

**PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76CH03073**

PPPL-3805
UC-70

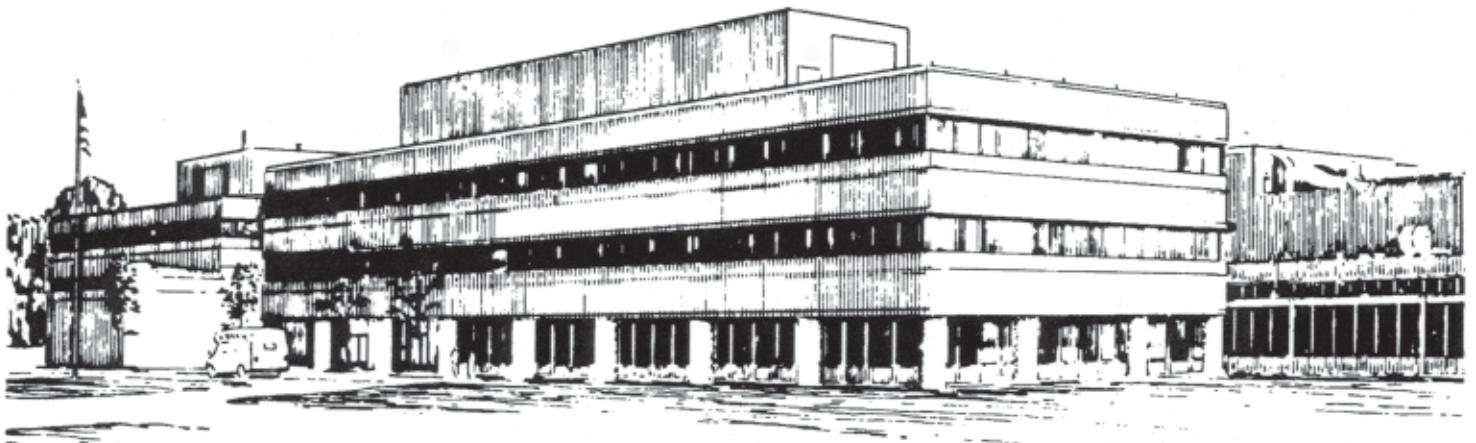
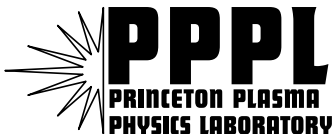
PPPL-3805

**GeV Electrons Acceleration in Focused Laser Fields
after Above-threshold Ionization**

by

I.Y. Dodin and N.J. Fisch

April 2003



**PRINCETON PLASMA PHYSICS LABORATORY
PRINCETON UNIVERSITY, PRINCETON, NEW JERSEY**

PPPL Reports Disclaimer

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Availability

This report is posted on the U.S. Department of Energy's Princeton Plasma Physics Laboratory Publications and Reports web site in Fiscal Year 2003. The home page for PPPL Reports and Publications is: http://www.pppl.gov/pub_report/

DOE and DOE Contractors can obtain copies of this report from:

U.S. Department of Energy
Office of Scientific and Technical Information
DOE Technical Information Services (DTIS)
P.O. Box 62
Oak Ridge, TN 37831

Telephone: (865) 576-8401
Fax: (865) 576-5728
Email: reports@adonis.osti.gov

This report is available to the general public from:

National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, VA 22161

Telephone: 1-800-553-6847 or
(703) 605-6000
Fax: (703) 321-8547
Internet: <http://www.ntis.gov/ordering.htm>

GeV electrons acceleration in focused laser fields after above-threshold ionization

I.Y. Dodin and N.J. Fisch

Princeton Plasma Physics Laboratory, Princeton, NJ 08543

Electrons produced as a result of above-threshold ionization of high- Z atoms can be accelerated by currently producible laser pulses up to GeV energies, as shown recently in [Hu and Starace, *Phys. Rev. Lett.* **88**, art. 245003 (2002)]. To describe electron acceleration by general focused laser fields, we employ an analytical model based on a Hamiltonian, fully relativistic, ponderomotive approach. Analytical expressions are derived and the applicability conditions of the ponderomotive formulation are studied both analytically and numerically. The theoretical predictions are supported by numerical simulations.

PACS numbers: 2.38.Kd, 52.35.Mw, 52.20.Dq, 52.27.Ny, 45.20.Jj

I. INTRODUCTION

Recent advances in high-power laser technology have resulted in the development of petawatt laser systems [1–3]. Those can deliver superstrong laser pulses with intensities of focused output radiation as high as 10^{21} W/cm², which makes them especially attractive from the standpoint of studying the interaction between electromagnetic radiation and matter. One of the aspects of this interaction is the problem of obtaining ultra-relativistic beams of charged particles. Results of numerical simulations [4–11] and experiments [12–15] show that there exists a mechanism of interaction between laser waves and charged particles, which allows particle acceleration up to ultra-relativistic energies. For electrons, the energies achievable with currently available laser systems can be as large as several GeV [4–7, 9].

In this paper, we propose derive analytical expressions describing acceleration of electrons produced as a result of above-threshold ionization up to ultra-relativistic energies [9, 10, 13, 16]. To start, let us introduce the dimensionless parameter $a_0 = eE_0/m\omega c$, conventionally used to characterize the strength of electromagnetic radiation. Here E_0 is the amplitude of the laser electric field, e and m are the electron charge and the mass correspondingly, ω is the frequency of the laser field, and c stands for the speed of light. The value of a_0 can be understood as the ratio of the momentum imparted by the wave field in a single oscillation to mc , meaning that relativistic effects become important at $a_0 \gtrsim 1$. (For the wavelength of the laser radiation equal to 1 μ m, the intensity corresponding to $a_0 \sim 1$ is about 10^{18} W/cm².) Thus, under the influence of currently available laser intensities ($a_0 \lesssim 10^2$), the ultra-relativistic electron oscillatory motion itself has gamma-factor ($\gamma = \mathcal{E}/mc^2$) of the order of a_0 , which though large is still small compared to the experimentally and numerically observed values $\gamma \sim a_0^2$ [4–15].

Under the plane wave approximation, the particle motion is exactly integrable (Sec. III) and the scaling $\gamma \sim a_0^2$ follows naturally. This can also be explained in terms of the ponderomotive formulation (Sec. IV, see also Refs. [10, 11, 17]), which can be applied to experimentally realizable conditions. For focused laser beams used in exper-

iments, the plane-wave model does not capture the important dynamics, because of the possibility for particles to escape from the interaction region in the direction perpendicular to the wavevector. However, ionization event itself cannot be described ponderomotively, because the electron leaves the ion on a time scale small compared to the oscillation period [9].

The surprise in this work is that the ponderomotive formulation can still give good results if the ionization is accounted for properly, and the laser field, in which the acceleration takes place changes, is smooth enough (for the nonrelativistic problem, see Ref. [18]). We show how ionization can be included into a model based on the relativistic ponderomotive approach. For certain cases of interest, we calculate the electron energy gain precisely. Also, we address the applicability conditions of the model, both analytically and numerically. These conditions turn out to be significantly more relaxed than those given in Ref. [11].

The paper is organized as follows. In the next section, we discuss the main principles of particle acceleration by intense laser pulses and formulate the actual analytical model studied in the present paper. In Sec. III, we revise the conventional model of particle acceleration under the plane wave approximation and derive the corresponding expression for the particle energy gain. In Sec. IV, we give the ponderomotive treatment of nonadiabatic particle acceleration in spatially nonuniform laser fields. In Sec. V, we compare our analytical results with those obtained from numerical simulations of particle motion and, in the end, summarize our main ideas.

II. PARTICLE ACCELERATION BY LASER PULSES

In order to describe the mechanism of charged particle acceleration by intense laser pulses, first, consider particle motion in laser waves of uniform or slightly nonuniform intensity. In a smooth laser field with a characteristic spatial scale of the wave envelope L , large compared to the wavelength λ , a charged particle experiences oscillatory “figure-eight” motion (in a linearly polarized wave) or circular motion (in a wave with circular polarization)

[19, 20]. Its guiding center drifts as a quasi-particle with an effective mass $m_{\text{eff}} = m\sqrt{1 + \overline{a^2}}$, where the bar above $a(\eta) = eE(\eta)/m\omega c$ stands for averaging over the phase $\eta = \omega t - \mathbf{k} \cdot \mathbf{r}$ [21–26]. Slow variations of the laser intensity “seen” by the particle result in variations of its effective mass and produce a ponderomotive force, proportional to the intensity gradient. Because of the conservative nature of this force, as the pulse passes over the particle, the particle is decelerated almost precisely down to its initial energy. In this case, the net acceleration appears to be exponentially small with respect to the parameter λ/L . In order to prevent adiabatic deceleration and extract the energy from the laser field *irreversibly*, one needs to violate the adiabatic approximation for particle drift. (See also the discussion on the so-called Lawson-Woodward theorem in Ref. [27] and references cited therein.)

From the technological point, pulses with $L \lesssim \lambda$, for which the net acceleration can be substantial [8, 28], are generally hard (yet possible) to produce. However, fields with sufficiently large *transverse* gradients of average intensity can be obtained by focusing laser radiation into a small spot, which size is only limited by diffraction spreading. The focal waist $w = N\lambda$ (with N often to be of the order of several units) is usually small compared to the longitudinal scale of the focal region $b = kw^2$. Thus, nonadiabatic acceleration in a focal spot can occur primarily due to particle escape in the transverse direction at $N \sim 1$ [4–7]. (For more precise condition, see Sec. IV.) However, in this case, charged particles must be delivered to the interaction region *after* the laser field has been turned on. Otherwise, as the laser intensity “seen” by a charged particle grows slowly, the particle is swept away by the very front of the pulse even before the intensity reaches its maximal value, which results in inefficient acceleration [9].

Additional efforts are necessary to keep charged particles in the focal region until the field reaches its maximal amplitude. In Ref. [9] (see also Refs. [10, 13, 16]), it was proposed to keep the electron trapped by the Coulomb field of a high- Z ion for the time needed for the laser field to reach its maximal intensity. During this time, the bounded electron practically remains at rest but, after the laser field intensity becomes large enough, it is swept off by the wave. After the electron has left the ion, the Coulomb field does not influence the electron dynamics significantly. Thus, the electron can be considered as accelerated in free space, assuming for the initial conditions that, effectively, it starts “seeing” the intense laser field instantaneously, immediately after ionization. The actual value of the ionization potential does not influence the acceleration but rather determines the amplitude of the laser field, at which ionization occurs.

Consider now electron acceleration by an instantaneously switched on laser field [9, 10]. Assume, for clarity, that the laser wave is linearly polarized and propagates in vacuum at the speed of light. On a quarter of the first period of the field “seen” by electron, the wave electric field $\mathbf{E}$ accelerates the electron up to the velocity

$\mathbf{v}$ close to the speed of light in the direction transverse to the wavevector $\mathbf{k}$. The magnetic part of the Lorentz force, $\mathbf{v} \times \mathbf{B}/c$, rotates the electron velocity vector in the direction of $\mathbf{k}$ and, since $v \approx c$ and $E = B$, reduces the transverse force applied to the particle by a factor about $1/\gamma^2 \ll 1$. Further, slower (but yet efficient) acceleration is provided by the electric force. The magnetic force tends to align the electron velocity vector in the direction of $\mathbf{k}$ providing that, in ultra-intensive laser fields, the accelerated electrons remain well-collimated. In other words, though it is the electric field that actually accelerates the particle, it is the magnetic force that allows “storing” the obtained energy in the longitudinal motion. In principle, this mechanism allows particles to be accelerated by purely transverse waves, contrary to the conventional accelerators utilizing the longitudinal component of a wave electric field to accelerate charged particles in the direction of wave propagation (see, e.g., Ref. [27, 29] and references therein).

This mechanism is quite robust, and the acceleration proceeds as long as the particle remains in phase with the wave. In vacuum, where the phase velocity is equal to the speed of light, phase mismatching is inevitable. On the other hand, phase matching can be sustained for arbitrarily long time if the acceleration takes place in a dispersive medium, where the laser wave is slowed down properly [29].

It is important to emphasize that, after a *single* nonadiabatic effect (such as instantaneous ionization) has been introduced into the acceleration scheme, an abrupt field structure in the focal region no longer remains crucial for effective acceleration. Indeed, if the laser intensity “seen” by the particle decreases adiabatically to zero at $t \rightarrow \infty$, the ponderomotive deceleration generally does not bring the particle to rest because the preceding acceleration did not follow the adiabatic law. Therefore, in this case, the particle can retain a significant part of its energy even if leaving the region of interaction with the laser field adiabatically.

In this paper, we consider such smooth laser field profiles (for precise conditions, see Sec. IV, V). We develop a ponderomotive formulation to study electron acceleration after ionization, which we model as an instantaneous jump of the laser intensity “seen” by the electron. The problem of particle acceleration by a *plane* laser pulse yields an exact solution, which we discuss in the next section. As shown in Sec. IV, the ponderomotive formulation generalizing this conventional treatment to the case of spatially nonuniform laser fields, which are relevant to real experiments.

III. ACCELERATION BY PLANE WAVES

The conventional approach to solving the problem of charged particle acceleration by a laser pulse implies approximating the latter by a plane wave [8, 19, 30]. In this case, the problem of particle motion can be solved

exactly for arbitrary polarization and longitudinal profile of the pulse. In this section, we give a simple derivation. A more detailed analysis follows in the next sections.

Consider particle motion under the action of a plane laser wave propagating in vacuum with vector potential given by

$$\mathbf{A}(\mathbf{r}, t) = (mc^2/e) \mathcal{A}(\eta), \quad (1)$$

where $\eta = \omega t - kz$ stands for the phase of the wave, $k = \omega/c$ represents the wavevector, and $\mathcal{A}(-\infty) = 0$. Several exact integrals of particle motion under the action of such a wave can be figured out readily. First, the canonical momentum $\mathbf{P}_\perp = \mathbf{p}_\perp + (e/c)\mathbf{A}$ is conserved because of the symmetry of the laser field with respect to the transverse displacement and equals the initial kinetic momentum of the particle before the interaction $\mathbf{p}_{\perp,0}$. This yields a solution for the kinetic momentum

$$\mathbf{p}(\eta) = \mathbf{p}_0 - (e/c)\mathbf{A}(\eta). \quad (2)$$

Also, because of the fact that the wave field depends on the phase (that is, on the combination $z - ct$, rather than z and t separately), there exists an additional integral of motion given by

$$u = \gamma - p_z/mc = \gamma_0 - p_{z,0}/mc = \text{const}, \quad (3)$$

where $\gamma_0 = \sqrt{1 + (p_0/mc)^2}$ is the initial normalized energy of the particle. The equations for the particle energy γ and the phase η

$$\frac{d\gamma}{dt} = \frac{e}{mc^2} (\mathbf{v} \cdot \mathbf{E}), \quad \frac{d\eta}{dt} = \omega - kv_z, \quad (4)$$

where $\mathbf{E} = -(1/c)(\partial\mathbf{A}/\partial t)$, with Eqs. (2), (3) taken into account, can be put in the dimensionless form

$$\frac{d\gamma}{d\tau} = \frac{1}{2\gamma} \frac{d\hat{\mathcal{A}}^2}{d\eta}, \quad \frac{d\eta}{d\tau} = \frac{u}{\gamma}, \quad (5)$$

where we introduce the dimensionless time $\tau = \omega t$ and the quantity $\hat{\mathcal{A}} = \mathcal{A} - \mathbf{p}_{\perp,0}/mc$. Solving those, one readily gets the function $\gamma(\eta)$ for arbitrary initial conditions and arbitrary shape of the laser pulse:

$$\gamma(\eta) = \gamma_0 + \frac{(\mathcal{A}(\eta) - \mathbf{p}_{\perp,0}/mc)^2 - (\mathbf{p}_{\perp,0}/mc)^2}{2(\gamma_0 - p_{z,0}/mc)}. \quad (6)$$

Let us now precisely point out the connection between $\mathcal{A}$ that enters the above formula and the actual laser electric field $\mathbf{E}$. From Eq. (1), it follows that

$$\mathcal{A}(\eta) = -\frac{e}{m\omega c} \int_{-\infty}^{\eta} \mathbf{E}(\eta') d\eta'. \quad (7)$$

In the case when the field “seen” by the charged particle is smooth enough ($\lambda/L \ll 1$), one has

$$\mathcal{A} \approx \mathbf{a} = \Re \left(-i \frac{e\mathbf{E}}{m\omega c} \right), \quad \mathbf{E} = \Re (\mathbf{E}_0(\eta)e^{-i\eta}), \quad (8)$$

with $\mathbf{E}_0(\eta)$ to be a slow function of η . For such a pulse, $\mathcal{A}(+\infty)$ is exponentially small (with respect to the parameter λ/L), since it represents the average of a rapidly oscillating function. Thus, after such a smooth laser pulse had passed over the particle, the particle energy γ reverts to its initial value γ_0 . This regime corresponds to the adiabatic motion, when no substantial acceleration takes place. As discussed above, in order to provide significant acceleration, one needs to make the laser field amplitude $\mathbf{E}_0(\eta)$ an abrupt function.

Consider the case when the particle initially rests, which will be the case of our further primary interest. Eq. (6) now simplifies to

$$\gamma(\eta) = 1 + \mathcal{A}^2(\eta)/2. \quad (9)$$

Suppose that the laser field “seen” by the particle grows instantaneously at some phase η_0 from zero value up to a finite amplitude. If, later, the pulse amplitude decreases to zero slowly, the value of the integral (7) at $\eta \rightarrow \infty$ is determined by the intensity only at $\eta = \eta_0$. Precisely, for the field (8), Eq. (9) can be written as

$$\gamma(\infty) = 1 + a^2(\eta_0)/2 = 1 + a_0^2 \sin^2(\eta_0)/2. \quad (10)$$

The model of instantaneous increase of the field “seen” by the particle can adequately describe the acceleration following an ionization event [9, 13] and, generally, represents a good qualitative model describing any nonadiabatic effects during acceleration. As mentioned above, the scaling $\gamma \sim a_0^2$ predicted by Eq. (10) matches well with the values of γ actually measured in experiments [12–15].

Using Eqs. (2), (3), and (10), one can also calculate the angular spectrum of accelerated particles (see also Refs. [10, 11, 15]). The angle, at which particle travels after the interaction with a plane laser pulse, is given by $\tan \theta = p_\perp(\infty)/p_z(\infty)$, or $\tan \theta = 2/a$. One can also represent the scattering angle in terms of particle energy, which, in the case $a_0 \gg 1$, leads to $\theta = \sqrt{2/\gamma}$, meaning that a beam of charged particles accelerated up to ultra-relativistic energies by a plane laser pulse must be well collimated.

Consider now what happens if the particle interacts with a train of pulses with equivalent intensity but random phases. (A related problem is the well-known Fermi acceleration of energetic particles in space plasmas where particles are scattered off randomly wandering magnetic shocks of various nature [31, 32].) Throughout the whole acceleration process, u remains an exact integral of particle motion. Thus, averaging over η_0 and assuming the particle remained at rest before acceleration ($u = 1$), one gets that each time the particle interacts with a new laser pulse, on average, it gets the additional energy $\langle \Delta\gamma \rangle = a_0^2/4$. Since this value does not depend on particle energy, the particle is accelerated steadily with uniform rate by the whole wave train, despite the fact that its energy may be changing by orders of magnitude during acceleration. Thus, we conclude that, as a charged

particle interacts with a train of laser pulses, its energy, in average, grows proportionally to the total number of pulses passed over the particle.

IV. PONDEROMOTIVE TREATMENT

The considered treatment of the acceleration process under the approximation of a plane laser pulse represents a useful tool providing a simple estimate for the energy gain obtained by a particle during the interaction with a laser wave. In reality, however, a different configuration of the laser field is of primary interest. In order to get the most efficient acceleration for a given fluence, the radiation is usually focused into a tiny spot, where the field gradients become essential for particle dynamics. As discussed in Sec. II, the particles energy gain depends on the transverse structure of the field because particles primarily escape from the focal region in the direction transverse to the wave propagation. In addition, in focused fields, the longitudinal component of the laser electric field appears, which, though small, can influence the acceleration process substantially [11, 33]. Therefore, more accurate model (rather than the one assuming the plane-wave approximation for a laser pulse) is needed to describe electron acceleration in real experiments.

In this section, we employ a ponderomotive formulation of acceleration of an electron produced as a result of above-threshold ionization in a smooth (though arbitrarily nonuniform) laser field. Again, below we will assume that the field “seen” by the particle is switched on instantaneously at $t = t_0$ (or $\eta = \eta_0$), when the particle remains at rest. Note that, if the spatial scale of the field is large enough (the exact conditions are discussed below), the particle dynamics is adiabatic at any time $t > t_0$. The nonadiabatic “jump” of the laser field intensity at $t = t_0$ can be attributed to proper initial conditions for the *drift* motion, which is superimposed on the *known* laser-driven oscillations and is considered only at $t \geq t_0$. The requirement on these proper initial conditions is that they must provide the particle *total* velocity to be zero at $t = t_0$ (similar approach for nonrelativistic particle energies is discussed in Ref. [18]).

At $t > t_0$, the particle average motion is determined by the effective ponderomotive force [11, 21, 22, 24–26] caused by inhomogeneity of a smooth laser field with the vector potential satisfying Eq. (8). As shown in Refs. [21, 25], the particle drift is described by the Hamiltonian function

$$\mathcal{H}(\mathbf{r}_d, \mathbf{p}_d; t) = \sqrt{m_{\text{eff}}^2 c^2 + p_d^2 c^2}, \quad (11)$$

where $\mathbf{r}_d = \langle \mathbf{r} \rangle$ stands for the guiding center location, $\mathbf{p}_d = m_{\text{eff}} \gamma_d \mathbf{v}_d$ equals the particle *phase*-averaged momentum $\bar{\mathbf{p}}$, $m_{\text{eff}}(\mathbf{r}_d, t) = m \sqrt{1 + \bar{a}^2}$ is the slowly variable effective mass, and $\gamma_d = 1/\sqrt{1 - v_d^2/c^2}$ is the gamma-factor associated with the drift velocity $\mathbf{v}_d$, which coincides with the *time*-averaged particle velocity $\langle \mathbf{v} \rangle$. (The

bar denotes averaging over the phase η , and the angle brackets stand for the time averaging [21].) As follows from the definition, the transverse drift momentum $\mathbf{p}_{d,\perp}$ at $t = t_0$ equals the canonical momentum $\mathbf{P}_\perp = (e/c)\mathbf{A}(\eta_0)$. The value of $p_{d,z}(t = t_0)$ can be readily obtained from [21, 25]

$$\mathcal{H} = m_{\text{eff}} \gamma_d c^2 = m \bar{\gamma} c^2, \quad \bar{\gamma} = \gamma_d \sqrt{1 + \bar{a}^2}, \quad (12)$$

considered together with the expressions $\bar{\gamma} = u + \bar{p}_{z,0}$, $u(t = t_0) = 1$. (Note that, in nonuniform fields, which are not only phase-dependent but may also vary in time and space independently, the quantities u and $\mathbf{P}_\perp$ are subject to slow but substantial variations as the particle travels across the laser field.) Finally, one gets the expressions for the drift momentum and the normalized energy:

$$\begin{aligned} \mathbf{p}_{d,\perp}(t = t_0) &= m c \mathbf{a}, \\ p_{d,z}(t = t_0) &= m c (a^2 + \bar{a}^2)/2, \\ \bar{\gamma}(t = t_0) &= 1 + (a^2 + \bar{a}^2)/2, \end{aligned} \quad (13)$$

where the right-hand side is assumed evaluated at $t = t_0$ (or $\eta = \eta_0$).

Eqs. (13) represent the initial conditions for the particle drift motion, which, at $t > t_0$, can be solved for in the framework of the ponderomotive canonical description with the Hamiltonian function given by Eq. (11). Using those, one can finally formulate the limitations of the ponderomotive treatment. (For the problem of free electron scattering off a laser pulse, those were addressed in Ref. [11].) In addition to the restriction that the amplitude of oscillations must remain small compared to the characteristic scale of the laser field, the particle needs to undergo at least several oscillations before it leaves the interaction region. The number of oscillations can be calculated as the ratio of the time T_{int} spent by the particle within the interaction region to the Doppler-modified period of oscillations $T \sim 2\pi a_0^2/\omega$. Denoting the transverse and the longitudinal scales of the laser field with $L_\perp$ and $L_\parallel$ correspondingly, one gets $T_{\text{int}} = \min\{L_\perp/v_{d,\perp}; L_\parallel/v_{d,\parallel}\}$. For $a_0 \gg 1$, from Eqs. (13), it follows that $v_{d,\perp} \sim c/a_0$ and $v_{d,\parallel} \approx c$. Thus, finally, the condition for the ponderomotive approach to be valid can be written as

$$L_\perp \gg \lambda a_0, \quad L_\parallel \gg \lambda a_0^2 \quad (14)$$

(see also Sec. V). The condition of small amplitude of particle oscillations (compared to $L_\perp$ and $L_\parallel$) can also be shown to coincide with Eqs. (14).

It is important to emphasize that, for the ponderomotive approach to stay valid, the laser pulse may not satisfy the plane wave approximation on the global scale. Note that it is the potential of the field that enters the drift motion equations rather than the field itself. Thus, the possible inhomogeneity of the laser intensity is automatically taken into account in the proposed treatment, and so is the particle acceleration by a small longitudinal

field component, which is hard to take into consideration in the conventional plane-wave model [33].

In the framework of the ponderomotive approach utilizing the concept of a quasi-particle (that is a guiding center drifting with the effective mass m_{eff}), the scaling for the retained energy $\gamma \sim a_0^2$ of an electron produced as a result of ionization becomes apparent. Indeed, consider the initial zero total velocity as a superposition of a drift velocity $\mathbf{v}_d$ in the laboratory frame of reference K and the quiver velocity $\mathbf{v}'$ in the frame K' moving relatively to K at the speed $\mathbf{v}_d$. From Lorentz addition of velocities, one finds for $t = t_0$ that $\mathbf{v}_d = -\mathbf{v}'$. Since the magnitude of a is relativistically invariant [21, 22], it follows that $\gamma_d = \gamma' \sim a_0$, where $\gamma' = 1/\sqrt{1 - v'^2/c^2}$ stands for the relativistic factor of quiver motion. Recall that it is the guiding center with an effective mass $m_{\text{eff}} \sim ma_0$ (rather than the true particle with mass m) that is accelerated up to the velocity v' . Thus, immediately after the laser field is switched on, the guiding center energy $m_{\text{eff}}\gamma_d c^2$ increases up to the value of the order of $mc^2 a_0^2$. On the other hand, as follows from Eq. (12), the guiding center energy coincides with the average energy of the true particle, which yields the predicted scaling $\gamma \sim a_0^2$.

The results obtained in the previous section for charged particles acceleration by a plane wave with $\mathbf{a} = \mathbf{a}(\eta)$ can be readily derived from the proposed ponderomotive formulation. Indeed, consider a particle instantaneously injected into a plane laser wave, which amplitude decreases adiabatically at $\eta \rightarrow \infty$ ($\mathbf{a}(\infty) = 0$). Using $d\mathcal{H}/dt = \partial\mathcal{H}/\partial t$ and $\partial(\overline{a^2})/\partial t = \omega \{d(\overline{a^2})/d\eta\}$, one gets equations similar to Eqs. (5):

$$\frac{d\overline{\gamma}}{d\tau} = \frac{1}{2\overline{\gamma}} \frac{d(\overline{a^2})}{d\langle\eta\rangle}, \quad \frac{d\langle\eta\rangle}{d\tau} = \frac{1}{\overline{\gamma}}. \quad (15)$$

Here $\langle\eta\rangle = \omega t - kz_d$ is the time-averaged phase, which we can approximately use as the argument of a slow function $\overline{a^2}(\eta)$, that stands for the normalized average intensity. From Eqs. (15), the previously obtained formula for the particle net energy gain (10) follows immediately: As follows from Eqs. (15), one has $\overline{\gamma} - \overline{a^2}/2 = \text{const}$, which yields

$$\overline{\gamma}(\infty) = \overline{\gamma}(\eta_0) + \frac{\overline{a^2}(\infty) - \overline{a^2}(\eta_0)}{2} = 1 + \frac{a^2(\eta_0)}{2}, \quad (16)$$

where we substituted Eq. (13) for $\overline{\gamma}(\eta_0)$ and used that $\overline{a^2}(\infty) = 0$.

On the other hand, from the ponderomotive treatment, more results of interest can be derived. For example, suppose the nonuniform intensity profile remains static during a time larger than required for the accelerated particle to leave the region of interaction (e. g., in a focal region). In this case, the phase-average particle energy $\overline{\gamma}$ is conserved, since $\partial\mathcal{H}/\partial t = 0$. Thus, assuming $\overline{a^2}(\infty) = 0$, one gets $\gamma(\infty) = \overline{\gamma}(\infty) = \overline{\gamma}(0)$, meaning that

$$\gamma(\infty) = 1 + \frac{1}{2}(a^2 + \overline{a^2})_0, \quad (17)$$

where the subindex 0 denotes the evaluation of the right-hand side of the formula at $t = t_0$ ($\eta = \eta_0$). The additional term $\overline{a^2}/2$ (compared to Eq. (10)) results from the particle ponderomotive acceleration out of the interaction region, which, for static intensity profile, replaces a similar deceleration by the tail of a plane laser pulse. The detailed structure of the intensity profile appears to be unimportant for the net energy gain because of the conservative property of the ponderomotive force. Also, it is worth noting that, by above-threshold ionization, electrons are primarily produced at the maximum of electric field [9], when the instantaneous magnitude of the vector potential is zero ($a(\eta_0) = 0$). Therefore, electrons produced in the same region of a static laser field at different moments of time eventually must retain approximately the same energy $\gamma(\infty) = 1 + \overline{a^2}_0/2$.

V. DISCUSSION

In this section, we discuss the possibility of ponderomotive acceleration by ultra-intense electromagnetic pulses, available with existing laser systems. We consider particle acceleration both by a plane laser wave and by a focused wave with a static average intensity profile. In these two cases, the energy gain can be obtained analytically, which allows us to compare the numerical results with our theoretical predictions.

To start, let us calculate the actual value of the parameter a_0 , which determines the characteristic energy of accelerated particles. In terms of the wavelength λ (measured in microns) and the laser intensity I (measured in W/cm²), a_0 is given by

$$a_0 = \lambda_{\mu\text{m}} \sqrt{I_{\text{W/cm}^2} / 2.74 \times 10^{18}}. \quad (18)$$

For the sake of definiteness, consider the parameters of laser radiation accepted in Ref. [9], namely, $\lambda = 1.054 \mu\text{m}$ and $I = 8 \times 10^{21} \text{ W/cm}^2$ for the peak field. For those, Eq. (18) yields $a_0 \approx 57$. In the peak field, the maximal possible value of the oscillating electron energy $\gamma(\eta) = 1 + a_0^2(\sin \eta - \sin \eta_0)^2/2$ is therefore $\gamma_{\text{max}} = 1 + 2a_0^2 \approx 6500$ (or $\mathcal{E} \approx 3.3 \text{ GeV}$).

As discussed above, a significant fraction of the electron energy $\gamma \sim \gamma_{\text{max}}$ can be retained by the particle after the interaction, only if the field amplitude “seen” by the particle changes abruptly in time. As in previous sections, we will assume a free electron to be born (for instance, by means of ionization) in a peak laser field rapidly in comparison with the wave period, so that instantaneous increase of the “seen” field represents an adequate approximation.

First, consider the interaction with a linearly polarized plane laser pulse switched on at $\eta = \eta_0$, with intensity decreasing slowly at $\eta \rightarrow \infty$ (see Fig. 1). From Eqs. (10) and (16), it follows that the maximal retained energy is given by $\gamma_{\text{ret}} = 1 + a_0^2/2 \approx 1630$ (or $\mathcal{E} = 0.83 \text{ GeV}$). This value matches precisely with the one obtained in

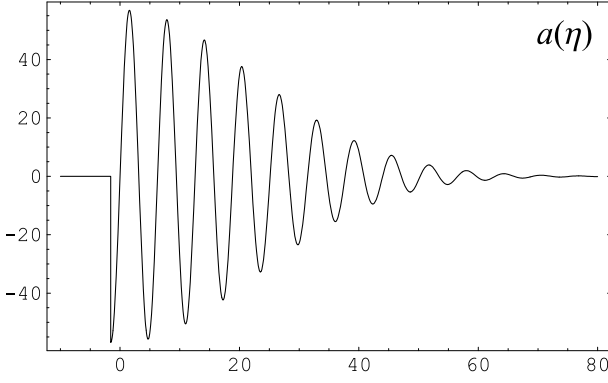


FIG. 1: Normalized vector potential a (“seen” by electron) of a linearly polarized laser pulse versus phase η . The field is switched on at $\eta_0 = -\pi/2$. For the peak intensity $I = 8 \times 10^{21}$ W/cm² and the wavelength $\lambda = 1.054$ μ m, the dimensionless field strength parameter is $a_0 \approx 57$.

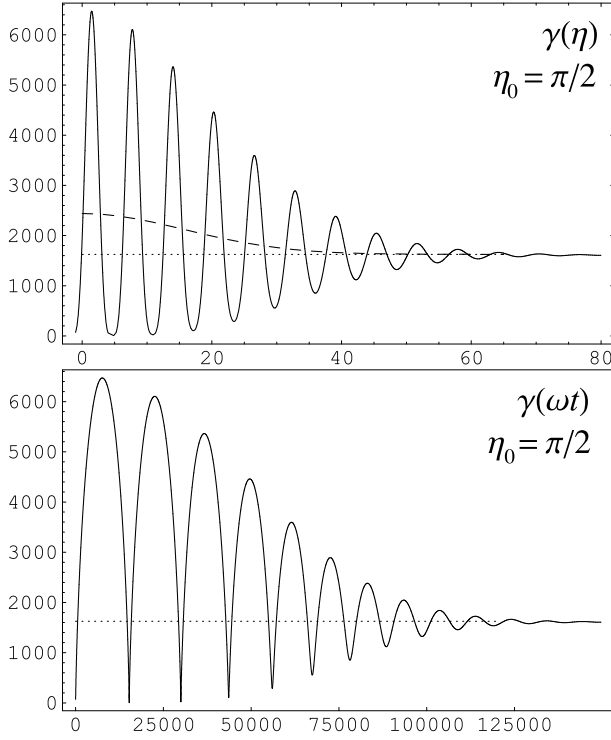


FIG. 2: Electron normalized relativistic energy $\gamma = \mathcal{E}/mc^2$ (solid line), normalized drift energy $\bar{\gamma} = \mathcal{H}/mc^2$ (dashed line), and predicted normalized retained energy $\gamma_{\text{ret}} = 1 + a_0^2/2 \approx 1630$ (dotted line) versus phase η (upper plot) and time $\tau = \omega t$ (lower plot) for electron acceleration by the laser field shown in Fig. 1 ($a_0 = 57$).

numerically. In Fig. 2, the normalized electron energy γ (solid line) is shown versus the normalized time ωt . It can be seen that $\gamma(\infty) = \gamma_{\text{ret}}$, where the asymptotic value is represented by a dotted line. The electron drift energy $\bar{\gamma} = \mathcal{H}/mc^2$ (dashed line) is found to follow Eqs. (15), as predicted above.

Lower acceleration is found for initial phases, other

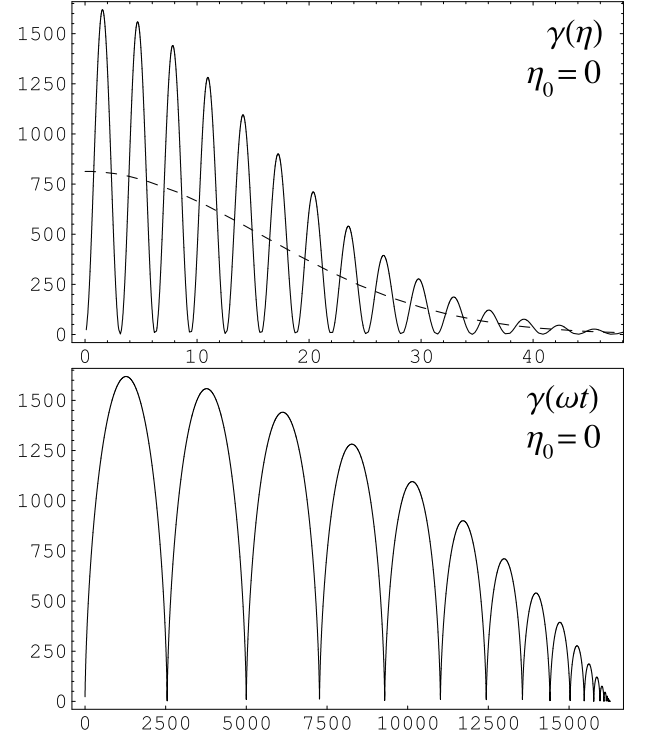


FIG. 3: Electron normalized relativistic energy $\gamma = \mathcal{E}/mc^2$ (solid line) and normalized drift energy $\bar{\gamma} = \mathcal{H}/mc^2$ (dashed line) versus phase η (upper plot) and time $\tau = \omega t$ (lower plot) for electron acceleration by the laser field shown in Fig. 1 ($a_0 = 57$) switched on at $\eta_0 = 0$. The theoretically predicted retained energy is $\gamma_{\text{ret}} = 1$.

than $\eta_0 = \pm\pi/2$. For the same laser field switched on at $\eta_0 = 0$, the electron energy after acceleration is negligible (compared to γ_{max}), exactly as predicted by Eq. (10) (Fig. 3). If the amplitude of the laser pulse decreases adiabatically at $\eta \rightarrow \infty$, substantial acceleration for $\eta_0 = 0$ can only be achieved in spatially nonuniform field (rather than the field which depends on the phase only), when the acceleration is provided by the ponderomotive force pushing the particle out of the region of strong field.

Simulations of electron acceleration of this type were performed for a linearly polarized focused field with $\mathbf{a} = \Re \alpha$, where

$$\alpha_x = a_0 \frac{ib}{b + 2iz} \exp\left(-\frac{k(x^2 + y^2)}{b + 2iz} + i\eta\right), \quad (19)$$

$\alpha_z = (i/k)(\partial\alpha_x/\partial x)$, and the y -component is neglected as one of a higher order with respect to b/λ . Here $b = kw^2$ sets the longitudinal scale of the focal area, and $w = N\lambda$ is the focal waist size. For the factor N , three different values were chosen to simulate different regimes determined by the conditions Eqs. (14), which, for the focal area, can be put in a simple form

$$N \gg a_0. \quad (20)$$

Note that this condition is significantly more relaxed than the one given in Ref. [11], namely $1 - v_z/c \gg 1/kw$,

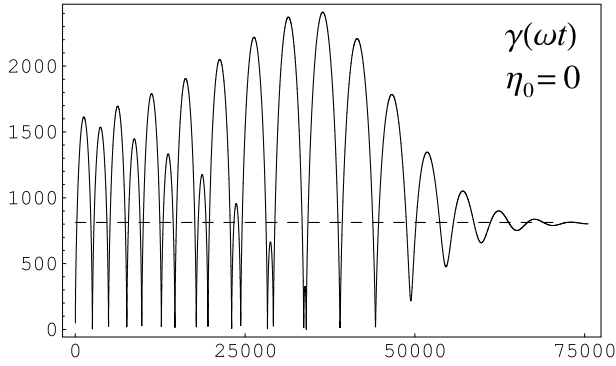


FIG. 4: Electron normalized relativistic energy $\gamma = \mathcal{E}/mc^2$ (solid line) versus time $\tau = \omega t$ for electron acceleration in a focused laser field with the focal waist $w = 140\lambda$ and $a_0 = 57$, instantaneously switched on at $\eta_0 = 0$. Initial particle location is in the center of the focal region. For $\eta > \eta_0$, the average intensity profile is assumed static. The condition (20) is satisfied, so that the ponderomotive description is valid. The normalized drift energy $\bar{\gamma} = \mathcal{H}/mc^2$ (dashed line) is conserved equal to the theoretically predicted retained energy $\gamma_{\text{ret}} \approx 810$.

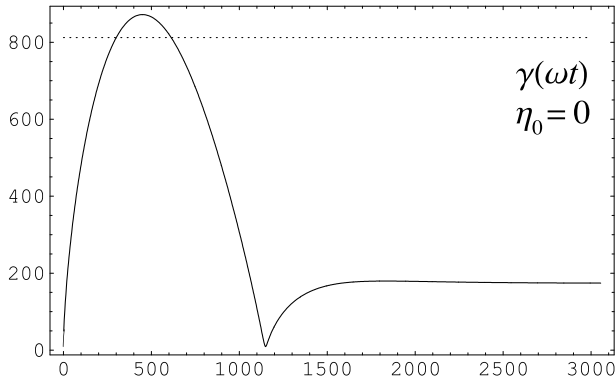


FIG. 5: Electron normalized relativistic energy $\gamma = \mathcal{E}/mc^2$ (solid line) versus time $\tau = \omega t$ for electron acceleration in a focused laser field with the focal waist $w = 10\lambda$ and $a_0 = 57$, instantaneously switched on at $\eta_0 = 0$. Initial particle location is in the center of the focal region. For $\eta > \eta_0$, the average intensity profile is assumed static. The condition (20) is not satisfied, and the electron motion is nonadiabatic. The theoretically predicted retained energy is $\gamma_{\text{ret}} \approx 810$ (dotted line).

which, for ultra-relativistic particles, yields $N \gg a^4$. As we show below, for the considered acceleration of electrons produced as a result of above-threshold ionization, the condition (20) is consistent with results of our numerical simulations.

In Fig. 4, the particle energy γ is shown versus time $\tau = \omega t$ for $N = 140$ and initial particle location $\mathbf{r}(t=0) = 0$, also assumed below. As predicted from the ponderomotive approximation, the phase-average energy is conserved throughout the acceleration process. Thus, the retained energy is $\gamma_{\text{ret}} = 1 + (a^2 + \bar{a}^2)/2$, where a is evaluated at the moment when the field was switched

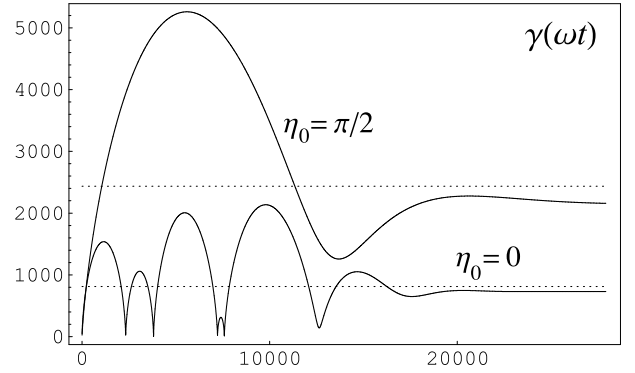


FIG. 6: Electron normalized relativistic energy $\gamma = \mathcal{E}/mc^2$ (solid line) versus time $\tau = \omega t$ for electron acceleration in a focused laser field with focal the waist $w = 50\lambda$ and $a_0 = 57$, instantaneously switched on at phase η_0 ($\eta_0 = 0, \pi/2$). Initial particle location is in the center of the focal region. For $\eta > \eta_0$, the average intensity profile is assumed static. Since $N \sim a_0$, the condition (20) is satisfied only marginally, though the ponderomotive description predicts the energy gain fairly well. The theoretically predicted retained energy (dotted lines) is $\gamma_{\text{ret}} = 1 + (a^2 + \bar{a}^2)/2$, which gives $\gamma_{\text{ret}} \approx 810$ for $\eta_0 = 0$ and $\gamma_{\text{ret}} \approx 2440$ for $\eta_0 = \pi/2$.

on. For the given field and $\eta_0 = 0$, one has $\gamma_{\text{ret}} \approx 810$. Note that despite the fact that the energy plots in Fig. 4 seem to demonstrate nonadiabatic behavior, in fact, the dynamics remains adiabatic: Each period of oscillations contains two peaks of $\gamma(\eta)$, and each of those changes slightly from one period to another. Because of the interchange of the two types of peaks, the function $\gamma(\eta)$ contains multiple Fourier harmonics, but the amplitudes of those evolve slowly. As the interaction region is decreased and the condition (20) is violated, the acceleration becomes nonadiabatic, as, for example, shown in Fig. 5 for $N = 10$. On the other hand, even for $N = 50 \sim a_0$, the ponderomotive treatment predicts the energy gain fairly well, as seen from Fig. 6.

VI. SUMMARY

In summary, we employed an analytical model to describe acceleration of electrons produced as a result of above-threshold ionization up to ultra-relativistic energies. The model is based on a fully relativistic ponderomotive treatment. The Hamiltonian formulation for the electron average motion allows us to simplify the problem of calculating the particle drift trajectory and the particle energy gain. Not only does it allow to reproduce the well-known results of electron acceleration by plane laser pulses, but it also gives the precise energy gain for acceleration in smooth transversely nonuniform fields, for which the conventional plane-wave approximation does not hold. The analytical predictions show a good agreement with the results of our numerical simulations.

The work was supported by the US DOE, under con-

-
- [1] M. Perry, D. Pennington, B. Stuart, G. Tiebohl, J. Britten, C. Brown, S. Herman, B. Golick, M. Kartz, J. Miller, et al., *Optics Lett.* **24**, 160 (1999).
 - [2] S. Hatchett, C. Brown, T. Cowan, E. Henry, J. Johnson, M. Key, J. Koch, A. Langdon, B. Lasinski, R. Lee, et al., *Phys. Plasmas* **7**, 2076 (2000).
 - [3] M. Key, M. Cable, T. Cowan, K. Estabrook, B. Hammel, S. Hatchett, E. Henry, D. Hinkel, J. Kilkenny, J. Koch, et al., *Phys. Plasmas* **5**, 1966 (1998).
 - [4] J. Wang, Y. Ho, Q. Kong, L. Zhu, L. Feng, S. Scheid, and H. Hora, *Phys. Rev. E* **58**, 6575 (1998).
 - [5] P. Wang, Y. Ho, X. Yuan, Q. Kong, N. Cao, A. Sessler, E. Esarey, and Y. Nishida, *Appl. Phys. Lett.* **78**, 2253 (2001).
 - [6] Q. Kong, Y. Ho, J. Wang, P. Wang, L. Feng, and Z. Yuan, *Phys. Rev. E* **61**, 1981 (2000).
 - [7] Y. Salamin and C. Keitel, *Phys. Rev. Lett.* **88**, art. 095005 (2002).
 - [8] Y. Cheng and Z. Xu, *Appl. Phys. Lett.* **74**, 2116 (1999).
 - [9] S. Hu and A. Starace, *Phys. Rev. Lett.* **88**, art. 245003 (2002).
 - [10] A. Maltsev and T. Ditmire, *Phys. Rev. Lett.* **90**, art. 053002 (2003).
 - [11] B. Quesnel and P. Mora, *Phys. Rev. E* **58**, 3719 (1998).
 - [12] P. Bucksbaum, M. Bashkansky, and T. McIlrath, *Phys. Rev. Lett.* **58**, 349 (1987).
 - [13] C. Moore, J. Knauer, and D. Meyerhofer, *Phys. Rev. Lett.* **74**, 2439 (1995).
 - [14] G. Malka and J. Miquel, *Phys. Rev. Lett.* **77**, 75 (1996).
 - [15] G. Malka, E. Lefebvre, and J. Miquel, *Phys. Rev. Lett.* **78**, 3314 (1997).
 - [16] C. Moore, A. Ting, T. Jones, E. Briscoe, B. Hafizi, R. Hubbard, and P. Sprangle, *Phys. Plasmas* **8**, 2481 (2001).
 - [17] P. Wang, J. Hua, Y.Z.Lin, and Y. Ho, *Phys. Lett. A* **300**, 76 (2002).
 - [18] N. Burnett and P. Corkum, *J. Opt. Soc. Am. B* **6**, 1195 (1989).
 - [19] L. Landau and E. Lifshitz, *The classical theory of fields* (Pergamon Press, New York, 1971).
 - [20] E. Sarachik and G. Schappert, *Phys. Rev. D* **1**, 2738 (1970).
 - [21] I. Dodin, N. Fisch, and G. Fraiman, submitted to *JETP Lett.* (2003).
 - [22] J. Moore and N. Fisch, *Phys. Plasmas* **1**, 1105 (1994).
 - [23] T. Kibble, *Phys. Rev.* **150**, 1060 (1966).
 - [24] P. Mora and T. Antonsen Jr., *Phys. Plasmas* **4**, 217 (1997).
 - [25] M. Tokman, *Plas. Phys. Rep.* **25**, 140 (1999).
 - [26] D. Bauer, P. Mulser, and W. Steeb, *Phys. Rev. Lett.* **75**, 4622 (1995).
 - [27] E. Esarey, P. Sprangle, and J. Krall, *Phys. Rev. E* **52**, 5443 (1995).
 - [28] G. Dudnikova, V. Bychenkov, A. Maksimchuk, G. Mourou, J. Nees, S. Bochkarev, and V. Vshivkov, *Phys. Rev. E* **67**, art. 026416 (2003).
 - [29] E. Bochove, G. Moore, and M. Scully, *Phys. Rev. A* **46**, 6640 (1992).
 - [30] F. Hartemann, S. Fochs, G. Lesage, L. Luhmann Jr., J. Woodworth, M. Perry, Y. Chen, and A. Kerman, *Phys. Rev. E* **51**, 4833 (1995).
 - [31] E. Fermi, *Phys. Rev.* **75**, 1169 (1949).
 - [32] R. Blandford and D. Eichler, *Phys. Rep.* **154**, 1 (1987).
 - [33] K. McDonald, *Phys. Rev. Lett.* **80**, 1350 (1998).

External Distribution

Plasma Research Laboratory, Australian National University, Australia
Professor I.R. Jones, Flinders University, Australia
Professor João Canalle, Instituto de Fisica DEQ/IF - UERJ, Brazil
Mr. Gerson O. Ludwig, Instituto Nacional de Pesquisas, Brazil
Dr. P.H. Sakanaka, Instituto Fisica, Brazil
The Librarian, Culham Laboratory, England
Mrs. S.A. Hutchinson, JET Library, England
Professor M.N. Bussac, Ecole Polytechnique, France
Librarian, Max-Planck-Institut für Plasmaphysik, Germany
Jolan Moldvai, Reports Library, MTA KFKI-ATKI, Hungary
Dr. P. Kaw, Institute for Plasma Research, India
Ms. P.J. Pathak, Librarian, Insitute for Plasma Research, India
Ms. Clelia De Palo, Associazione EURATOM-ENEA, Italy
Dr. G. Grosso, Instituto di Fisica del Plasma, Italy
Librarian, Naka Fusion Research Establishment, JAERI, Japan
Library, Plasma Physics Laboratory, Kyoto University, Japan
Research Information Center, National Institute for Fusion Science, Japan
Dr. O. Mitarai, Kyushu Tokai University, Japan
Dr. Jiangang Li, Institute of Plasma Physics, Chinese Academy of Sciences, People's Republic of China
Professor Yuping Huo, School of Physical Science and Technology, People's Republic of China
Library, Academia Sinica, Institute of Plasma Physics, People's Republic of China
Librarian, Institute of Physics, Chinese Academy of Sciences, People's Republic of China
Dr. S. Mirnov, TRINITI, Troitsk, Russian Federation, Russia
Dr. V.S. Strelkov, Kurchatov Institute, Russian Federation, Russia
Professor Peter Lukac, Katedra Fyziky Plazmy MFF UK, Mlynska dolina F-2, Komenskeho Univerzita, SK-842 15 Bratislava, Slovakia
Dr. G.S. Lee, Korea Basic Science Institute, South Korea
Institute for Plasma Research, University of Maryland, USA
Librarian, Fusion Energy Division, Oak Ridge National Laboratory, USA
Librarian, Institute of Fusion Studies, University of Texas, USA
Librarian, Magnetic Fusion Program, Lawrence Livermore National Laboratory, USA
Library, General Atomics, USA
Plasma Physics Group, Fusion Energy Research Program, University of California at San Diego, USA
Plasma Physics Library, Columbia University, USA
Alkesh Punjabi, Center for Fusion Research and Training, Hampton University, USA
Dr. W.M. Stacey, Fusion Research Center, Georgia Institute of Technology, USA
Dr. John Willis, U.S. Department of Energy, Office of Fusion Energy Sciences, USA
Mr. Paul H. Wright, Indianapolis, Indiana, USA

The Princeton Plasma Physics Laboratory is operated
by Princeton University under contract
with the U.S. Department of Energy.

Information Services
Princeton Plasma Physics Laboratory
P.O. Box 451
Princeton, NJ 08543

Phone: 609-243-2750
Fax: 609-243-2751
e-mail: pppl_info@pppl.gov
Internet Address: <http://www.pppl.gov>