PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76CH03073

PPPL-3612
UC-70

PPPL-3612

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by

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September 2001

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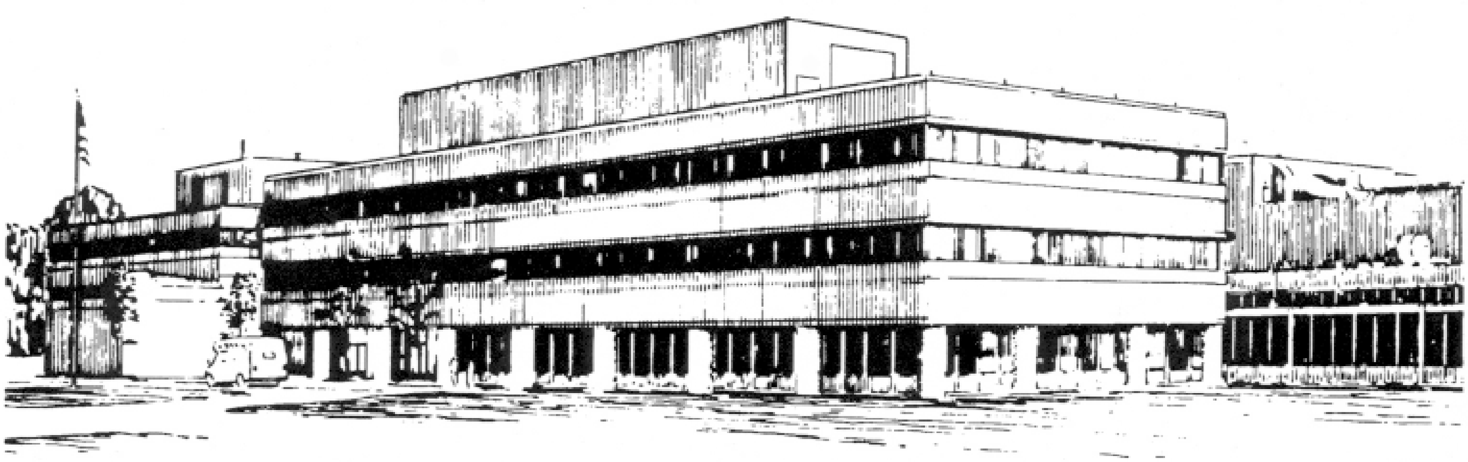

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BALLOONING STABILITY OF THE COMPACT QUASIAXIALLY SYMMETRIC STELLARATOR

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The magnetohydrodynamic (MHD) ballooning stability of a compact, quasiaxially symmetric stellarator (QAS), expected to achieve good stability and particle confinement is examined with a method that can lead to estimates of global stability. Making use of fully 3D, ideal MHD stability codes, the QAS beta is predicted to be limited above 4% by ballooning and high-*n* kink modes. Here MHD stability is analysed through the calculation and examination of the ballooning mode eigenvalue isosurfaces in the 3-space (s, a, q_k); s is the edge normalized toroidal flux, a is the field line variable, and q_k is the perpendicular wave vector or ballooning parameter. Broken symmetry, *i.e.*, deviations from axisymmetry, in the stellarator magnetic field geometry causes localization of the ballooning mode eigenfunction, with new types of non-symmetric, eigenvalue isosurfaces in both the stable and unstable spectrum. The isosurfaces around the most unstable points in parameter space (well above marginal) are topologically spherical. In such cases attempts to use ray tracing to construct global ballooning modes lead to a k-space runaway. Introduction of a reflecting cutoff in $k_{\wedge}$ to model numerical truncation or finite Larmor radius (FLR) yields chaotic ray paths ergodically filling the allowed phase space, indicating that the global spectrum must be described using the language of quantum chaos theory. However, the isosurface for marginal stability in the cases studied are found to have a more complex topology, making estimation of FLR stabilization more difficult.

Introduction

Understanding of ballooning mode stability boundaries may lead to performance improvement of toroidal devices through control of disruptions. Toroidally localized ballooning modes have been found as precursors to high beta disruptions on TFTR arising in conditions of *n=1* kink mode asymmetry. Recent optimization has shown that magnetohydrodynamic stability as well as good particle confinement are likely to be achievable in the proposed National Compact Stellarator Experiment (NCSX), a compact, quasiaxially symmetric stellarator (QAS) for values of the plasma near b = 4%.[1] The configuration, with a major radius of 1.42 m, an aspect ratio of 4.4, a toroidal magnetic field 1.2-1.7 T and 6MW of neutral beam heating, is stable to MHD instabilities, with b expected to be limited by high-*n* kink and ballooning modes. This paper describes the ballooning eigenvalue isosurfaces obtained for NCSX above the design beta, the first step in examining kinetic stabilization of the ballooning beta limit using a hybrid WKB approach.[2,3]

Eigenvalue Isosurfaces of the Quasiaxially Symmetric Stellarator

The VVBAL module of the TERPSICHORE code suite[4] has been used to calculate the ballooning instability for several NCSX equilibria (Fig. 1) above the design point (b = 4.1%). The displacement of the flux surface grows with growth rate g; x μ $exp\,(iw\,t)$ μ $exp\,(g\,t)$. We define the eigenvalue $l = -w^2$; positive values of l denote instability, while negative values denote stability. For b = 4.3% and b = 6.8% we have assembled a datacube of ballooning eigenvalues $l(s,a,q_k)$, of size (126,101,21). s is the toroidal flux, α is the field line variable. Roughly, within $\pm\pi$, the ballooning parameter q_k determines where the eigenfunction is a maximum. Figure 2 shows the plasma iota of the two equilibria. These are weak shear plasmas, with vanishing shear near the edge.

The isosurfaces of l constrain the possible trajectories of rays of the eikonal equation. Consequently they help determine the quantization conditions that are used to find the maximum wave vector, and thereby kinetic stabilization of the ballooning mode at the beta limit. The QAS isosurfaces are found to exhibit unusual topologies for the two equilibria. Distinct and unique structures at 4.3% b occur for different ranges of l in the stable spectrum for Alfven waves: a) at $l = -0.15$, a helical structure is found near the plasma edge, rotating about an axis nearly parallel to the q_k axis and open toward the plasma center; b) at $l = -0.45$, cylinders are found nearly constant in q_k, localized in s and a. At b = 6.8%, similar structures in the stable spectrum occur, although more global in extent.

The unstable spectra are less complex, consisting primarily of planes and topologically cylindrical and spherical isosurfaces near the outer edge of the plasma, where shear goes to zero and the instability is more easily driven. In general, there is a weak dependence on the ballooning angle q_k, stronger dependence on the field line a and quite strong dependence on the radial parameter s. At b = 4.3% topologically spherical isosurfaces are found for the maximum eigenvalues, indicative of strong quantum chaos.[2] This description "quantum chaos" for the paths of rays of the ballooning equation does not mean that the plasma behavior is chaotic, but that the mathematics of a quantum chaotic scattering problem can be used for instabilities for high values of l, far above the marginal point of the equilibrium. As the eigenvalue l drops to zero, isolated unstable cylindrical and planar isosurfaces conjoin and the isosurface is no longer simply connected. At 6.8% b, the surfaces break up at maximum eigenvalues. The configuration is Mercier stable at both values of b. Comparison with a related tokamak shows that the rich structure of the QAS spectra arise from the complexity of the magnetic configuration.

Finite Larmor Radius Stabilization of the Ballooning Mode at the Beta Limit

In practice, only finite-*n* modes can be unstable due to finite ion Larmor radius (FLR) stabilization, so that the infinite-*n* ballooning calculation may underpredict the actual MHD limiting beta. The validity of the hydrodynamic, fluid model for MHD breaks down and kinetic corrections are required if the condition $(k_\wedge r_i)^2 << 1$ is not satisfied. Here $k_\wedge$ is the wave vector perpendicular to the field line, and r_i is the ion Larmor radius, which for the QAS is ~ 1cm. Finite-*n* ballooning mode stability calculations with a 3D linear MHD code for a two-field period QAS configuration showed that the finite-*n* ballooning modes (n~20) are significantly more stable than the infinite-*n* results. For H1 and for a 10 field period stellarator, finite-*n* ballooning modes have been examined by applying the WKB ballooning formalism and semi-classical quantization or quantum chaos theory, depending on the topology of the isosurfaces.[2,3] Near the QAS beta limit the ballooning rays at the marginal point ($l=0$) will propagate on an isosurface having a new and complex topology, determining $k_\wedge$. It remains to be determined whether the ray orbits are regular, and how to use the orbit results to estimate $k_\wedge$ *via* the Einstein-Brillouin-Keller semiclassical quantization or the quantum chaos method.[3]

Anderson localization

Toroidal localization of the ballooning mode in stellarator plasmas has been identified for H1[3], LHD[5] and HSX[6]. This localization is analogous to Anderson localization[7] of electron eigenfunctions in condensed matter. For the QAS, we find that localization increases toward the edge of the plasma where the ballooning potential is increasingly aperiodic (Fig. 3) and there is stronger effective field ripple (Fig. 4). Each flux surface has a different shape, changing the poloidal angle, q_k, at which the eigenfunction is maximized. The most localized modes in this geometry occur in the region where global magnetic shear is weakest, including at the shear reversal surface itself, demonstrating the existence of Anderson localization in the QAS.

Conclusion

We find Anderson localization of the ballooning mode in the QAS and have obtained eigenvalue isosurfaces with which to examine kinetic stabilization of b. A new method of regularizing the eigenfunction to estimate $k_\wedge$ may be needed for the QAS at the beta limit, because of the complex topology of the marginal point isosurfaces.[3] The WKB method of high *n* ballooning stability calculations may break down for the QAS at the marginal point, requiring fully 3D, ideal or resistive MHD codes such as CAS3D, TERPSICHORE and Spector3D. Finally, while drift orbit optimization has allowed neoclassical particle transport to be kept low in the QAS, the detailed relationships between ballooning mode isosurface structures and drift mode growth rates need to be examined. Microinstability-based drift wave calculations based on radially local ballooning representations for LHD and QAS configurations can already predict drift mode growth rates in the electrostatic limit[8]. The toroidal dependence of anomalous transport will be sensitive to the localized ballooning structures. An investigation of stable magnetosonic structures would require relaxation of the incompressibility condition and integration of a 4th order system of equations. Further work will be needed in all these aspects of stellarator configuration design.

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* Research supported by U. S. DOE Contract DE-AC02-76CH0373.

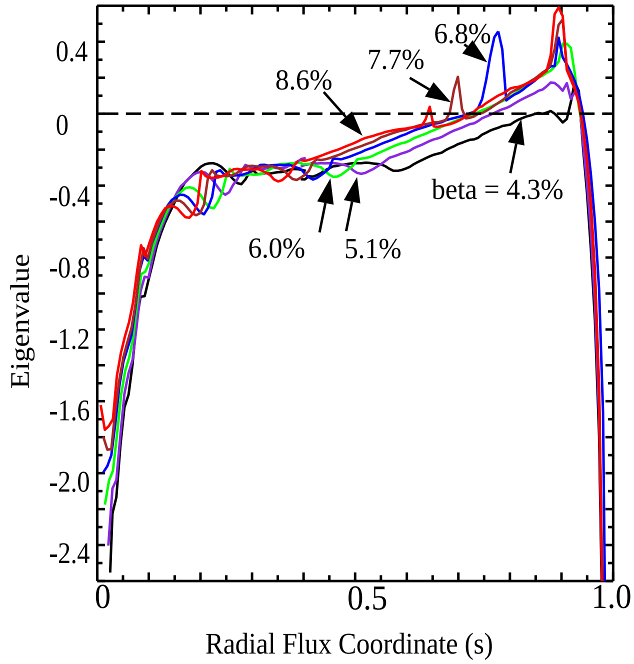

Figure 1. Ballooning eigenvalues for NCSX above the design point. α=0, θk=0.

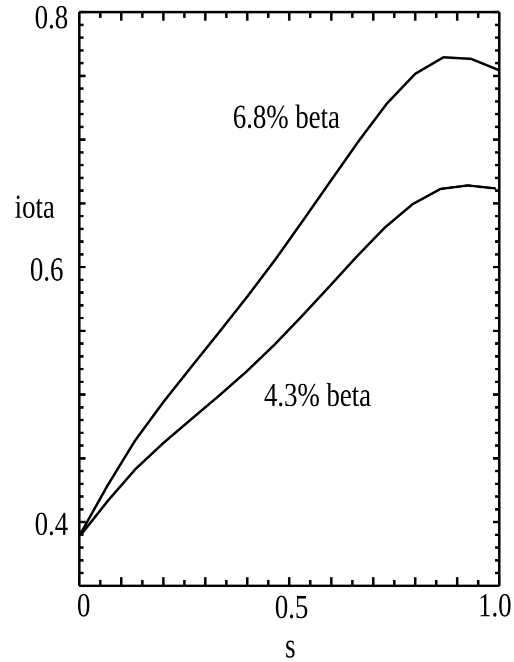

Figure 2. Iota profiles for equilibria above the design point beta.

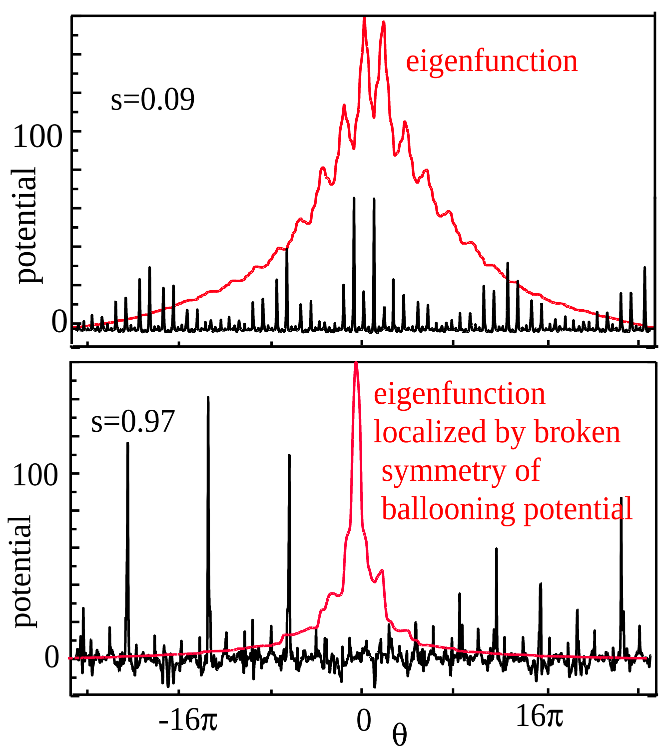

Figure 3. Eigenfunction localization in poloidal angle near the plasma edge, labeled by s, the edge normalized toroidal flux.

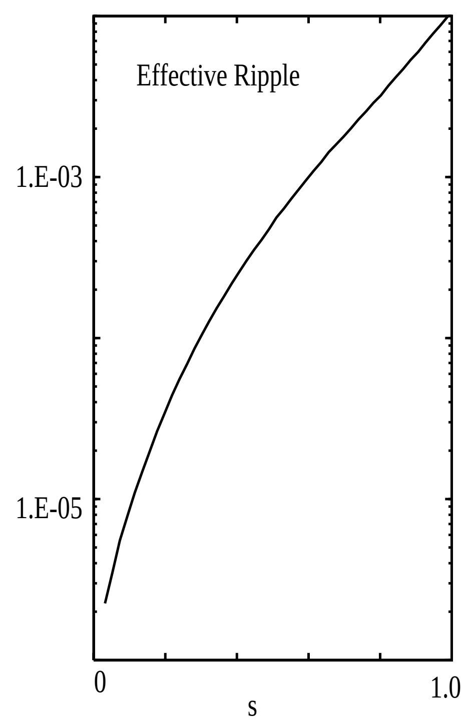

Figure 4. Effective ripple of QAS, calculated with NEO code.

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