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Suppression of the Richtmyer-Meshkov Instability in the Presence of a Magnetic Field

by

Ravi Samtaney

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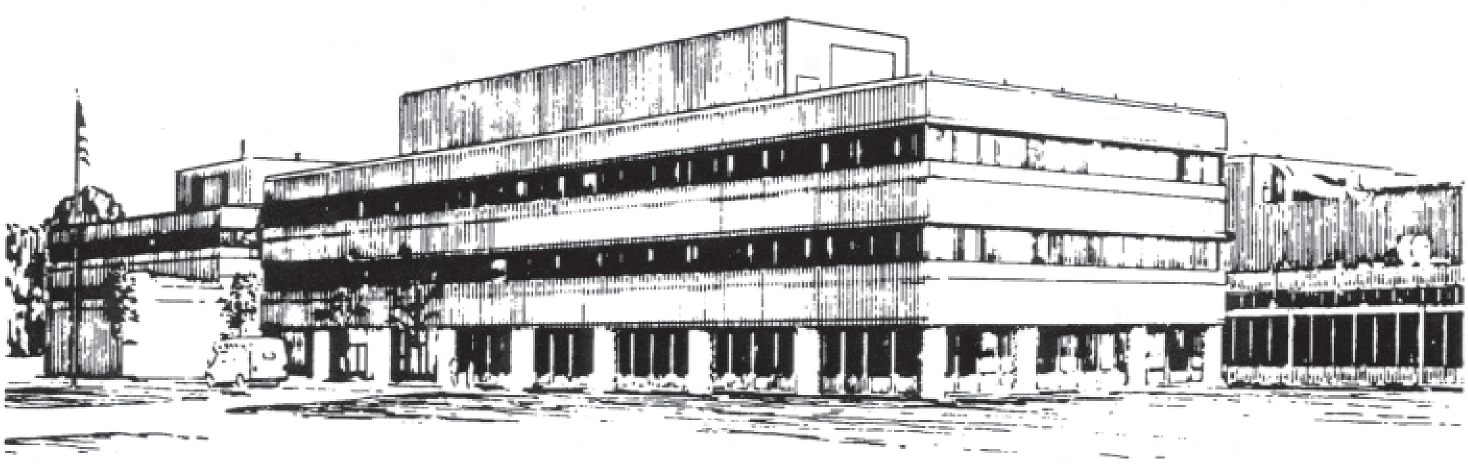

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Suppression of the Richtmyer-Meshkov instability in the presence of a magnetic field

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(May 7, 2003)

Abstract

We present numerical evidence from two dimensional simulations that the growth of the Richtmyer-Meshkov instability is suppressed in the presence of a magnetic field. A bifurcation occurs during the refraction of the incident shock on the density interface which transports baroclinically generated vorticity away from the interface to a pair of slow or intermediate magnetosonic shocks. Consequently, the density interface is devoid of vorticity and its growth and associated mixing is completely suppressed.

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Keywords: Richtmyer-Meshkov instability, MHD, shocks.

The Richtmyer-Meshkov (RM) instability is the subject of extensive experimental, theoretical and computational research [1] due to it's importance in technological applications such as inertial confinement fusion, as well as astrophysical phenomena such as supernova blast waves interacting with surrounding matter. A linear stability analysis was performed originally by Richtmyer [2], followed by experimental confirmation by Meshkov [3]. Richtmyer's work, which deals with the interaction of a shock wave with a perturbed contact discontinuity separating gases of different densities, concluded that the perturbations on the contact discontinuity grew linearly with time. Nonlinearly, it is found both experimentally and computationally that the perturbations grow as a power law in time. The determination of a universal power law exponent is still an area of active research. A related hydrodynamic instability is the Rayleigh-Taylor instability wherein perturbations at a fluid interface grow exponentially in time during the linear phase [4]. It is also well-known that the linear growth rate of the Rayleigh-Taylor instability with constant acceleration is mitigated at high-wave numbers in the presence of a magnetic field [4]. The effects of a magnetic field on the growth of the Richtmyer-Meshkov instability have not yet been investigated. In the ensuing paragraphs, we demonstrate, via state-of-the-art numerical simulations, that the growth of the Richtmyer-Meshkov instability is suppressed in the presence of a magnetic field.

To set the stage of this demonstration, we first make several simplifying assumptions. First, we assume that the medium under investigation is a conducting fluid which is further assumed to be quasi-neutral, i.e., the number density of charged ions and electrons is the same. A second assumption is that diffusive, resistive and heat conduction time scales are much longer than the Alfvén and convective time scales. Under these assumptions, the mathematical model describing the evolution are the equations of ideal magneto-hydrodynamics (MHD) which are written in conservation form as,

$$\frac{\partial U}{\partial t} + \frac{\partial F_j(U)}{\partial x_j} = 0,$$

where the solution vector $U \equiv U(x_i, t)$ is

$$U = \{\rho, \rho u_i, B_i, e\}^T.$$

Here ρ is the density, p is the pressure, u_i is the velocity, B_i is the magnetic field, and $e = p/(\gamma - 1) + 1/2(\rho u_k u_k + B_k B_k)$ is the total energy per unit volume. The flux vectors $F_j(U)$ are

$$F_j(U) = \left\{ \begin{array}{c} \rho u_j \\ \rho u_i u_j + p\delta_{ij} + \frac{1}{2} B_k B_k \delta_{ij} - B_i B_j \\ u_j B_i - B_j u_i \\ \left(e + p + \frac{1}{2} B_k B_k\right) u_j - B_i u_i B_j \end{array} \right\}.$$

An additional constraint which must be satisfied is $\nabla \cdot \mathbf{B} = 0$. Finally, we assume the domain to be two dimensional. The above equations are solved using the 8-wave upwinding formulation [5] with an unsplit upwinding method [6]. The solenoidal property of the magnetic field is enforced at each time step using a projection method which is solved using a multigrid technique. We further use adaptive mesh refinement (AMR) of the Berger-Colella type [7] using the Chombo framework [8]. Details of the numerical method will be presented elsewhere.

The physical domain is $[-2, 6] \times [0, 1]$, discretized with a base mesh of 256×32 mesh points and three levels of mesh refinement with refinement ratio of 4 in each direction, yielding an effective uniform mesh resolution of 16384×2048. The refinement criterion is $|\nabla \rho| > 0.2\rho_0/W$, where ρ_0 is the unshocked gas to the left of the interface and W is the half-width of the sawtooth perturbed interface. The physical setup and boundary conditions are depicted schematically in Figure 1. A shock propagating from left to right is initialized at $x = -0.2$ which is upstream of the density interface whose lower end is initialized at $x = 0$. The principal parameters are the strength of the incident shock characterized by it's Mach number M, the density ratio across the interface η, the angle between the incident shock and the density interface θ, and the non-dimensional strength of the magnetic field written as $\beta = 2p_0/B_0^2$. The four-tuple (M, η, θ, β) completely characterizes the problem. The magnetic field is initially chosen to be $B(x, y, t = 0) = (B_0, 0)$, i.e., initially the magnetic field is uniform in the (x, y) plane and perpendicular to the incident shock front. This is chosen so that the hydrodynamics is decoupled from the magnetic field until the propagating incident shock strikes the interface at which time two-dimensionality sets in and the magnetics and the hydrodynamics get coupled. Numerical results shown below are for parameters $M = 2$, $\theta = 45^o$, $\eta = 3$ and $\beta^{-1} = 0$ (no magnetic field) and $\beta^{-1} = 0.5$ (magnetic field present).

A time sequence of the density field is shown in Figure 2 for the non-magnetic RM instability (Figure 2:a1,b1,c1), as well as the evolution in the presence of a magnetic field (Figure 2:a2,b2,c2). The top two images depict the early refraction process of the incident shock at the contact discontinuity which are described in detail later. During this early time, the interface is compressed by the incident shock and baroclinic vorticity generation takes place. The middle two images (Figure 2:b1,b2) show the development of the instability at a later time $t \approx 1.8$. In the absence of the magnetic field ($\beta^{-1} = 0$), the interface, which is a vortex layer, rolls-up as expected for the usual Richtmyer-Meshkov instability. In the presence of the magnetic field ($\beta^{-1} = 0.5$), the interface remains smooth and no evidence of roll-up is observed. The bottom two images show the density field at $t \approx 3.3$. The interface, in the absence of a magnetic field, has grown in extent and shows considerable amount of mixing (which is due to numerical viscosity), with an average increase in the interface extent by 54% compared with initial unshocked extent. On the other hand, in the presence of the magnetic field, the average extent of the interface shows no difference between this time and the earlier one.

Hawley and Zabusky [9] have given a vortex dynamical interpretation regarding the growth of the interface, in which the baroclinic vorticity generation on the density interface drives the instability. This is easily seen by examining the vorticity evolution equation,

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + (\mathbf{u} \cdot \nabla)\boldsymbol{\omega} = -\boldsymbol{\omega}(\nabla \cdot \mathbf{u}) + (\boldsymbol{\omega} \cdot \nabla)\mathbf{u} + \frac{\nabla\rho \times \nabla p}{\rho^2} + \nabla \times (\rho^{-1}\nabla \times \mathbf{B} \times \mathbf{B}).$$

The term $\nabla\rho \times \nabla p$ is the baroclinic source term and is solely responsible for the generation of vorticity on the interface during the transit phase of the incident shock over the interface. The vorticity on the interface causes the interface to roll-up and feeds the growth of the instability. The last term in the above equation does not contribute to the vorticity generation during the shock transit phase. In Figure 3 we plot the total circulation in the domain as a function of time. The initial rapid rise is the baroclinic generation of circulation which is about the same with or without the presence of the magnetic field, and yet the interface shows no evidence of instability in the presence of the magnetic field. This fact can be reconciled by examining the details of the shock refraction process on the density interface.

For $\beta^{-1} = 0$, the details of the refraction process are shown in Figure 4-a. For the chosen parameters, the incident shock (I) bifurcates into a reflected (R) and a transmitted shock (T) (i.e., the refraction is regular). The shocked interface is a vortex sheet (VS) or rather a vortex layer due to numerical diffusion, which essentially drives the instability. In the presence of the magnetic field, the shock refraction gives rise to MHD shocks. The details of the refraction are shown in Figure 4-b. For the chosen parameters, there is a pair of reflected shocks and a pair of transmitted shocks. RS and RF are the slow reflected and fast reflected magnetosonic shocks, respectively; whereas TS (either slow or intermediate shock) [1] and TF (fast shock) are the transmitted magnetosonic shocks. Both reflected shocks, TS and RS, are closer to the density interface than their fast counterparts. It is well known that MHD shocks support velocity slip on the shock front [12]. Furthermore, it is also well known that contact discontinuities are unable to sustain velocity slip in MHD if they are initially vorticity free(Section 9 pages 35-37 in Reference [13]). While the baroclinic

[1] Recently, Wheatley and Pullin [10] performed a local shock-polar analysis (for parameters $M = 2,\ \eta = 3,\ \theta = 45^o,\ \beta^{-1} = 0.4$) in the neighborhood of the point where all discontinuities meet. In their analysis, the Alfvènic Mach number ahead of TS was 1.00488. This implies that TS is an intermediate shock of the "2-4" variety [11]. Analysis of our numerical results, wherein shock fronts are smeared due to shock-capturing, are not conclusive due to the near sonic Alfvènic Mach number ahead of TS. Hence, we leave the possibility open that in our simulations TS is either an intermediate shock, or a slow shock which is close to being a "switch-off" shock [11].

vorticity generation during the shock refraction phase is unaffected by the magnetic field, the vorticity migrates away from the contact discontinuity and a bifurcation occurs: the vortex sheet splits and the vorticity is transported away from the density interface on to the slow or intermediate MHD shocks (See Figure 5:a-b). Furthermore, it is interesting to note that the MHD shock fronts, which in our simulations are themselves stable to perturbations, become sites of current sheets (Figure 5:c). A complete taxonomy of the shock refraction patterns at the density interface in the presence of a magnetic field is beyond the scope of the present work.

In conclusion, we have shown, via numerical simulations, that the growth and associated mixing of the Richtmyer-Meshkov instability is suppressed by the presence of a magnetic field. The baroclinic generation of vorticity remains the same with or without the magnetic field. However, in the presence of the magnetic field the vorticity is transported away from the density interface effectively suppressing it's growth. This has obvious consequences for turbulent interfacial mixing which occurs in the classical Richtmyer-Meshkov instability at late times. It also suggests that perhaps externally applied magnetic fields could decrease mixing in inertial confinement fusion. We have examined the influence of the magnetic field for one parameter ($\beta^{-1} = 0.5$) in detail. Other numerical simulations were performed for $\beta^{-1} = 5, 0.05$ which are not reported here. The instability is completely suppressed for the higher $\beta^{-1} = 5.0$ value. For the smaller $\beta^{-1} = 0.05$ case, we observed the bifurcation into pairs of slow and fast shocks, and the fact that the slow shocks were in very close proximity to the interface lead to an entrainment effect, i.e., the vorticity on the slow shock fronts is close enough to the interface that the vortex layers influence the interface motion by local churning of the interface causing the instability to grow, albeit at a smaller rate than in the complete absence of the magnetic field.

Lastly, we present a few conjectures. In an experiment, a magnetic field perpendicular to the incident shock front can be created using coils which carry current azimuthally in a shock tube of circular cross-section. Even in three dimensions, the interface will be devoid of vorticity as long as there is a non-zero component of the magnetic field normal to the density interface. Therefore, we conjecture that the growth of the instability will be suppressed even in three dimensions, and in spherical/cylindrical geometries encountered in inertial confinement fusion. Upwind numerical methods have an intrinic viscosity and resitivity, so it is conjectured that inclusion of viscous and resistive terms in our mathematical model will have a negligible influence on the suppression of the instability. Whether or not the inclusion of the Hall effect will play a significant role in changing the main suppression effect observed is part of future work.

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List of Figures

FIGURES

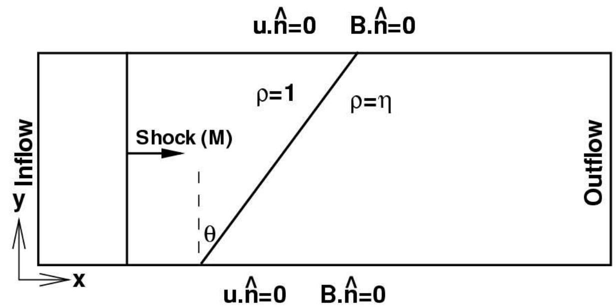

FIG. 1. Setup of the physical domain and boundary conditions for the Richmyer-Meshkov simulations. The magnetic field, if present, is initially aligned along the x-direction. The initial pressure in the unshocked regions is $p_0 = 1$.

Fig. 1, Samtaney, Physics of Fluids

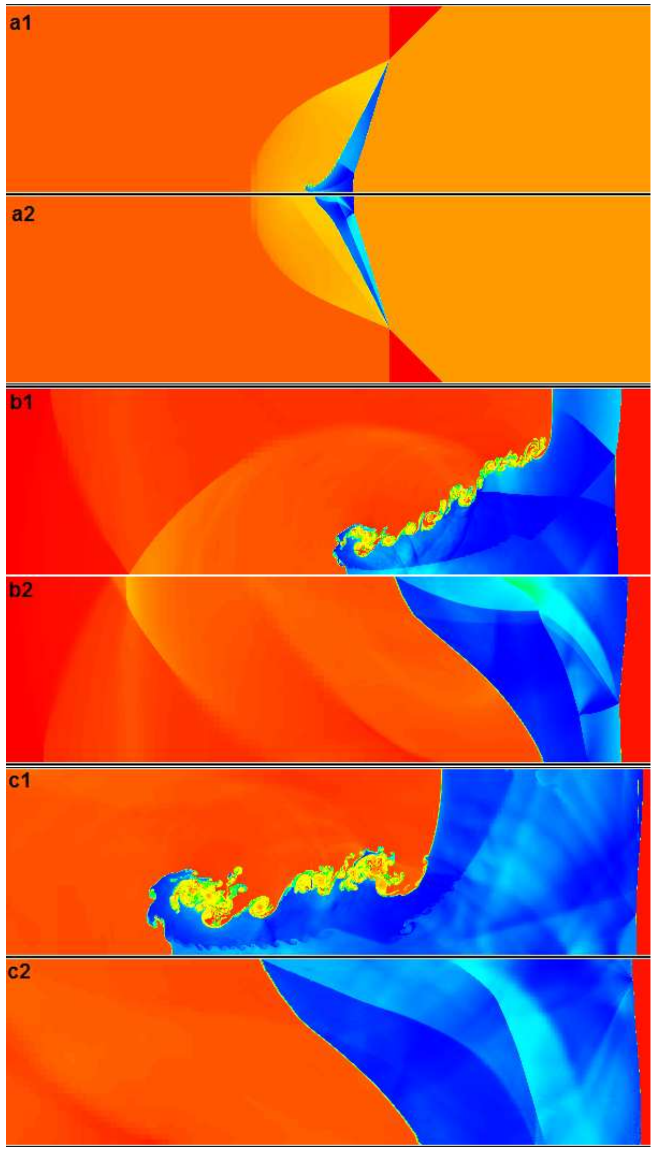

FIG. 2. Time sequence of the density field at times $t = 0.385$ (a1,a2), $t = 1.82$ (b1) ,$t = 1.86$ (b2), $t = 3.33$ (c1), and $t = 3.37$ (c2). The images with nonzero magnetic field $\beta^{-1} = 0.5$ (a2, b2, c2) are reflected about the x-axis. Domain shown is $[-1.38, 2.09] \times [0, 1]$ (a1,a2), $[-0.31, 3.16] \times [0, 1]$ (b1,b2) and $[2.06, 5.53] \times [0, 1]$ (c1,c2). Simulation parameters: $M = 2$, $\eta = 3$, $\theta = 45^o$.

Fig. 2, Samtaney, Physics of Fluids

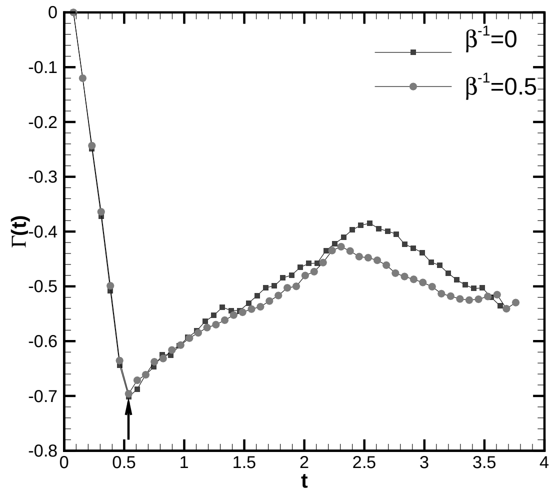

FIG. 3. Time history of the total circulation in the domain. The arrow indicates the time at which the incident shock has completely traversed the interface. Simulation parameters: $M = 2$, $\eta = 3$, $\theta = 45^o$. Plots shown for magnetic field absent $\beta^{-1} = 0$ (square) or present $\beta^{-1} = 0.5$ (circle).

Fig. 3, Samtaney, Physics of Fluids

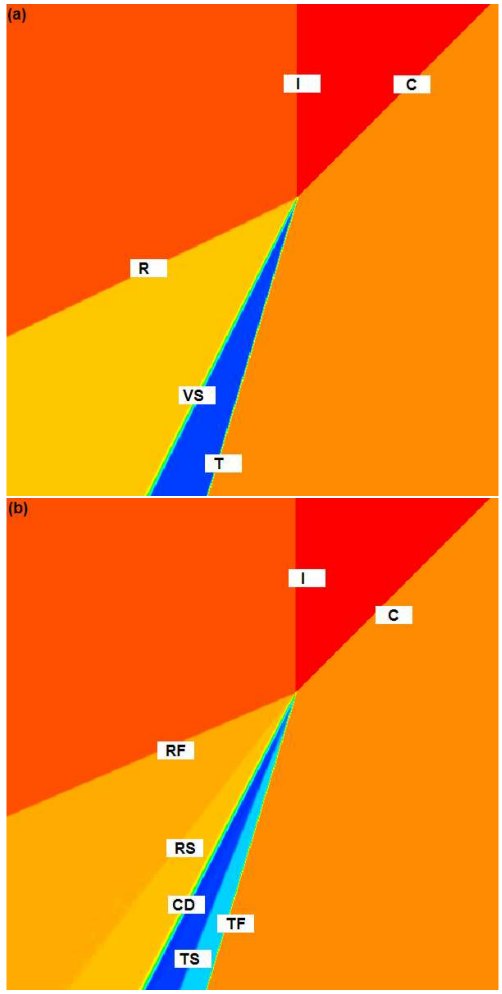

FIG. 4. (a) Refraction of the incident shock (I) at the contact discontinuity (C) gives rise to a vortex sheet (VS), a reflected shock (R) and a transmitted shock (T) in the absense of a magnetic field ($\beta^{-1} = 0$), and (b) The refraction pattern in the presence of a magnetic field ($\beta^{-1} = 0.5$): (I) is the incident shock, (C) is the unperturbed contact, (CD) is the shocked contact (deviod of vorticity), (RS) and (RF) are the slow and fast reflected magnetosonic shocks, respectively; (TS) and (TF) are the slow and fast transmitted magnetosonic shocks, respectively. The density field in (a) and (b) is shown in the domain $[0.55, 0.81] \times [0.55, 0.81]$ at time $t = 0.385$. Simulation parameters: $M = 2$, $\eta = 3$, $\theta = 45^o$.

Fig. 4, Samtaney, Physics of Fluids

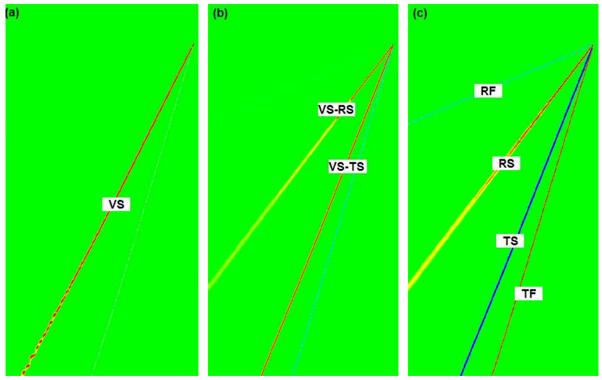

FIG. 5. Vorticity field at $t = 0.385$ (a) $\beta^{-1} = 0$ and (b) $\beta^{-1} = 0.5$. (c) Current field for $\beta^{-1} = 0.5$. Baroclinic generation of vortex layer (VS) during shock refraction on the contact discontinuity shown in (a) bifurcates into (b) two vortex layers (VS-RS and VS-TS) in the presence of a magnetic field which is accompanied by current layers on the MHD shock fronts shown in (c). Domain shown is $[0.40, 0.72] \times [0.16, 0.78]$. Simulation parameters: $M = 2$, $\eta = 3$, $\theta = 45^o$.

Fig. 5, Samtaney, Physics of Fluids

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