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Near-Earth thin current sheets and Birkeland currents during substorm growth phase

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Abstract. Two important phenomena observed during the magnetospheric substorm growth phase are modeled: the formation of a near-Earth (|X| ∼ 9 R_E) thin cross-tail current sheet, as well as the equatorward shift of the ionospheric Birkeland currents. Our study is performed by solving the 3-D force-balance equation with realistic boundary conditions and pressure distributions. The results show a cross-tail current sheet with large current (J_φ ∼ 10 nA/m^2) and very high plasma β (β ∼ 40) between 7 and 10 R_E. The obtained region-1 and region-2 Birkeland currents, formed on closed field lines due to pressure gradients, move equatorward and become more intense (J_||_{max} ∼ 3 μA/m^2) compared to quiet times. Both results are in agreement with substorm growth phase observations. Our results also predict that the cross-tail current sheet maps into the ionosphere in the transition region between the region-1 and region-2 currents.

1. Introduction

Two important phenomena are associated with the substorm growth phase. One is the appearance of a thin cross-tail current sheet in the near-Earth (7 – 10 R_E) plasma sheet [e.g., Sergeev et al., 1990]. The other consists in an intensity increase and an equatorward shift of the region-1 and region-2 ionospheric field-aligned (Birkeland) currents (with a similar shift being observed in the auroral arc structures [e.g., Samson et al., 1992]). Using our 3-D quasi-equilibrium model [Cheng, 1995; Zaharia et al., 2003] we investigate here these 2 effects, i.e. the formation of the cross-tail current sheet and the Birkeland current configuration, as well as their relationship during the substorm growth phase.

There is a consensus in the space physics community that the inner and middle magnetosphere on closed field lines is a “slow-flow” region [Wolf, 1983] at most times, including during the growth phase. Thus the inertial terms can be neglected in the plasma equation of motion and the magnetospheric evolution can be depicted as a temporal series of “snapshots”, each of them a “quasi-static equilibrium” state in which force balance is maintained between the magnetic (Lorentz) force and the plasma pressure gradient force. Within the “slow flow” approximation there have been several theoretical efforts trying to explain thin current sheet formation. Most approaches [e.g., Wiegelmann and Schindler, 1995; Birn and Schindler, 2002] investigate the currents at X < −20 R_E (from here on X, Y and Z are the usual GSM coordinates), where the so-called tail approximation [Birn et al., 1975] is valid, and the problem is sometimes even analytically tractable. Only a few studies [e.g., Becker et al., 2001] look at the current sheets closer to Earth, where the tail approximation becomes inaccurate, and these studies assume 2-D axisymmetry, missing the formation of the field-aligned currents (a 3-D effect, as explained by Cheng [1995]). Most studies consider the magnetospheric evolution during the growth phase to be dictated by “adiabatic convection” [e.g., Wolf, 1983] whereby the entropy, related to the quantity S = PVγ, is conserved (P is the pressure, V the magnetic flux tube volume per unit flux, V = ∫ ds/B, with the integral performed along a magnetic field line; γ = 5/3). With entropy conservation constraints a very thin current sheet can form for example due to deformations of the magnetopause boundary [Birn and Schindler, 2002].

In this letter we discuss 3-D force-balanced magneto-
spheric configurations, focusing on the formation during the substorm growth phase of a thin current sheet in the near-Earth plasma sheet and on the changes in Birkeland currents compared to a quiet time state. Our configurations are obtained by solving numerically the 3-D force.balance equation \( \mathbf{J} \times \mathbf{B} = \nabla P \) in a flux coordinate system [Cheng, 1995; Zaharia et al., 2003], subject to realistic flux boundary conditions and pressure distributions. The cause of the current sheet formation is the large value of \( |\partial P/\partial \psi| \) caused by plasma and flux transport during the growth phase. The Birkeland currents, formed on closed field lines due to pressure gradients, move equatorward and become more intense \( (J_{\|})_{\text{max}} \sim 3 \mu A/m^2 \) compared to quiet times. The cross-tail current sheet region maps into the ionosphere in the transition region between the region-1 and region-2 currents.

2. Modeling approach

We work in a geomagnetic flux coordinate system \( \{\psi, \alpha, \chi\} \), in which two of the coordinates, \( \alpha \) and \( \psi \), are Euler potentials for the magnetic field: \( \mathbf{B} = \nabla \psi \times \nabla \alpha \). We choose \( \psi \) to label the magnetic flux and \( \alpha \) an azimuthal angle-like function. The third coordinate, \( \chi \), is a function of the distance along the field line. The computation of equilibria in the \( \{\psi, \alpha, \chi\} \) system, described in detail elsewhere [Cheng, 1995; Zaharia et al., 2003], consists in solving the 3-D equation \( \mathbf{J} \times \mathbf{B} = \nabla P \) iteratively, subject to input pressure distribution and boundary conditions. The boundary \( \psi \) surfaces delimiting the computational domain have specified shapes, usually obtained [Zaharia et al., 2003] from empirical models such as T96 [Tsygannenko and Stern, 1996]. We only briefly describe here the changes implemented in our method for a more accurate computation of configurations with strong current sheets. The first change was using as “planet boundary” a sphere of radius \( 2R_E \) instead of the Earth’s surface. While the code can perfectly handle a computation from \( 1R_E \), this was done in order to save grid points for the important plasma sheet region. The second change was relaxing the “equal arc length” choice [Cheng, 1995] for \( \chi \), instead concentrating the grid points near the equatorial plane. The grid points are also non-uniformly distributed in azimuth [Zaharia et al., 2003], with a concentration near the midnight meridian. The number of grid points used is \( N_\psi \times N_\alpha \times N_\chi = 75^3 \), leading to a spatial resolution near the equatorial plane at \( R \sim 8R_E \) and midnight of about \( \Delta X = 0.25R_E, \Delta Y = 0.25R_E \) and \( \Delta Z = 0.05R_E \).

3. Results: quiet time vs. substorm growth phase

While our emphasis is on the substorm growth phase, we will discuss first a 3-D force-balanced quiet-time configuration in order to facilitate a discussion of differences between the two. For the quiet-time case we use inner and outer boundary shapes for \( \psi \) obtained by field-line trac-

![Figure 1](image-url)  
Figure 1. For the quiet-time state: (a) Contours of constant ionospheric \( J_\parallel(\mu A) \); solid (dashed) lines represent currents into (out of) the ionosphere; (b) Equatorial plane contours of \( P(\text{nPa}) \) (solid) and \( V \) (dashed); also shown are \( \nabla P \) and \( \nabla V \) at two points mapping into regions of opposite \( J_\parallel \) in the ionosphere; the dotted lines show const. \( \psi \) contours.

ing using the T96 model, with parameters \( DST = -5 \text{nT} \), \( P_{SW} = 2.1 \text{ nPa} \), \( B_{zIMF} = 0 \) and \( B_{xIMF} = 1 \text{ nT} \), representing average quiet-time parameters as obtained from the OMNI solar wind database. For the pressure \( P \) we choose the following form in the equatorial plane:

\[
P(R, \phi, Z = 0) = 89e^{-0.59R} \left[ A + Be^{-\left(\frac{Z}{58}\right)^2}\right] + 8.9R^{-1.53} \left[ C + De^{-\left(\frac{\phi}{\pi}\right)^2}\right] \tag{1}
\]

where \( R, \phi, Z \) define the usual cylindrical coordinate system with Earth as origin and \( \phi = \pi \) at midnight, while \( A, B, C, D \) and \( \Delta \phi \) are constants. We choose \( A = B = 0.5, \)
Figure 2. (a) Profiles of $P$, $B$ and $\beta$ along the Sun-Earth axis for the growth phase (solid) and quiet-time (dashed); For the growth phase, plots in the noon-midnight meridian plane of: (b) Magnetic field lines; (c) Constant $J_\phi$ (solid) and $\psi$ (dotted) contours; (d) Constant $\beta$ (solid) and $\psi$ (dotted) contours.

$C = 2$, $D = -1$ and $\Delta \phi = 0.5\pi$, such that for $\phi = \pi$ Eq. (1) recovers the Spence-Kivelson empirical formula [Spence and Kivelson, 1993], which is based on observations at midnight. At the same time, since the first term on the RHS of Eq. (1) dominates close to Earth ($R < 10 R_E$), while the second term farther in the tail, Eq. (1) also simulates for a given $R$ an azimuthal maximum in $P$ at midnight close to Earth, and an azimuthal minimum farther in the tail. This qualitative local-time dependence, seen in the equatorial $P$ contours in Fig. 1(b), is justified by observations showing a maximum in $P$ at midnight close to Earth [De Michelis et al., 1999, e.g.], but a slight minimum at midnight for $R > 10 R_E$ (see Fig. 11 of [Tsyganenko and Mukai, 2003]).

We will only briefly summarize the physical parameters of the computed quiet-time state. The cross-tail current ($J_\phi = \mathbf{J} \cdot \nabla \phi \parallel (\nabla \phi)$) has a maximum $J_\phi \approx 2.4 \text{nA/m}^2$. Dashed lines in Fig. 2(a) show the profiles along the Sun-Earth axis of $P$, $B$ and $\beta$ for this case, while the Birkeland currents are shown in Fig. 1(a). The region-2 currents span a broad area, but are very weak ($J_{12\text{max}} = 0.07 \mu\text{A/m}^2$) — consistent with observations [Uijma and Potemra, 1976] showing their virtual disappearance during quiet times. On the other hand, a more narrow region-1 current pattern exists at higher latitudes ($\sim 68^\circ$), with maximum densities ($\approx 0.5 \mu\text{A/m}^2$) at 11:00 and 2:00 local times, again agreeing very well with quiet-time observations [Uijma and Potemra, 1976]. The region-1 and region-2 current formation mechanism is easily understood from Vasyliunas relation [Vasyliunas, 1970], in the form

$$\frac{J_\phi}{B} \bigg|_{\text{iono}} = \frac{B_{eq}}{B_{eq}} \cdot (\nabla V \times \nabla P_{eq})$$

(2)

The quantity $B_{eq} \cdot (\nabla P_{eq} \times \nabla V)$ has opposite signs for region-1 vs. region-2 current formation, as seen in Fig. 1(b), which shows $\nabla P$ and $\nabla V$ at two equatorial plane locations that map into the ionosphere in regions of opposite $J_1$.

For modeling a substorm growth phase, the $\psi$ boundary shapes are obtained again from T06, this time with $P_{SW} = 5 \text{nPa}$, $B_{ZMF} = -5 \mu\text{T}$, $B_{YMF} = 0.5 \mu\text{T}$ and $DST = -50 \mu\text{T}$, typical for disturbed times. There are only scarce plasma pressure observations during the growth phase. While $P$ generally increases with activity throughout the plasma sheet [e.g., Tsyganenko and Mukai, 2003], observations [e.g. Spence et al., 1989] as well as convection simulations [Wang et al., 2003] show that the pressure enhancement is larger at smaller radial distances. Another property, both observed [Wing and Newell, 1998; Tsyganenko and Mukai, 2003] and apparent in simulations [Wang et al., 2003, e.g.], is the Earthward expansion of regions with azimuthal minimum $P$ at midnight (for fixed $R$). We thus choose the $P$ distribution in the equatorial plane as

$$P(R, \phi, Z = 0) = 12.5 e^{-0.25R} \left[ A + B \tan \left( \frac{x_1 - R}{\Delta R} \right) \right. + e^{-\left( \frac{2\Delta R}{x_1} \right)^2} + 8.9 R^{-1.53} \left. \left[ C + D e^{-\left( \frac{2\Delta R}{x_1} \right)^2} \right] \right]$$

(3)

We choose $A = 1.25$, $B = 0.75$, $C = 3$, $D = -2$, $\Delta \phi = 0.3\pi$, $x_1 = 10$ and $\Delta R = 1.25$ in Eq. (3). The resulting $P$ profile along the Sun-Earth axis, shown by solid lines in Fig. 2(a), is about twice the quiet-time value tailward of $10 R_E$, and even more enhanced at the inner edge of the plasma sheet. The equatorial $P$ contours, shown in Fig. 3(b), show the Earthward expansion of regions with $P$ minimum (for a given $R$) at midnight, and at the same time the more pronounced azimuthal minima in $P$ as compared to the quiet-time case.
Figure 3. For the growth phase: (a) Ionospheric $J_\parallel$; (b) Equatorial plane $P$(nPa) contours (thick solid lines), $V$ (dashed) and $\psi$ (dotted), over a color plot of $J_\phi$; the thin solid contour shows the region inside which $J_\phi > 0.5 J_{\phi\text{max}}$; also shown are $\nabla P$ and $\nabla V$ at three locations.

Figure 2 shows several quantities in the the obtained force-balanced state. The solid lines in Fig. 2(a) show profiles of $P$, $B$ and $\beta$ along the Sun-Earth axis. We notice the appearance of a local magnetic well, with $B_{\text{min}} \approx 15$ nT, between $X = -7 R_E$ and $X = -9 R_E$. In the magnetic well, plasma $\beta$ peaks at $\beta \approx 45$ near $X = -8 R_E$. The magnetic field is extremely tail-like in the near-Earth plasma sheet, as seen in Fig. 2(b). The tail-like field suggests a thin current sheet, which can indeed be seen in Fig. 2(c), which shows noon-midnight meridian plane contours of $J_\phi$. The maximum current density is $J_{\phi\text{max}} \approx 11$ nA/m$^2$, and the sheet has a minimum half-width of $0.6 R_E$ at $X = -9 R_E$. Finally, from Fig. 2(d) one notices that plasma $\beta$ is very large in the vicinity of the equatorial plane.

The Birkeland currents in this state are shown in Fig. 3(a). Both the region-1 and region-2 currents have moved to lower latitudes compared to the quiet-time case shown in Fig. 1(a), and are much more intense (with the intense $J_1$ regions quite peaked in latitudinal extent). The region-2 current has a maximum density of $1.2 \mu$A/m$^2$ at 22:00 and 2:00 local times, and stretches between $60^\circ$ and $62^\circ$ in latitude. The region-1 current is found between $62^\circ$ and $65^\circ$ and has a maximum of $3.5 \mu$A/m$^2$ closer to midnight (22:30 and 1:30 local times). Again, the different signs of $B$ ($\nabla P \times \nabla V$) in the region-2 and region-1 current regions, respectively, are readily seen in Fig. 3(b), which shows the orientation of the vectors $\nabla P$ and $\nabla V$ over a color plot of $J_\phi$ in the equatorial plane. From Fig. 3 one observes that the cross-tail current sheet maps into the ionosphere into the transition area between region-1 and region-2 currents.

4. Discussion and Summary

We have modeled a quasi-static equilibrium magnetospheric state during the substorm growth phase, by solving the 3-D force-balance equation with realistic pressure and flux boundaries. The obtained configuration includes a thin current sheet with $J_\phi \sim 10$ nA/m$^2$ in the near-Earth plasma sheet between $X = -7 R_E$ and $X = -9 R_E$. The configuration is also characterized by the region-1 and region-2 Birkeland currents moving toward lower latitudes ($60^\circ - 65^\circ$), and being more intense (region-1 $J_{\phi\text{max}} \sim 3 \mu$A/m$^2$) compared to quiet times. The cross-tail current sheet region maps into the ionosphere in the transition area between the region-1 and region-2 currents.

The near-Earth cross-tail current sheet has a half-thickness $\sim 0.6 R_E$, in good agreement with observations [Sanny et al., 1994] showing the current sheet being wider than $1 R_E$ throughout the growth phase. This result differs from the popular belief that the sheet thickness is on the order of an ion gyro-radius ($\rho_i \sim 1000$ km). It is unlikely that such currents can be found in a force-balanced configuration in the transition region between the dipole-like and tail-like magnetic field. Among the reasons for this, we note that direct evidence of extremely thin current sheets is scarce — most observations measure the B-field and try to fit it with an unrealistic very thin Harris current sheet, without discussing whether a thicker sheet might suffice. Secondly, observations in the near-Earth plasma sheet at $X \approx -8 R_E$ by AMPTE/CCE [Lui et al., 1992] show that the angle between the $B_z$ and $E_z$ components of $B$ is never less than $40^\circ$ at any time during the substorm growth phase, therefore precluding the existence of a sheet of width $\sim \rho_i$. Finally, we note that the current sheet does not need to become thinner than $0.5 R_E$ in order to lead to substorm onset; indeed, in our current sheet region plasma $\beta$ ($\sim 45$) is already sufficiently large for a kinetic ballooning instability [Cheng and Lui, 1998] to be excited and lead to onset.

Based on our study, the scenario for current sheet formation near Earth ($|X| < 10 R_E$) is the following: during
the growth phase, the larger solar wind $P_{SW}$ and increased flux merging at the magnetopause leads to enhanced tail stretching. At the same time, plasma pressure in the near-Earth plasma sheet greatly increases due to enhanced convection, leading to larger pressure gradients. Due to the strong stretching of the tail flux tubes, the difference $\Delta \psi$ between $\psi_{out}$ (the flux on the outer boundary at $R \approx 18.5 R_E$) and $\psi_{in}$ (on the inner boundary at $R \approx 3.5 R_E$) becomes smaller compared to quiet times. The increase in $|\partial P/\partial R|$ coupled with the decrease in $\partial \psi/\partial R$ leads to very large $|\partial P/\partial \psi|$ and thus current densities (at midnight $\alpha = \phi$ and $J_0 = J \cdot \nabla \phi / |\nabla \phi| = \nabla P / (\partial \psi)$) localized in the near-Earth plasma sheet. In our study a large gradient in the flux volume $V = \int ds / B$ is not needed in order to have large $|\partial P/\partial \psi|$, unlike in adiabatic formalisms of current sheet formation such as the “gradient of flux volume mechanism” (GFVM) [Wiegelmann and Schindler, 1995].

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References


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