PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76CH03073

PPPL-3759
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PPPL-3759

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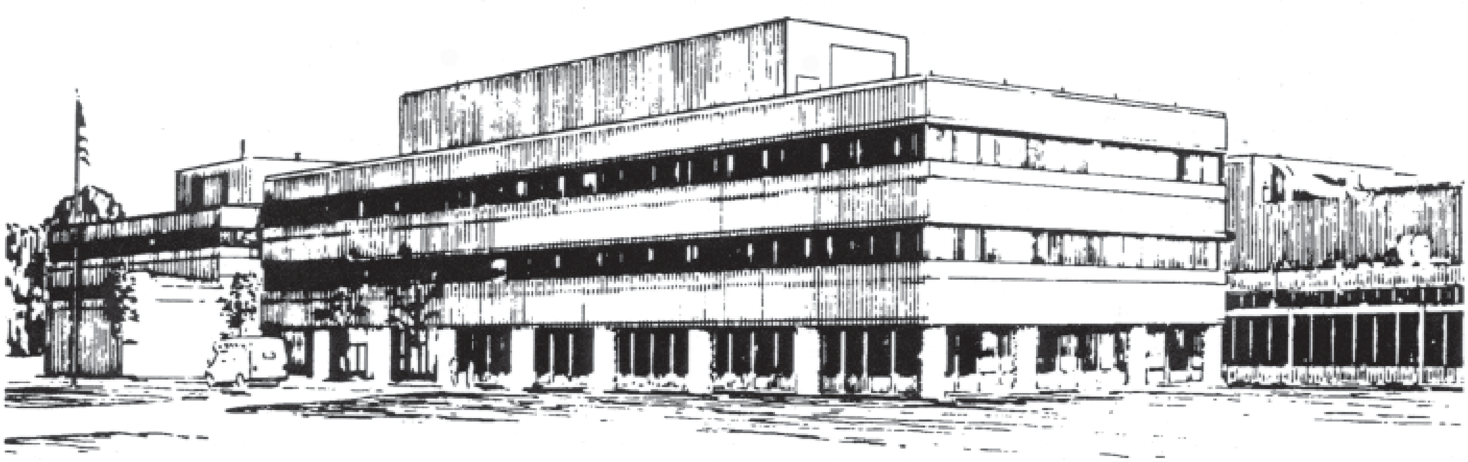
by

E.V. Belova, R.C. Davidson, H. Ji, and M. Yamada

October 2002



PRINCETON PLASMA PHYSICS LABORATORY
PRINCETON UNIVERSITY, PRINCETON, NEW JERSEY

2002 US-Japan Workshop "New directions in controlling and sustaining compact toroids"
September 9-11, 2002, Osaka, Japan

Nonlinear and Non-ideal Effects on FRC Stability

E. V. Belova, R. C. Davidson, H. Ji, M. Yamada
Princeton Plasma Physics Laboratory, P. O. Box 451, Princeton, NJ 08543, USA
E-mail: ebelova@pppl.gov

Abstract

New computational results are presented which advance the understanding of the stability properties of the Field-Reversed Configuration (FRC). We present results of hybrid and two-fluid (Hall-MHD) simulations of prolate FRCs in strongly kinetic and small-gyroradius, MHD-like regimes. The $n = 1$ tilt instability mechanism and stabilizing factors are investigated in detail including nonlinear and resonant particle effects, particle losses along the open field lines, and Hall stabilization. It is shown that the Hall effect determines the mode rotation and change in the linear mode structure in the kinetic regime; however, the reduction in the growth rate is mostly due to the finite Larmor radius effects. Resonant particle effects are important in the large gyroradius regime regardless of the separatrix shape, and even in cases when a large fraction of the particle orbits are stochastic. Particle loss along the open field lines has a destabilizing effect on the tilt mode, and contributes to the ion spin up in toroidal direction. The nonlinear evolution of unstable modes in both kinetic and small-gyroradius FRCs is shown to be considerably slower than that in MHD simulations. Our simulation results demonstrate that a combination of kinetic and nonlinear effects is a key for understanding the experimentally observed FRC stability properties.

1. Introduction

The field-reversed configuration (FRC) is an innovative confinement approach that offers a unique fusion reactor potential because of its compact and simple geometry, translation properties, and high plasma beta. One of the most important issues is FRC stability with respect to low-n (toroidal mode number) MHD modes. According to an empirical scaling, based on the experimental data for prolate FRCs, stability with respect to global MHD modes is observed for $S^*/E \lesssim 3 - 4$ [Tuszewskii,1998]; where E is the separatrix elongation, and S^* is a kinetic parameter which measures the number of thermal ion gyro-radii in the configuration (equals to the ratio of the separatrix radius to the ion skin depth).

Despite more than two decades of efforts, these results has not been explained theoretically. A number of numerical calculations utilizing various physical models find that the $n = 1$ tilt mode remains linearly unstable in the prolate configurations even in highly kinetic regimes: $S^*/E \sim 1$ (for references, see [1,2]). Our recent linearized hybrid simulations have shown a significant reduction in the growth rates due to kinetic effects, but not a complete linear stability [1]. However, in the nonlinear simulations, the saturation of the $n = 1$ tilt mode instability has been found [3], and the unstable configuration has been shown to evolve nonlinearly into a new equilibrium with smaller S^*, larger E and the increased separatrix beta. The nonlinear stabilization of the $n = 1$ tilt mode explains the observation in the low S^* experiments of initial

$n = 1$ tilt motion that does not result in a total loss of confinement [4]. However, the reported FRC stability for larger values of S^* has not been explained so far.

Here we present new results of hybrid and two-fluid (Hall-MHD) simulations of the $n = 1$ tilt mode in prolate FRCs. The linear instability mechanism and the nonlinear evolution of unstable modes, as well as the effects of the particle loss along the open field lines, and Hall stabilization have been studied using a 3D nonlinear hybrid and MHD simulation code (HYM) [1].

2. Linear stability

In order to assess the importance of different stabilizing factors and driving forces on the $n = 1$ tilt mode, we have focussed on the three kinetic effects: finite Larmor radius (FLR), Hall effects, and resonant ion effects. The first two are stabilizing, whereas the third one can be destabilizing, and tends to obscure the FLR and Hall stabilization in fully kinetic calculations. Two classes of equilibria have been considered: equilibria with an arbitrary chosen pressure profile, which, for large elongations, usually corresponds to a configuration with a racetrack-like separatrix shape; equilibria with large E and an elliptic separatrix shape for special pressure profile as proposed by Barnes [5].

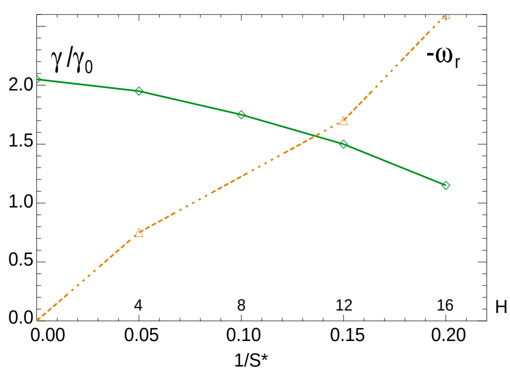


Figure 1: *Growth rate and real frequency of the tilt mode from Hall-MHD simulations with E=6.*

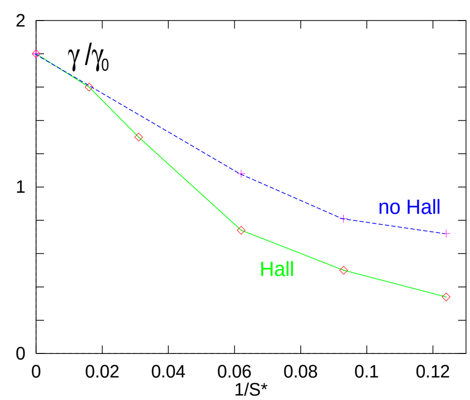


Figure 2: *Growth rate of the tilt mode from hybrid simulations with Hall term (solid line) and without Hall term (dashed line) for E=4.*

A. Hall effects

To isolate Hall effects from other kinetic effects, we have performed two-fluid (Hall-MHD) simulations of the $n = 1$ tilt mode. The MHD version of HYM code has been modified to include the Hall term in the Ohm's law, and sub-cycling in the induction equation has been used to insure the numerical stability. Calculations with E=6 and elliptical separatrix shape show a reduction of the tilt mode growth rate for small S^* (Fig. 1) and a significant change in the mode structure. As S^* decreases, the tilt mode becomes more localized, both radially and in the axial directions, compared to that of the MHD. The unstable mode has a negative real frequency, and rotates in the direction opposite to that of the equilibrium current. Since the reduction in the linear growth rate in the strongly kinetic regime, $S^*/E \sim 1$, is rather modest (50% at most), Hall stabilization alone cannot account for the experimentally observed stability. Similar results for different equilibria were obtained in [6].

B. FLR effects

FLR effects can be studied within fluid description by including gyroviscous force in the ion momentum equation [7]. However, this description implies an FLR expansion, which is not valid in the large-Larmor-radius (small-S^*), kinetic cases. In order to separate FLR effects from Hall-term effects, we have chosen to perform hybrid simulations including full ion dynamics, but turning off the Hall term in the Ohm's law. Figure 2 shows the results of these simulations for an equilibrium with E=4 and elliptic separatrix shape. It is seen that the strong reduction in the growth rate for $S^* \lesssim 8$ is mostly due FLR effects. In the simulations without Hall term, mode rotates in the positive (diamagnetic) direction, and the structure of the mode is similar to that of the MHD. Full hybrid simulations including Hall term, on the other hand, show mode structure similar to that of the Hall-MHD mode, and the real frequency is positive for large S^*, and negative for small S^*. Thus a comparison of the results of the Hall-MHD and hybrid simulations with and without Hall term has shown that the Hall term is responsible for the mode rotation in the negative direction and the change in the linear mode structure, however, the reduction in the growth rate is mostly due to finite Larmor radius effects.

C. Profile effects

A number of MHD calculations [1,5,7] have shown that the commonly assumed scaling of the tilt instability growth rate with elongation: $\gamma_{mhd} \sim 1/E$ is applicable only to configurations with elliptic shape. For racetrack equilibria, growth rate becomes independent of elongation, when the elongation is large enough: $E \gtrsim 5$. This difference is related to the differences in the mode structure: in elliptic configurations, tilt mode is always a global mode with a maximum amplitude near the midplane, whereas in the racetrack configurations, tilt mode is localized near the ends, where the curvature is large. The scaling of the MHD growth rate with elongation defines a condition for the transition to the kinetic regime [7]. In particular, FLR effects significantly modify the growth rate if $\gamma_{mhd}/\omega^* \lesssim 1$, where ω^* is diamagnetic frequency. This condition depends on the ratio S^*/E when elliptic configurations are considered, but it is a function of S^* alone for the racetrack equilibria.

We have studied the effects of elongation in kinetic FRCs for a range of S^* values using linearized hybrid simulations. Figure 3 shows the dependence of the normalized growth rate on parameter S^*/E for three different elliptic equilibria with $E = 4, 6, 12$. It is seen that the growth rate is indeed a function of S^*/E at least for $S^*/E > 2$. (In contrast, this scaling is not observed for FRCs with racetrack-like separatrix shape.) Vertical dashed line in Fig. 3 denotes the empirical experimental stability threshold at $S^*/E = 3.5$. The region to the right of this line corresponds to a parameter regime where experimental stability has been reported. It is seen that linear theory, while predicting reduced instability, does not explain the observed macroscopic stability threshold.

As can be seen from Fig. 3, the growth rate scaling with the S^*/E parameter is not valid in the strongly kinetic regime, when $S^*/E < 2$. The deviation from this scaling indicates that effects other than two-fluid effects are important for highly kinetic FRCs.

D. Resonant particle effects

Numerical simulations of the tilt instability for various FRC equilibria and separatrix shapes have shown that common to all cases is the change of the instability from a reactive one (with $\gamma \gg |\omega|$) into a weakly unstable one with $\gamma/|\omega| \ll 1$ as S^*/E decreases. The linear growth rate remains finite even for $S^*/E \sim 1$ cases due to resonant particle effects [8]. The mode

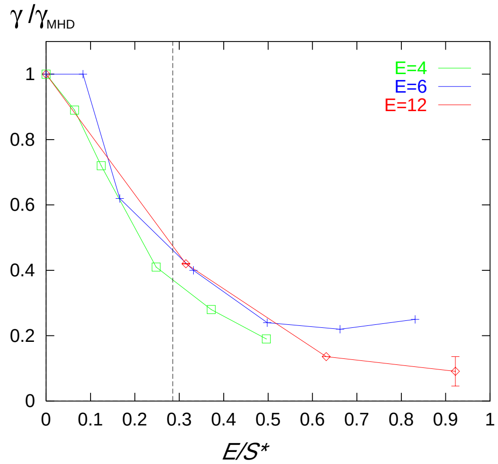


Figure 3: *Growth rate of the tilt mode from hybrid simulations with elliptic separatrix and different elongations:* $E = 4, 6, 12$.

with $\omega_r < 0$ (in ion frame) is a negative energy wave, which can be driven unstable via Landau damping on resonant ions when $\partial F_0/\partial\varepsilon < 0$. The resonant ions satisfy condition:

$$\Omega - \omega = l\omega_\beta, \tag{1}$$

where Ω is particle toroidal rotation frequency, ω_β is axial betatron frequency, and l is an odd integer. The growth rate of this resonant instability depends on the number of resonant particles, the slope of the ion distribution function F_0, and the stochasticity of ion orbits.

Our hybrid simulations employ the delta-f method for numerical noise reduction. In this method, equilibrium ion distribution function is assumed to be known analytically (in this paper it is taken $F_0 = n_0 \exp(-\varepsilon/T_0)$, where $\varepsilon = m_i v^2/2 + e\phi_0$ - particle energy), and equation for the perturbed distribution function $\delta F = F - F_0$ is integrated along equilibrium particle trajectories. Each simulation particle is assigned weight $w = \delta F/F_0$, which satisfy equation:

$$\frac{dw}{dt} = -(\mathbf{v}\cdot\delta\mathbf{E})\frac{\partial(\ln F_0)}{\partial\varepsilon}. \tag{2}$$

Large-weight particles indicate region in phase space, where the change in the ion distribution function is the largest, and resonances are likely to occur.

Analysis of ion orbits shows that in the configurations with large S^* and elliptic separatrix shape, the ion distribution in (Ω,ω_β) frequencies is narrow, and very few particles can satisfy the condition (1). Also, in this regime most of ion orbits are stochastic. Figure 4 shows scatter plots of particles in ($w = \delta F/F_0$, $(\Omega - \omega)/\omega_\beta$) plane from four simulations with elliptic separatrix, $E = 6$, and different values of S^*. For $S^*/E \gtrsim 6$ (MHD-like regime, Fig. 4a,b), very few particles can be at resonance with the mode. As the configuration size (S^*) reduces, particle distribution in frequencies broadens, and number of particles at resonance increases (fig. 4c,d). In addition, at smaller S^*, larger fraction of ions have regular orbits.

Simulation results show that the resonant particles effects are negligible in the small-Larmor-radius regime, but then the configuration is MHD unstable. In the kinetic regime, the MHD tilt

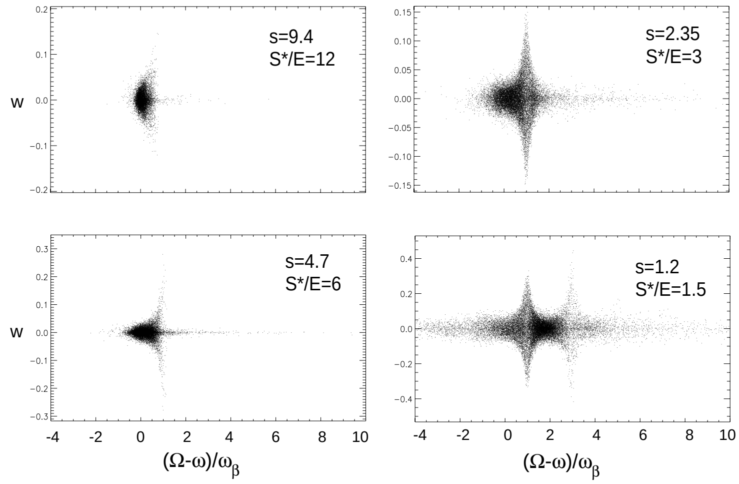


Figure 4: *Scatter plots in* $(w = \delta F/F_0, (\Omega - \omega_r)/\omega_\beta)$ *plane from linearized delta-f simulations for* $E = 6$ *and different values of* $S^*/E = 1.5, 3., 6., 12.$.

instability is completely stabilized or greatly reduced by FLR effects, but the residual instability persists due to the resonant particles drive. The importance of the resonances is also seen from the analysis of the energy balance, which shows that a relatively small number of resonant particles (about 5%) contributes more than 50% to the total energy balance.

Resonant instability can be avoided, if the equilibrium ion distribution is flattened in the region of the phase space where resonances occur. Therefore, a linearly stable, small S^*/E configuration is possible for a non-Maxwellian F_0. The localization of the resonant particles in the phase space indicates also that the instability can saturate nonlinearly through the modification of the initially unstable distribution function. This saturation has been observed in the hybrid simulations with initial $E = 4$, as reported in [3].

2. Nonlinear simulations at low and high S^*

The saturation of the $n = 1$ tilt instability, observed at low S^* [3], occurs due to the nonlinear change in the ion distribution function. The unstable configuration evolves nonlinearly into a new equilibrium (with smaller S^*, larger E and an increased separatrix beta), which does not show any MHD instability till the end of the simulation at $t \approx 70t_A$. On the other hand, hybrid calculations with large values of S^* show MHD-like linear growth of the tilt mode and no nonlinear saturation. However, surprisingly, the nonlinear evolution of the instability in these runs has been found to be considerably slower than that in similar MHD simulations. To make sure that periodic boundary conditions, used in our runs, have no effect on the observed nonlinear behavior, we have modified boundary conditions to allow the particle loss at the simulation boundaries.

Hybrid simulations with large values of S^* (small gyroradius regime) and loss boundary conditions (Fig. 5) show a significant loss of particles (about 30 %) during linear phase of the instability. It is found that particle loss along the open field lines has a destabilizing effect on the $n = 1$ tilt mode, increasing the linear growth rate by ≈10%, probably due to a reduction of plasma pressure on the open field lines. In these simulations ($E = 4$ and $S^* \approx 60$), the linear growth rate of the $n = 1$ tilt mode is very close to that of the MHD with $\gamma \gtrsim 0.9\gamma_{mhd}$. However, the nonlinear evolution of the instability is significantly slower than that in the MHD

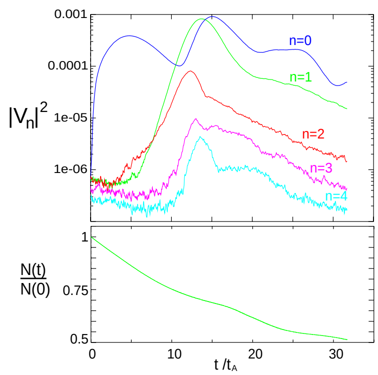


Figure 5: *Energy plots for n=0-4 modes (a), and total number of particles (b) from simulations with E=4 and S^*=60.*

simulations. Thus, despite the loss of about half of the particles, the field reversal of $B_z \approx -0.5B_{ext}$ is still present by $t = 32t_A$ in the simulations shown in Fig. 5. In contrast, the configuration is destroyed in $\sim 10t_A$ in the MHD run with the same equilibrium parameters. Significant ion spin up in toroidal (diamagnetic) direction is also observed at $t > 20t_A$ with $V_i \approx 0.3v_A$, which agrees with experiments.

The slow nonlinear evolution of the tilt mode in the large-S^* hybrid simulations can be related to the reduction of S^* (due the partial loss of flux), and the ion spin up in the nonlinear phase of instability. Both of these effects are missing from the MHD model. Our simulations have not included the end mirror coils, which are likely to improve the particle confinement and further slow down this nonlinear evolution.

Overall, hybrid simulations show the importance of the nonlinear effects, which are responsible for the saturation of instabilities in low S^* configurations, and also, for the significant increase of the FRC life-time compared to the MHD models in high-S^* configurations.

Acknowledgments

The authors acknowledge many useful discussions with D. C. Barnes, and thank him for providing equilibrium data for long elliptic separatrix FRCs. This work is supported by the US DOE Contract No. DE-AC02-76CH03073.

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