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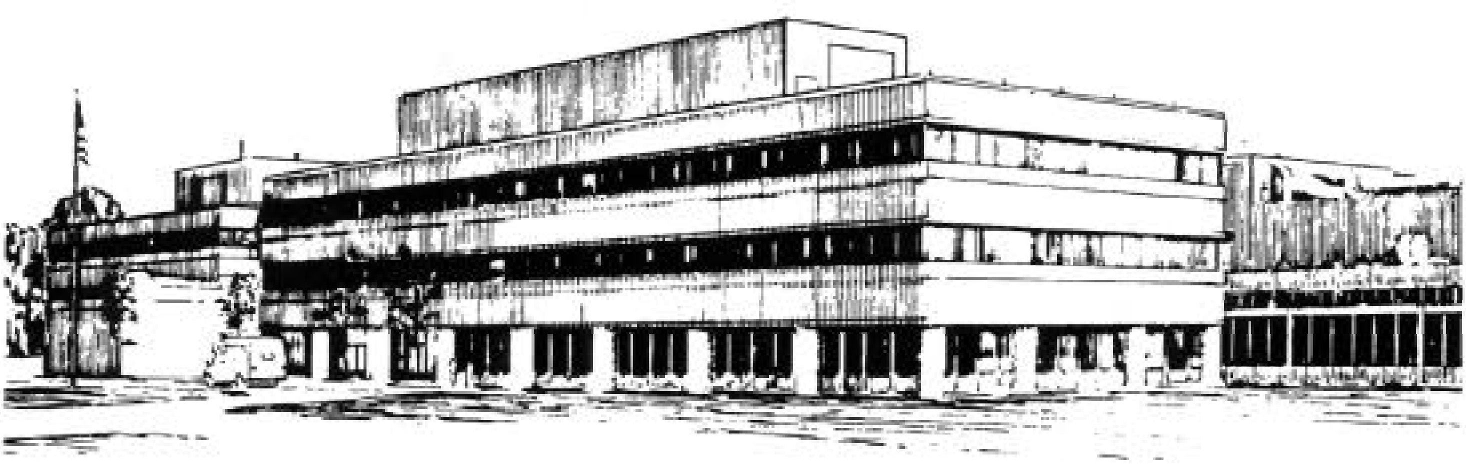

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Motion of Charged Particles near Magnetic Field Discontinuities

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The motion of charged particles in slowly changing magnetic fields exhibits adiabatic invariance even in the presence of abrupt magnetic discontinuities. Particles near discontinuities in magnetic fields, what we call "boundary particles", are constrained to remain near an arbitrarily fractured boundary even as the particle drifts along the discontinuity. A new adiabatic invariant applies to the motion of these particles.

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The classical description of particle behavior in non-uniform magnetic field consists of approximating the particle trajectory by a circular orbit that slowly drifts in space (see e.g. [1-2]). The adiabatic invariance of the magnetic flux through particle's Larmor orbit follows from the assumption that the fields seen by the particle change little during the Larmor orbit. The particle guiding center acquires a drift in slowly changing fields. Thus, both the conventional guiding-center formalism [3-8] and its high-order corrections [9-11] describe the dynamics of guiding centers when the particle gyroradius r_g is much smaller than the characteristic spatial scale L of the magnetic field, which makes the theory inapplicable to discontinuous fields.

In discontinuous fields, we identify and calculate the motion of what we call "boundary particles". For these particles, the guiding centers are located no further than a distance of one gyroradius from the magnetic field discontinuities, i. e., magnetic boundaries where $r_g / L >> 1$. The present work identifies and describes new and unusual properties of the motion of boundary particle along plane, broken and branching boundaries.

Consider then motion in a magnetic field $\vec{B} = \vec{z}^{\,0} B(x)$, which changes abruptly in space:

$$B(x) = \begin{cases} B_1, & x < 0 \\ B_2, & x > 0 \end{cases}. \tag{1}$$

The plane $x = 0$ represents a magnetic boundary - an infinitesimally narrow region where the magnetic field magnitude spatial derivative is infinite, such that a finite jump of B across the boundary exists. In practice, such a model is appropriate if the characteristic width of $B'(x)$ profile is much less than the particle gyroradius.

The motion of a "boundary particle" in the magnetic field given by (1) is shown in Fig.1. The particle crosses the magnetic field discontinuity with different gyroradii on different sides of the magnetic boundary, Thus, after one period of transverse oscillations (i.e. after two crossings of the boundary), it is displaced along the boundary, in complete analogy to the classical ∇B-drift in smooth non-uniform fields. The 1D particle oscillation transverse to the boundary can be described by the Hamiltonian

$$H = \frac{p_x^2}{2m} + \frac{1}{2m}\left(p_y - \frac{q}{mc} x B(x) \right)^2 = \frac{mV^2}{2} = const\,, \quad p_y = const\,,$$

where *x* is the direction perpendicular to the boundary. If the parameters of the system are changing slowly in time or in space, the particle motion has an adiabatic invariant proportional to the area confined inside the phase-space particle trajectory (see Fig. 2). This new invariant can be written as

$$\mu = \frac{mV^2}{2}\psi(\alpha) = \text{const}, \tag{2}$$

$$\psi(\alpha) = \frac{1}{2\pi}\cdot\begin{cases}\pi(1/\omega_1 + 1/\omega_2) + (1/\omega_1 - 1/\omega_2)\cdot(\pi - 2\alpha + \sin 2\alpha), & B_1B_2 > 0\\ (1/\omega_1 + 1/\omega_2)\cdot(2\pi - 2\alpha + \sin 2\alpha), & B_1B_2 < 0\end{cases},$$

which reduces to the well-known particle magnetic moment ($mV^2/2\omega = \text{const}$) for uniform magnetic fields ($B = B_1 = B_2$, $\omega = qB/mc$). Here $\alpha = \arccos(p_y/mV)$ is the angle that determines the guiding center transverse displacement relative to the boundary (see Fig.1), and $\omega_k = |qB_k|/mc$ are gyrofrequencies in the corresponding regions.

The adiabatic invariance of μ conservation enters in considering, for example, a static magnetic field smoothly changing in space along the boundary ($r_g\, d\ln|B|/dy << 1$). For simplicity, assume $B_1 = \text{const}$ and for definition consider the region where $B_2 > B_1$. Then Eq. (2) can be rewritten as

$$(1/B_1 - 1/B_2(y))\cdot\Theta(\alpha) = \text{const} > 0,$$

where $\Theta(\alpha) = 2\alpha - \sin 2\alpha$ is a monotonically increasing function of the angle α. The latter equation defines the relation between α and the magnitude of magnetic field, and determines the transverse drift of the particle. Since α increases with the decrease of B_2, a positively charged particle will drift in the positive *x* direction, with $\alpha < \pi$ (Fig.3). The direction of this drift coincides with the direction given by the classic ∇B-drift, which is applicable, however, for smooth magnetic fields only. Similarly, for a magnetic field that is homogeneous along the boundary but slowly changing in time, the adiabaticity of μ relates the particle energy to the magnetic field magnitude.

The adiabatic invariance theorem can also be generalized to include boundary particle motion in crossed static magnetic and electric fields. To do so, consider an external uniform electrostatic field directed along the boundary $\vec{E} = \vec{y}^{\,0} E$ with an abrupt magnetic field configuration of the form of Eq. (1). For simplicity, say $B_1 = -B_2 = B_0$. The $\vec{E} \times \vec{B}$-drift velocity is then always directed towards the boundary, so we can expect the magnetic boundary to attract particles not allowing them to leave the magnetic field discontinuity surface.

In the presence of friction, the particle motion is described by

$$\ddot{\vec{r}} = \omega_0 \left(\dot{\vec{r}} \times \vec{z}^{\,0} \right) \operatorname{sgn} x + q/m \cdot \vec{E} + \vec{R} , \quad \omega_0 = qB_0/mc ,$$

with a friction force term $\vec{R} = -\nu \dot{\vec{r}}$. In both the collisionless ($\nu = 0$) and collisional ($\nu \neq 0$) cases, the only possible equilibrium trajectory is the trajectory $x(t) \equiv 0$. Particles with non-zero initial *x* will eventually be attracted to the boundary, with transverse oscillations decaying in both the collisional and collisionless case. After sufficient time has passed ($t >> \tau$, where τ is the period of transversal oscillations, and $t >> mV_y(0)/qE$ in case $\nu = 0$, or after the amplitude of oscillation becomes much less than $qE\omega_0/m\nu^3$ in case $\nu \neq 0$, $\nu \tau(t) << 1$), the rate of decay can be obtained from the generalized adiabatic invariant's conservation law and given by

$$x_M \sim t^{-1/3}, \qquad \dot{x}_M \sim t^{1/3}, \qquad \tau \sim t^{-2/3}, \qquad (\nu = 0)$$

$$x_M \sim \exp(-2\nu t/3), \quad \dot{x}_M \sim \exp(-\nu t/3), \quad \tau \sim \exp(-\nu t/3), \qquad (\nu \neq 0)$$

where x_M and $\dot{x}_M$ are the amplitudes of transversal coordinate and velocity oscillations correspondingly.

Although charged particle motion along plane magnetic boundaries is relatively simple and can be described analytically, more complex 2D magnetic field profiles give rise to complicated motion. Yet, boundary particles retain an important property: if a particle crosses the boundary once, it will never be able to go away from it on a distance of a gyroradius or more within a finite amount of time. This can be seen as follows. Consider that after a particle has left the magnetic boundary, it must undergo purely rotational motion in the absence of an electric field applied and for uniform magnetic fields away from the boundary. However, the adiabatic invariant associated with this motion is clearly different than that for particles at the magnetic boundary. Hence, these particles could never intersect the magnetic boundary.

Formulating the theorem in different words, we can say that boundary particle always stays trapped by magnetic field discontinuity, no matter how complicated the discontinuity is. In Fig. 4, we show how boundary particles turn corners if the magnetic discontinuity has corners. Essentially, boundary particles "wet" the surfaces of field discontinuities, following them like liquid follows wetting surfaces due to surface tension.

Should the parameters of plane boundary change smoothly in the tangential direction, which means that the boundary has curvature small compared to the particle gyroradius, the conservation of μ, as given by Eq. (2), remains valid. However, if particle faces a sharp change of boundary direction (a "corner"), the validity conditions for the adiabatic invariance of μ are violated. As a result of "scattering on the corner", the particle undergoes changes of both μ and phase of oscillation phase. Nonetheless, as depicted in Fig. 4, the boundary nature of these particles is retained; i.e., they still wet the surface.

Only very simple two-dimensional magnetic field configurations, like the one given in Fig. 4, allow purely analytical quantitative description. However, other systems can still be explored qualitatively due to the wetting effect. For example, consider the situation when the boundary particle comes to a point where the boundary branches into two or more new boundaries. In the case when the drift velocities corresponding to all boundary branches are directed outward from the branching point, the particle will choose one of new boundaries to drift along. Each path depends in very fine detail on initial conditions. Since the probability to start drifting along a certain branch strongly depends on particle's initial phase, stochasticity of guiding centers motion can be found in relatively simple magnetic field configurations.

To see this, consider for example the four-field configuration given in Fig. 5. The magnitudes of magnetic field on different sides of the boundaries can be chosen to let a particle approach the scattering region from $B_1 \div B_3$ boundary only and leave it through channels $B_1 \div B_2$ and $B_2 \div B_3$. Due to the special choice of parameters ($B_1 > B_2 > B_3 > B_4$), particles can drift only counter clockwise along the central boundary loop. (The magnetic field here is directed out of the page and the particle charge is positive.)

After a particle has come to the central loop, it has a probability to "leak out" along the boundary $B_1 \div B_2$, but it can also scatter on the triple point $B_1 B_2 B_4$ and remain on the loop. The same applies to the next triple point $B_1 B_3 B_4$. Therefore, although a particle may leak out from the central part of the system at the first or second scattering events, it also may stay on the loop for more than one period of drift rotation, depending on the initial parameters of the particle motion. Since the probability of leaking out strongly depends on the particle's initial phase, which abruptly changes in each scattering event, the particle motion becomes quite complicated (Fig. 5). It is practically impossible to say along which of two boundaries the particle will eventually leave. According to numerical simulations performed, the characteristic initial phase step needed to distinguish alternative paths is less than 0.01 rad for $B_1 \sim B_2 \sim B_3 \sim B_4$ and loop radius close to r_{g4}. The latter property provides two output streams along $B_1 \div B_2$ and $B_2 \div B_3$, with uniform distribution over the particle phases.

Certian practical devices might be envisioned from these unsusual properties of boundary particles. The four-field configuration can be considered as a boundary-particle beam separator. However, the same, but field-reversed, system can be used for merging of streams coming from channels $B_1 \div B_2$ and $B_2 \div B_3$ into the single output stream $B_1 \div B_3$. Effective mixing caused by stochastic rotational drift along the central loop could provide the output beam of particles with uniform phase distribution as well.

In summary, we have identified a new class of charged particles undergoing adiabatic motion near abrupt magnetic boundaries. Magnetic discontinuities abrupt compared to a particle gyroradius are easily produced in the laboratory, and may occur naturally, for example, in fields undergoing magnetic reconnection. The classical adiabatic invariant for motion in slowly varying fields is generalized to account for the abrupt discontinuities. The complexity of the motion of boundary particles is constrained by an unusual "wetting effect", which is a profound property of the boundary particles. Apart from academic interest in the wetting effect, there may be practical consequences in the manipulation of particles with such constrained motion, including the directed transportation of boundary particles, as well as the merging or separation of magnetic boundary plasma flows.

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Figure captions

FIG. 1. Boundary particle motion along a plane magnetic boundary.

FIG. 2. Phase-space trajectory of boundary particle’s transverse oscillations.

FIG. 3. Transverse adiabatic drift of a boundary particle (scales of the *x* and *y* axes are not equal).

FIG. 4. Boundary particle scattering on a straight corner of a magnetic boundary: the “wetting effect” on a broken boundary .

FIG. 5. Boundary particle trajectories along branching magnetic boundaries.

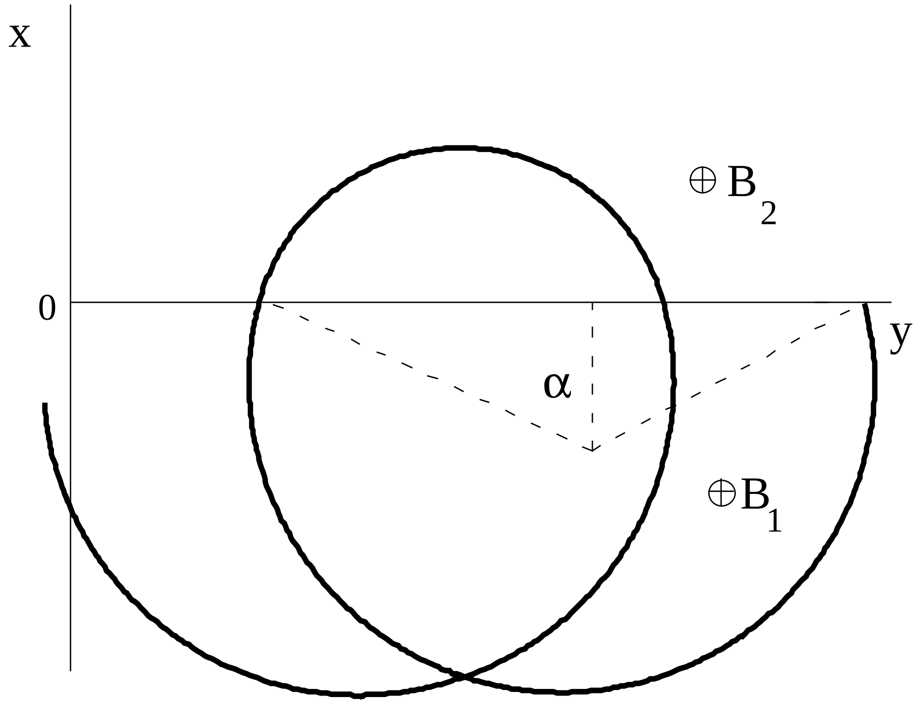

FIG.1

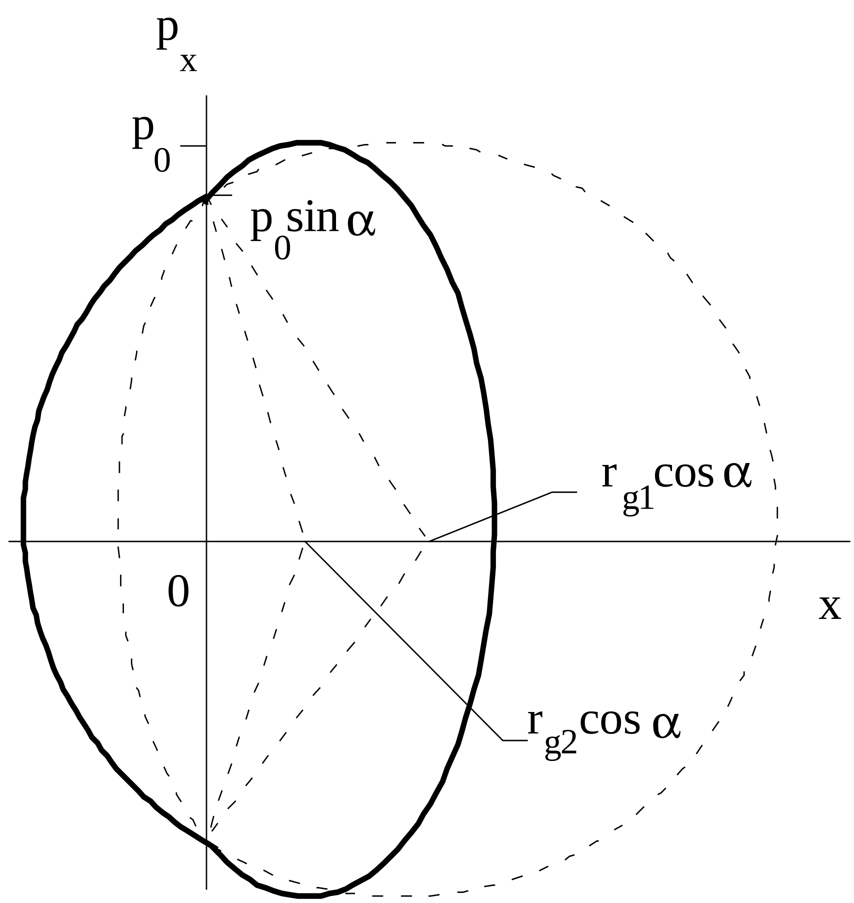

FIG.2

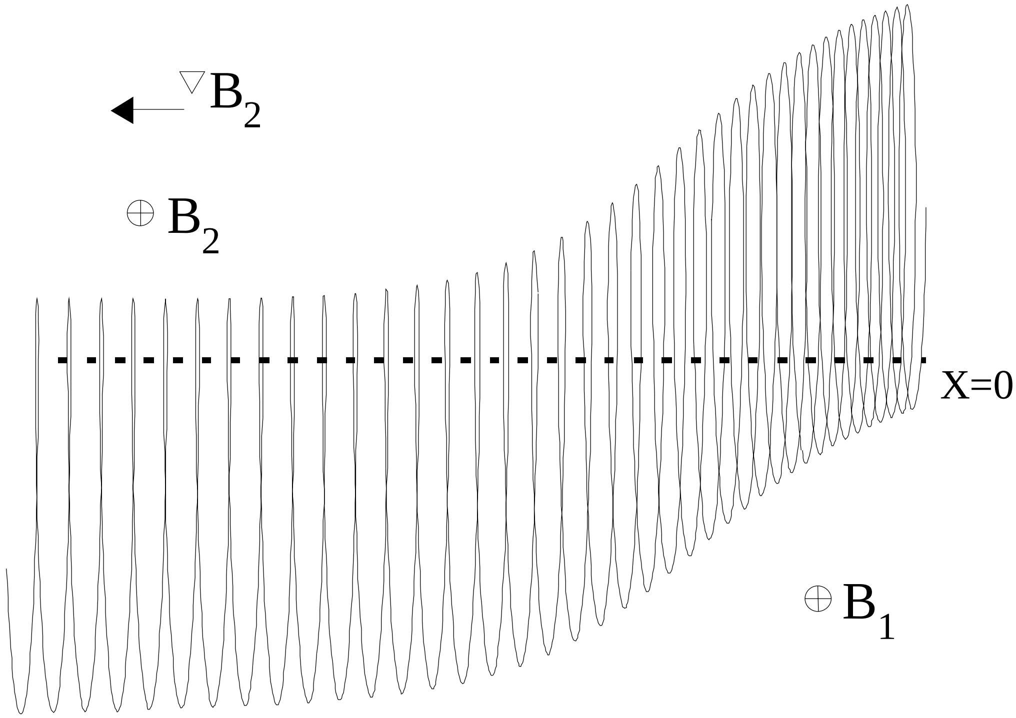

FIG.3

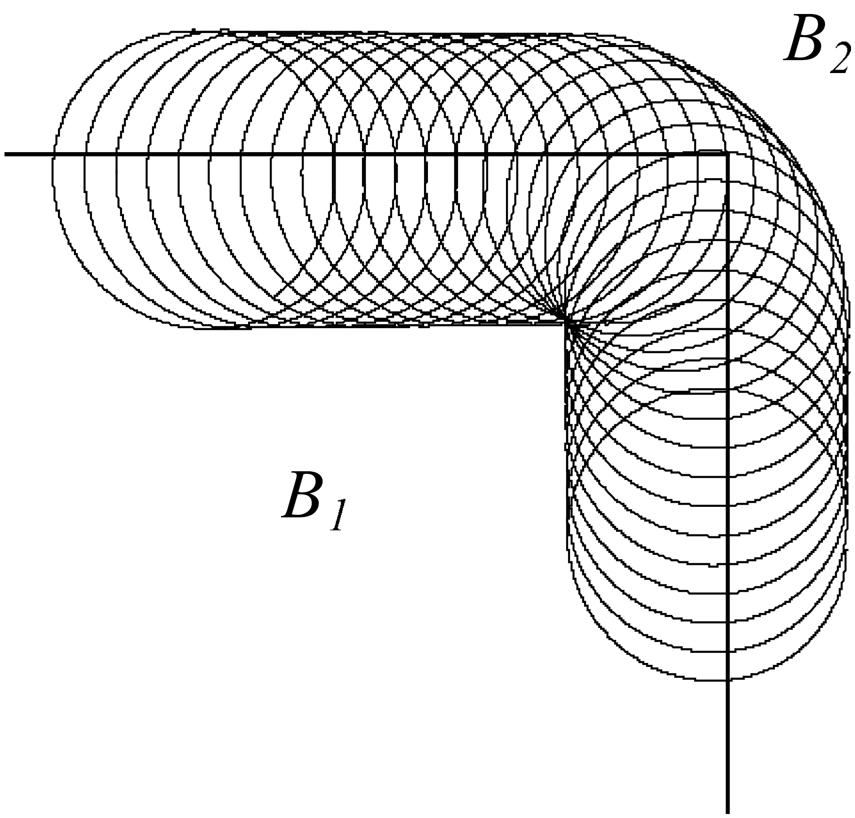

FIG.4

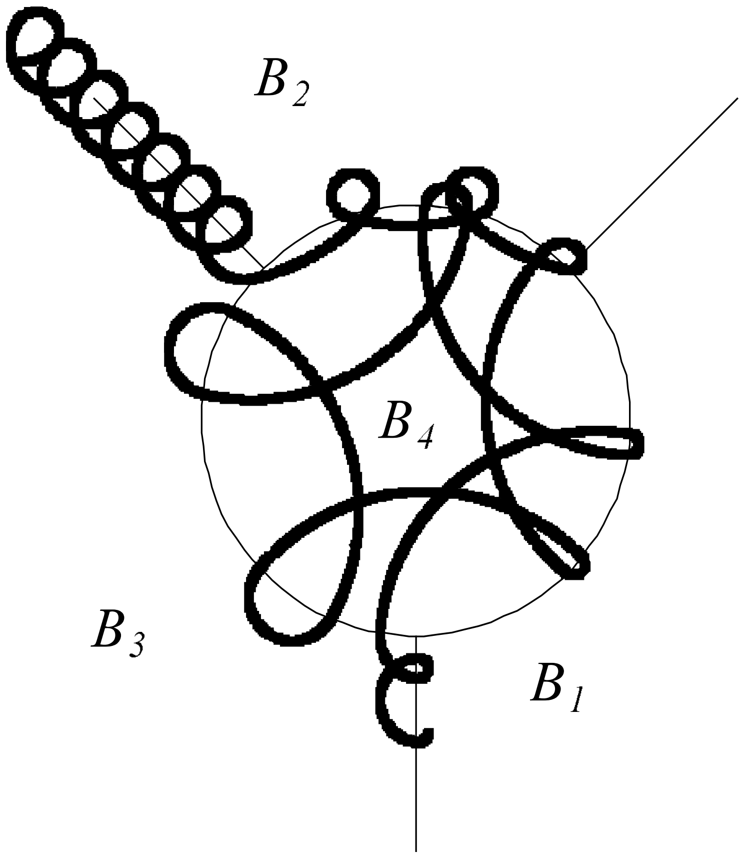

FIG.5

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