them to the following standard form for  $k_{\perp}^2 \rho_i^2 \ll 1$  [1, 3]:

$$\begin{aligned}
\mathcal{L}_P P &= 2S \partial_x Z \\
\mathcal{L}_S S &= -P \partial_x Z \\
\mathcal{L}_Z Z &= 2\text{Re} [P^* \partial_x S - S \partial_x P^*] \ .
\end{aligned} \tag{4}$$

Here, the normalized fields  $P, S, Z$  are related with  $A_0, A_+, A_z$  as

$$\begin{aligned}
\frac{e [A_0, A_+]}{T_e} &= \frac{\omega / \omega_{ci}}{(nq \rho_i / r)^2} \left( \frac{1.6q^2}{\alpha_0 \epsilon^{1/2}} \right)^{1/2} \frac{[P, -iS]}{s \Gamma^{1/2}} \ , \\
\frac{e A_z}{T_e} &= \frac{\omega / \omega_{ci}}{(nq \rho_i / r)^2} \frac{Z}{s \Gamma^{1/2}} \ ,
\end{aligned} \tag{5}$$

where  $\Gamma = \omega/|\gamma_{LP}(x = \infty)|$  is the mode frequency normalized to the drift wave growth rate at the boundaries of the radial domain,  $s = (r/q)(dq/dr)$ ,  $\epsilon = r/R_0$ ,  $R_0$  is the tokamak major radius,  $\alpha_0 = 1 + \delta p_{\perp i}/(en_0\delta\phi)$  [1],  $\delta p_{\perp i}$  is the perpendicular ion pressure fluctuation,  $n_0$  the equilibrium density and other notations are standard. Note that Eqs. (4) have the Reynolds-stress like antisymmetric non-linearity, as it is generally expected for electrostatic drift waves in the  $k_{\perp}^2\rho_i^2 \ll 1$  limit [5, 7]. The linear operators  $\mathcal{L}_P, \mathcal{L}_S, \mathcal{L}_Z$ , meanwhile, are defined in terms of the local drift wave dispersion function  $D = D_R(r, \omega, \theta_k) + iD_I(r, \omega, \theta_k)$ ; with the wave frequency  $\omega_0$  and envelope radial wave-number  $\theta_{k0}(r)$  given by  $D_R(r, \omega_0, \theta_{k0}(r)) = 0$ . More precisely:

$$\begin{aligned}\mathcal{L}_{P,S} &= \partial_{\tau} - \bar{\gamma}_{P,S} - 2\delta^{1/2}\partial_x + i\Gamma(\lambda + \xi) + i\partial_x^2, \\ \mathcal{L}_Z &= (\partial_{\tau} + \gamma_z),\end{aligned}\tag{6}$$

where time is normalized as  $\tau = |\gamma_{LP}(x = \infty)|t$ ,  $[\bar{\gamma}_{P,S}, \bar{\gamma}_z] = [\gamma_{LP,S}, \nu_z]/|\gamma_{LP}(x = \infty)|$ ,  $\nu_z$  being the zonal flow collisional damping,  $\nu_z \approx (1.5\epsilon\tau_{ii})^{-1}$  [8]. The normalized radial coordinate  $x$  and the other quantities to be defined in Eq. (6) are given by:

$$\begin{aligned}\xi &= \frac{\theta_{k0}\partial D_R/\partial\theta_{k0} - \theta_{k0}^2\partial^2 D_R/\partial\theta_{k0}^2}{\omega_0\partial D_R/\partial\omega_0}, \\ \lambda &= \frac{\theta_{k0}^2}{2} \frac{\partial^2 D_R/\partial\theta_{k0}^2}{\omega_0\partial D_R/\partial\omega_0}; \quad \delta^{1/2} = \frac{\xi\Gamma^{1/2}}{2\lambda^{1/2}}, \\ \gamma_L &= -\frac{D_I}{\partial D_R/\partial\omega_0}; \quad \frac{\partial}{\partial x} = \frac{\lambda^{1/2}\Gamma^{1/2}}{\theta_{k0}n(dq/dr)} \frac{\partial}{\partial r}.\end{aligned}\tag{7}$$

In the cylindrical limit,  $\partial D_R/\partial\theta_{k0} = 0$  and  $\xi = \lambda = \delta^{1/2} = 0$  as well as  $\partial_x = 0$  in Eqs.(6) and (7), demonstrating the crucial importance of toroidal geometry.

Equations (4) generally require numerical solutions. As a simple but relevant paradigm, we take Gaussian non-uniformity profiles and quadratic dispersiveness for numerical studies of Eqs. (4). That is,  $D_R = \omega/\omega_0 - 1 + \theta_k^2 + V(x)$ , with the potential well  $V(x) = 1 - \exp(-x^2/L^2)$ , where  $L$  is related with the equilibrium profile scale as  $L = |ndq/dr|L_p/\Gamma^{1/2}$ . We also choose  $D_I = -(\bar{\gamma}_P(x)/\Gamma) = -(1/\Gamma)(A\exp(-x^2/L^2) - 1)$  for the pump and  $D_I = (\bar{\gamma}_d/\Gamma)$  for the sidebands in order to have

$$\begin{aligned}\mathcal{L}_P &= \partial_{\tau} - \bar{\gamma}_P(x) - i\Gamma V(x) + i\partial_x^2, \\ \mathcal{L}_S &= \partial_{\tau} + \bar{\gamma}_d - i\Gamma V(x) + i\partial_x^2, \\ \mathcal{L}_Z &= (\partial_{\tau} + \bar{\gamma}_z).\end{aligned}\tag{8}$$

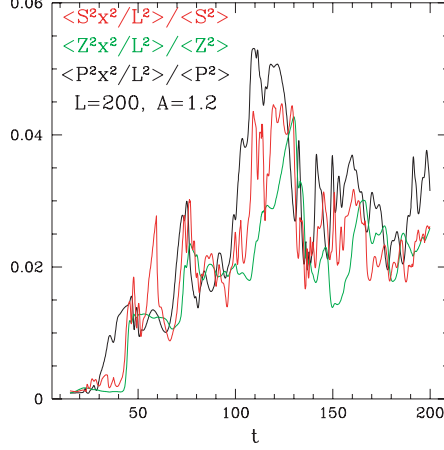


FIG. 2: Characteristic squared width of the wave fields normalized to  $L^2$  as a function of time, for  $A = 1.2$  and  $L = 200$ . Color code is the same as in Fig. 1.

From Eqs. (4) and (8), we readily recover the local limit considered in Ref. [1] by taking  $L \rightarrow \infty$ ,  $P = P_0(t)$ ,  $S = S_0(t) \cos(\kappa_z x)$  and  $Z = Z_0(t) \sin(\kappa_z x)$ . In that case, maximum zonal flow growth rate for fixed pump amplitude,  $\Gamma_{z,Max} = |P_0|^2 - \bar{\gamma}_z$ , is expected for  $\kappa_z^2 = \bar{\gamma}_d + \Gamma_{z,Max}$  i.e. for  $\theta_{kz} = \Gamma^{-1/2}(\bar{\gamma}_d + \Gamma_{z,Max})^{1/2}$ [1]. In the same  $L \rightarrow \infty$  limit, Eqs. (4) also admit a fixed point solution that, for minimum zonal flow amplitude,  $Z_{0,min}^2 = 2\bar{\gamma}_{P0}\bar{\gamma}_d/|\bar{\gamma}_d - \bar{\gamma}_{P0}|$  is characterized by  $P_{0,f} = \bar{\gamma}_z^{1/2}\bar{\gamma}_d^{-1/2}/|\bar{\gamma}_d - \bar{\gamma}_{P0}|^{1/2} \exp(-i\bar{\gamma}_{P0}\tau)$ ,  $S_{0,f} = -(1+i)(\bar{\gamma}_{P0}/2\bar{\gamma}_d)^{1/2}P_{0,f}$ , with  $\bar{\gamma}_{P0} = A - 1$  and  $\kappa_{z,f}^2 = |\bar{\gamma}_d - \bar{\gamma}_{P0}|$ .

Fast radial non-linear oscillations of sidebands and zonal flow on the characteristic scale  $\approx \kappa_z^{-1}$  are a general feature of the solutions of Eqs. (4) also for finite  $L$  [6]. The spatially averaged drift wave intensity on this short scale is  $\bar{I} = \overline{|P + 2S|^2} = |P_0|^2 + 2|S_0|^2$  and, for the fixed point solution as  $L \rightarrow \infty$ ,  $\bar{I}_f = |P_{0,f}|^2 + 2|S_{0,f}|^2 = \bar{\gamma}_z(\bar{\gamma}_d + 2\bar{\gamma}_{P0})|\bar{\gamma}_d - \bar{\gamma}_{P0}|^{-1}$ . Assuming drift waves are responsible for anomalous transport and that anomalous diffusion in an infinite system has gyro-Bohm scaling,  $\chi_\infty = \chi_{GB} \approx \rho_* \omega_{ci} \rho_i^2$ , the present model yields  $\chi = \chi_{GB}(\bar{I}/\bar{I}_f)$ . Thus, any size-scaling of anomalous transport can be reduced to the dependence of  $\bar{I}$  on  $L$ , and ultimately on  $\rho_*$ . In order to investigate this aspect, we have solved Eqs. (4) and (8) numerically, keeping  $\bar{\gamma}_z = 0.1$ ,  $\bar{\gamma}_d = 1$  and  $\Gamma = 4$  fixed, while changing both  $\bar{\gamma}_{P0} = A - 1$  and  $L$  to assess  $\bar{I}$  dependencies on these parameters. Snapshots of simulation results for the wave fields at different times are shown in Fig. 1 for  $A = 1.15$  and  $L = 200$ . They clearly demonstrate outward radial dispersion of pump, assisted by the non-linear modulation interaction and leading to radial spreading of the drift



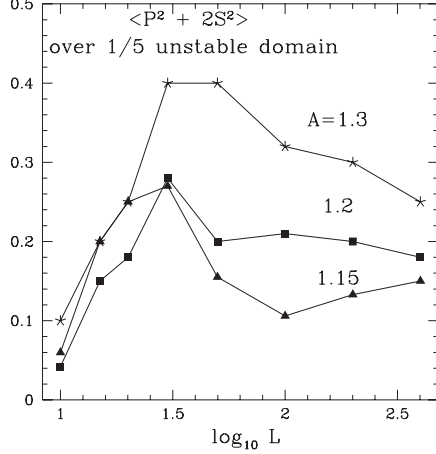


FIG. 3: Drift wave intensity  $\langle \bar{I} \rangle$  vs.  $L$  after spatial averaging on one fifth of the linearly unstable domain. The three curves refer to  $A = 1.15$ ,  $A = 1.2$  and  $A = 1.3$ .

wave turbulence qualitatively similar to that observed in recent simulations by Lin et al. [2]. Pump radial spreading is then followed by similar spreading of zonal flow and sidebands, as indicated in Fig. 2. Numerical solutions can be understood via asymptotic analyses of Eqs. (4) and (8); employing the optimal ordering  $\partial_\tau \approx \bar{\gamma}_z \approx \bar{\gamma}_{P0} \ll \bar{\gamma}_d$ . Here, we omit the details and only report some of the main results. The leading order solution can be represented as  $P \sim P_0(x_1, \tau)$ ,  $S \sim S_0(x_2, \tau) \cos(\kappa_z x)$ ,  $Z \sim Z_0(x_2, \tau) \sin(\kappa_z x)$ ,  $x_1 \approx L^{1/2}$ ,  $x_2 \approx L^{1/4}$ , with  $\kappa_z = \bar{\gamma}_d^{1/2}$  for the fastest growing zonal flow. Equations (4) and (8) then reduce to  $S_0 = -(1+i)P_0 Z_0 / 2\bar{\gamma}_d^{1/2}$  and

$$\begin{aligned} & [\partial_\tau - \bar{\gamma}_P(x) + Z_0^2/2 - i\Gamma V(x) + iZ_0^2/2 + i\partial_x^2] P_0 = 0 \quad , \\ & [\partial_\tau + \bar{\gamma}_z - |P_0|^2 - 2\bar{\gamma}_d^{-1}|P_0|^2 \partial_x^2] Z_0 = 0 \quad . \end{aligned} \quad (9)$$

Zonal flows, thus, act both as non-linear damping as well as anti potential well on the drift wave pump. Meanwhile, the pump drives zonal flows non-linearly but it generates non-linear diffusion as well, that manifests itself in numerical simulations as turbulence spreading [2, 9]. In the early non-linear phase,  $P_0 = \Pi_0(\tau) H_p(y) \exp(-y^2/2)$ , with  $y = \Gamma^{1/4} L^{-1/2} (1 + i/4\Gamma)x$ ,  $H_p$  are Hermite polynomials and  $\Pi_0 \sim \exp[i(2p+1)(\Gamma^{1/2}/L)\tau + \bar{\gamma}_{P0}\tau - (p+1/2)\tau/(L\Gamma^{1/2})]$ . The maximum order of excited radial modes will then be  $p \approx \Gamma^{1/2} L \bar{\gamma}_{P0}$ , while the fastest growing mode is the ground state, for which  $Z_0 = \zeta_0(\tau) \exp(-\bar{\gamma}_d^{1/2} \Gamma^{1/4} x^2 / \sqrt{8L})$  and  $\zeta_0 \sim \exp[|\Pi_0|^2 (1 - \sqrt{2\Gamma^{1/2}/\bar{\gamma}_d L}) - \bar{\gamma}_z] \tau$ , as in Fig. 1 (left). When radial spreading stops and the fluctuation intensity has reached a time asymptotic value (see right frame in Fig. 1),

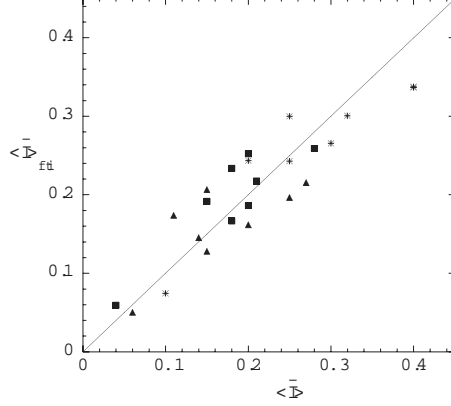


FIG. 4: Values of  $\langle \bar{I} \rangle_{fit} = 24.0(L - 9.6)^{1/2}(\bar{\gamma}_{P0}^{1/2} + .03\bar{\gamma}_{P0}L)/(98.3 + L^{3/2})$  are shown vs. the same simulation data of Fig. 3. Same markers are maintained.

both pump and sideband are characterized by complex radial structures on intermediate scales between fast non-linear oscillations on  $\approx \bar{\gamma}_d^{-1/2}$  and the size of the linearly unstable region  $\approx \bar{\gamma}_{P0}^{1/2}L$ . To adequately evaluate the drift wave intensity  $\bar{I}$ , we have taken a further spatial average of its value to make the result reasonably independent on the averaging method itself. Figure 3 shows the results of the spatially averaged drift wave intensity  $\langle \bar{I} \rangle$  on 1/5 of the linearly unstable domain [10]. In the  $L \rightarrow \infty$  limit, numerical results reflect well the values for the fluctuation intensity expected from the fixed point solution, i.e.  $\langle \bar{I}_f \rangle \simeq 0.15, 0.18, 0.23$  respectively for  $A = 1.15, 1.2, 1.3$ . The scaling of  $\langle \bar{I} \rangle$  with the system size is evident: it sharply increases with  $L$  for  $L < 30$ , suggesting a Bohm scaling of anomalous transport, and it eventually reaches the asymptotic value set by the fixed point solution for  $L > 100$ , where gyro-Bohm scaling is indeed expected. Due to the definition of  $L = nq(\rho_i/r)(|s|/\rho_*)\Gamma^{-1/2}$ , values obtained from simulation results depend intrinsically on dimensionless physical parameters such as magnetic shear and normalized poloidal wavelength. With the parameters of global gyrokinetic simulations reported in Ref. [2], and defining  $a$  as the tokamak minor radius, present results would predict a Bohm to gyro-Bohm transition for  $a/\rho_i > 420$  and saturation to gyro-Bohm transport for  $a/\rho_i > 1400$ , in remarkable agreement with the results therein [2]. From dimensional analyses of Eqs. (4) and (8), we note that the dependence of  $\langle \bar{I} \rangle$  on  $\bar{\gamma}_{P0}L$  is expected, with  $\bar{\gamma}_{P0}L$  representing the ratio between the size of the unstable region and the characteristic scale of pump wave-packets as well [6]. In particular, Fig. 3 shows that  $\langle \bar{I} \rangle \approx \bar{\gamma}_{P0}^{1/2}(L - L_{crit})^{1/2}$  as  $\bar{\gamma}_{P0}L \rightarrow 0$  and  $\langle \bar{I} \rangle \approx \bar{\gamma}_{P0}$  as  $\bar{\gamma}_{P0}L \rightarrow \infty$ . We have therefore fitted the simulation data on  $\langle \bar{I} \rangle$  with

$\langle \bar{I} \rangle_{fit} = 24.0(L - 9.6)^{1/2}(\bar{\gamma}_{P0}^{1/2} + .03\bar{\gamma}_{P0}L)/(98.3 + L^{3/2})$ , and the results are plotted in Fig. 4.

In summary, we have demonstrated that the coherent 4-wave drift wave-zonal flow modulation interaction model of Chen, Lin and White [1] not only captures the essential features observed in global gyrokinetic simulations in the  $\rho_i/L_p \rightarrow 0$  limit, but, by allowing non-uniform equilibrium, accounts as well for size scaling of drift wave intensity and ultimately of turbulent diffusion. This model sets a hierarchy among the relevant non-linear interactions; making it possible to consistently derive equations for the slow space-time evolution of drift wave radial envelope and zonal flow structures. The predicted size scaling of drift wave intensity is remarkably similar to that of global gyrokinetic simulations [2].

This work was partly done with support of U.S. D.o.E. Grant DE-FG03-93ER54271 and Contract No. DE-AC02-76-CH03073 to University of California at Irvine and Princeton Plasma Physics Laboratory.

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