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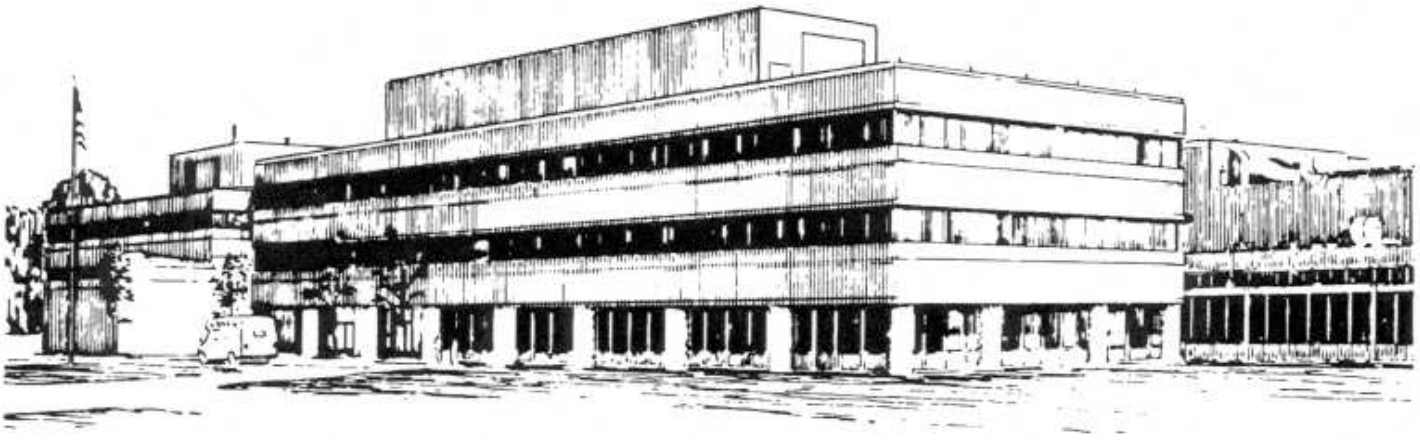

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DIAMAGNETIC FISHBONE MODE ASSOCIATED WITH CIRCULATING FAST IONS IN SPHERICAL TOKAMAKS

Ya.I. Kolesnichenko[1], V.S. Marchenko[1], R.B. White[2]

[1]*Scientific Centre "Institute for Nuclear Research", Kyiv, 03680, Ukraine*

[2]*Princeton Plasma Physics Laboratory, P.O. Box 451, Princeton, New Jersey, 08543, USA*

I. INTRODUCTION

Recently it was shown theoretically that high β (β is the ratio of the plasma pressure to the magnetic field pressure) inherent to plasmas of Spherical Tokamaks (ST) stabilizes the fishbone mode associated with the trapped particles [1,2]. This prediction agrees with the experimental observations of the fishbone behavior on Small Tight Aspect Ratio Tokamak (START) [3]. However, in the mentioned experiments the circulating particles rather than the trapped ones were dominant in the energetic ion population. Therefore, the theory of Refs. [1,2] is not sufficient to explain the START experiment and predict the behavior of the circulating-particle-induced fishbone mode in future experiments on STs. Thus, a new theory is required, which stimulated the fulfilment of this present work.

There are two fishbone branches: the high frequency (precession) branch and low frequency (diamagnetic) one [4,5]. In this work we restrict ourselves with the study of the low-frequency branch. The stability of this branch associated with the circulating particles in a low-beta plasma was studied in Ref. [6]; no attempts to consider high beta plasmas were done yet.

II. STABILIZING EFFECT OF HIGH β: QUALITATIVE ANALYSIS

As in Ref. [6], we assume that the safety factor $q(r)$ is a monotonic function and that $m = n = 1$ kink perturbation dominates, where m and n are the poloidal and toroidal mode number, respectively. However, in contrast to Ref. [6], we assume that the diamagnetic frequency of the bulk ions, ω_{*i}, is much less than the frequency of the toroidal drift motion of the energetic ions, ω_D, which implies that $\mathcal{E}_b/T \gg A^2$, where $\mathcal{E}$ is the particle energy, subscript "b" labels the beam ions, T is the plasma temperature, A is the aspect ratio of the torus (we used the estimate $\omega_D \sim \rho_\| v_b/R_0^2$ with $\rho_\| = |v_\||/\Omega_b$ the "parallel Larmor radius", Ω is the gyrofrequency, v is the particle velocity, and R_0 is the radius of the magnetic axis). Note that the latter condition is well satisfied in STs and can be fulfilled even in conventional tokamaks. When it is satisfied, the circulating energetic ions interact with the mode through the resonance $k_\| v_\| + \omega_D \approx 0$ (rather than through the resonance $\omega \approx k_\| v_\|$ considered in Ref. [6]). Because $v_\|/R_0 \gg \omega_D$ and $k_\| R_0 = q^{-1} - 1$,

this resonance takes place only at a certain radius (r_*) in the vicinity of the radius of the $q = 1$ surface (r_s), but not so close to r_s as in the case of $\omega_D \ll \omega_{*i}$. Below we will show that ω_D grows with β, so that $\omega_D = \xi\rho_\parallel v_b/R_0^2$, where $\xi(\beta) > 1$. Taking into account this fact and that $k_\parallel \approx s_1(\epsilon_s - \epsilon)/r_s$, where $s_1 = s(r_s)$, s is the magnetic shear, $\epsilon_s = r_s/R_0$, we obtain $|r_* - r_s| = \xi_s\rho_\parallel\epsilon_s/s_1$ with $\xi_s = \xi(r_s)$, which determines r_*. On the other hand, only particles crossing the $q = 1$ surface can lead to strong instability (the energy exchange between other particles and the waves is small) [6]. Therefore, the instability arises provided that the resonance radius satisfies the condition $|r_* - r_s| < \Delta_b$, where Δ_b is the half-width of the orbits of the beam ions. In order to see whether it is satisfied in STs, we use r_* found above and $\Delta_b \sim \rho_\parallel$. Then we obtain the condition $s_1 > \xi_s(\beta)\epsilon_s$, which is difficult to satisfy in STs when β is sufficiently large. Thus, we conclude that high β and small aspect ratio of STs are the factors which tend to stabilize the fishbone instability associated with the circulating particles.

III. CALCULATION OF THE PRECESSION FREQUENCY AND STABILITY ANALYSIS

Following the canonical description of the orbits introduced in Ref. [7], we can write for the well circulating particles:

$$\dot{\theta} = \frac{v_\parallel}{qR} - \frac{cmv_\parallel^2}{eR}\frac{\partial R}{\partial \psi}, \tag{1}$$

where R is the distance from the major axis of the torus, ψ is the toroidal flux, θ is the poloidal coordinate related to the corresponding Shafranov coordinate (labeled by S) in accordance with the expression

$$R = R_0 - \Delta + r\cos\theta_S = R_0 - \Delta + r\cos\theta + \eta r(\cos 2\theta - 1) + R_0 O(\epsilon^3) \tag{2}$$

with $\eta(r) = 0.5(\Delta' + r/R_0) \sim \epsilon$, $\Delta' = d\Delta/dr$, and $\Delta(r) > 0$ is the Shafranov shift, $\Delta(0) = 0$. Combining Eqs. (2), (1) and carrying out the orbit averaging, we obtain:

$$\langle\dot{\theta}\rangle = \frac{v_\parallel}{qR_0} + \omega_D, \tag{3}$$

where

$$\omega_D = \frac{\rho_\parallel|v_\parallel|}{2rR_0}\left(2\epsilon + 3\Delta' + r\Delta''\right), \tag{4}$$

the radial coordinate r is defined by $\psi(r) = \int^r drrB_0$, B_0 is the magnetic field at the magnetic axis. When deriving this equation, it was assumed that the orbit width is small compared to the shear length. Theory of the tokamak equilibrium provides an ordinary differential equation for Δ. Corresponding expressions for Δ' and $r\Delta''$ are well known [8]:

$$\Delta' = \epsilon(\beta_\theta + 0.5 l_i), \tag{5}$$

$$r\Delta'' = \epsilon\left[1 - (3 - 2s)\left(\beta_\theta + 0.5 l_i\right)\right] + \alpha_p, \tag{6}$$

where $l_i = 2/(r^2 B_\theta^2)\int_0^r B_\theta^2 r dr$ is the internal inductance per unit length, $\beta_\theta = (8\pi/B_\theta^2)(\bar{p} - p)$ with $\bar{p} = (2/r^2)\int_0^r p r dr$ the average pressure, and $\alpha_p = -(8\pi p'/B_0^2)R_0 q^2$. Substituting Eqs. (5), (6) into Eq. (4) we find that $\omega_D = \xi\rho_\parallel |v_\parallel|/(R_0^2)$, where

$$\xi = \frac{3}{2} + s\left(\beta_\theta + \frac{l_i}{2}\right) + \frac{\alpha_p}{2\epsilon}. \tag{7}$$

It follows from Eq. (7) that the Shafranov shift strongly increases the frequency of the toroidal drift motion in STs with high-β plasmas (for parabolic profiles $\xi \sim 3$).

In order to analyse the plasma stability we use an expression for the kinetic part of the potential energy of the perturbations, δW_k, which differs from that in Ref. [6] by including a more general resonance condition having the form:

$$\omega + s_1 \frac{v_\parallel^2 z}{\Omega_b R_0 r_s} - \frac{\xi_s v_\parallel^2}{\Omega_b R_0^2} = 0. \tag{8}$$

Concerning the injected ions we assume that they are characterized by the distribution function $F_b \sim p_b(r)\delta(\mu)/(\mathcal{E}^{1.5} + \mathcal{E}_c^{1.5})\eta(\mathcal{E}_\alpha - \mathcal{E})$, where $p_b(r)$ is the beam ion pressure, $\mathcal{E}_\alpha$ the injection energy, $\mathcal{E}_c \sim (m_i/m_e)^{1/3} T$, μ is the particle magnetic moment, $\eta(x) = \int_{-\infty}^x \delta(t) dt$. Assuming $\mathrm{Re}\omega \gg \mathrm{Im}\omega$, we find:

$$\mathrm{Im}\,\delta W_k = -\epsilon_s \left[\frac{\Delta_b^\alpha}{r_{pb}} \frac{\beta_{b\theta}}{s^3}\right]_{r_s} I(\kappa, \mathcal{E}_s/\mathcal{E}_\alpha), \tag{9}$$

where $\kappa \equiv \xi_s \epsilon_s / s_1$, $\beta_{b\theta}$ is the poloidal beta of beam ions, $\mathcal{E}_s \equiv m_b \Omega_b \omega R_0 r_s/(2 s_1)$, and $r_{pb}^{-1} = -d\ln p_b/dr$.

When $\xi_s \epsilon_s \ll s_1$ ($\kappa \ll 1$), the quantity I is maximum. This corresponds to the limit case of Betti and Freidberg (BF) [6], therefore we denote this magnitude by I^{BF}. But in STs typically $\kappa \gtrsim 1$, in which case I can be much less than I^{BF}: $I/I^{BF} \sim (\mathcal{E}_s/\mathcal{E}_\alpha)^{1.5} \sim (T R_0)^{1.5}/(\mathcal{E}_\alpha s_1 r_{pi})^{1.5}$, where $r_{pi}^{-1} = -d\ln p_i/dr$. $I \to 0$ for $\kappa \to \infty$, $I \neq 0$ for finite κ. The reason why $\mathrm{Im}\,\delta W_k$ does not vanish for the finite κ is the presence of particles with $\mathcal{E} \ll \mathcal{E}_\alpha$ for which $|r_* - r_s| < \Delta_b$ due to the resonance $\omega \approx k_\parallel v_\parallel$.

A simple estimate shows that when $\mathcal{E}_s/\mathcal{E}_\alpha \simeq 0.1$, $\kappa = 1.2$ is sufficient for more than the tenfold decrease of the BF response. Using Eq. (7) we find that the condition $\kappa \geq 1.2$ yields the following restriction on β for the plasma with the parabolic profile of the pressure (in which case $\alpha_p = 2\epsilon_s \beta(0) A^2$, $\beta_\theta(r_s) = \beta(0)A^2/2$ with $\beta(0) = 2\bar{\beta}$, $\bar{\beta}$ is the volume averaged β):

$$\bar{\beta} \geq \frac{1}{(2 + s_1)A^2}\left(1.2\frac{s_1}{\epsilon_s} - \frac{3}{2} - \frac{s_1 l_i}{2}\right). \tag{10}$$

When this condition is satisfied, the amplitude of fishbones is small or the instability is completely stabilized due to the presence of weak damping mechanisms.

It is of interest to see whether Eq. (10) is satisfied in experiments on START where fishbone oscillations were weak or disappeared in high-β discharges with $\langle\beta\rangle \sim 30\%$) [3]. To make an estimate we take $A = 1.5$, $\mathcal{E}_\alpha = 30keV$, $T = 300eV$, $l_i = 0.7$, $s_1 \sim 0.5$, and $r_s \sim a/3$. Then we can write Eq. (10) as $\bar{\beta} \geq 20\%$. For the used parameters $\mathcal{E}_s/\mathcal{E}_\alpha \sim 0.03$, which leads to $I/I_{BF} \sim 10^{-2}$. This result together with the prediction of the complete stabilization of the trapped-particle-induced fishbone mode at high β [1,2] may explain the disappearance of fishbones in START.

IV. SUMMARY AND CONCLUSIONS

In conclusion, we considered for the first time the stability of the fishbone mode associated with the circulating ions in high-β plasmas of spherical tokamaks. We have shown that well-circulating energetic ions undergo strong toroidal drift motion when the aspect ratio of the torus is small and the plasma pressure is high. Because of this enhanced drift motion, the radius r_* (where the resonance between the energetic ions crossing the $q = 1$ surface and the internal kink perturbation occurs) may be shifted for the distance exceeding the particle orbit width, which stabilizes the instability.

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