MHD Stability Calculations of High-β Quasi-Axisymmetric Stellarators

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Abstract

The MHD stability of quasi-axisymmetric compact stellarators is investigated. It is shown that bootstrap current driven external kink modes can be stabilized by a combination of edge magnetic shear and appropriate 3D plasma boundary shaping while maintaining good quasi-axisymmetry. The results demonstrate that there exists a new class of stellarators with quasi-axisymmetry, large bootstrap current, high MHD beta limit, and compact size.

1 Introduction

The design of quasi-axisymmetric stellarators[?](QAS) aims at combining the best features of both tokamaks and stellarators to achieve reactor-relevant plasma performance: high beta, good particle confinement, disruption-free steady state operation with little need for current drive, and compact size. A key issue in the design of high-beta compact stellarators is the stability of beta-limiting MHD modes, such as Mercier modes, ballooning modes and external kink modes. In conventional stellarators, the external kink modes were thought to be unimportant because the net toroidal current is zero or very small. However, in high-beta compact stellarators, such as QAS, the amount of the pressure-driven bootstrap current can be substantial. A large bootstrap current can help to generate rotational transform necessary for particle confinement, but it can also drive external kink modes unstable. Early work had considered current-driven external kink modes where the stabilizing role of magnetic shear was recognized[?, ?, ?]. More recently, Mikhajlov and Shafranov[?] have shown analytically that a sufficient magnetic shear generated by helical coils can stabilize the external kink modes. Most previous work has assumed cylindrical geometry and zero beta.

In this work, extensive calculations have been carried out to evaluate stability limits imposed by external kink modes and high-n ballooning modes for fully three dimensional equilibria at finite beta. We find that the external kink modes in QAS with high edge shear are significantly more stable than a corresponding tokamak without a conducting wall. It is shown that 3D shaping also plays a significant role in the stability.

2 Stability of External Kink Modes and Ballooning Modes

The stability of external kink modes driven by large bootstrap current is studied by using a 3D equilibrium code VMEC[?] and a 3D free boundary global MHD stability code TERPSICHORE[?]. We have considered a large number of QAS configurations with aspect ratio $A = R/\langle a \rangle = 2.1 \sim$

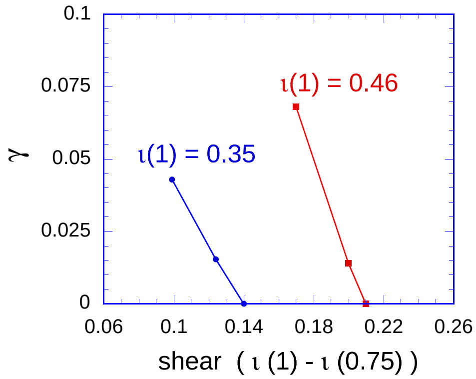

Figure 1: *Growth rate of the $n = 1$ external kink mode v.s. edge magnetic shear.*

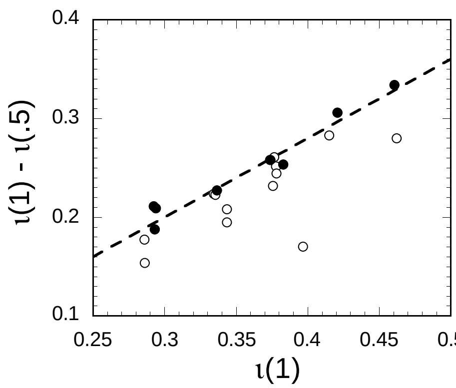

Figure 2: *The stability diagram of the $n = 1$ external kink mode as functions of edge shear and transform. The solid dots/circles denote stable/unstable configurations.*

3.5 and number of field periods $N_p = 2 - 4$. These QAS configurations have been obtained by adding appropriate 3D modifications to the plasma boundary of an optimized tokamak equilibrium while maintaining quasi-axisymmetry[?]. The equivalent tokamak equilibrium at $A = R/\langle a \rangle = 2.1$ has reversed shear profile with 90% bootstrap current, ballooning-stable 7% beta, and stable external kink modes with conducting wall at $b/a = 1.3$. However, the beta limit of the tokamak due to the external kink modes drops to 2.5% without a conducting wall. We show that, in QAS, the kink modes can be stabilized at beta up to 7.5% without a conducting wall by choosing appropriate externally generated transform with sufficient stellarator-like shear ($d\iota/dr > 0$) near the plasma edge.

Systematic analysis of the effects of iota profile on kink stability has been carried out for $N_p = 4$ and $A = R/\langle a \rangle = 2.1$ by varying the 3D shaping. The current and pressure profiles are fixed. The actual variation in rotational transform is entirely controlled by the specifications for the external helical coils. It is found that the kink stability is strongly dependent on values of iota and shear near the plasma edge. Figure 1 shows the calculated $n = 1$ external kink growth rate as a function of edge shear defined by $\iota(1) - \iota(0.75)$ at fixed value of edge transform $\iota(1)$, where the argument is the normalized toroidal flux. We see that the effects of edge shear are strongly stabilizing. The critical shear needed for stabilization increases with edge transform. Physically, the external kink modes are driven by current gradient and pressure gradient. The magnetic shear is stabilizing because it enhances the positive field line bending energy in the plasma.

We have also studied parameter dependence on plasma current, pressure, and the axisymmetric shaping. It is found that the plasma current is strongly destabilizing because it is a driving source and it generates part of the transform. The current increases edge transform

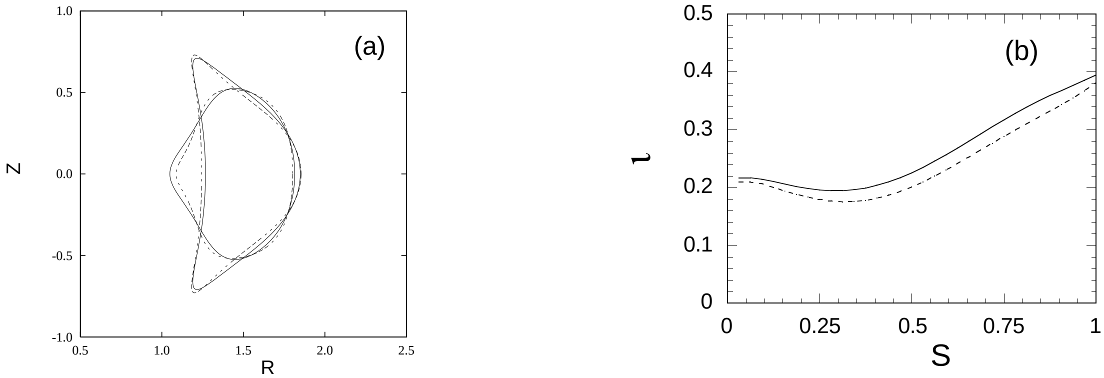

Figure 3: *(a) Plasma cross-sections at $\phi = 0$ and $\pi/2$ and (b) iota profile of an unstable QAS before (solid lines) and after (dashed lines) stability optimization.*

and reduces edge shear since the current-induced rotational transform has tokamak-like shear near the edge which cancels part of externally generated shear. The plasma pressure can be either destabilizing or stabilizing because of its two competing effects. On one hand, the plasma pressure gradient is an instability driving source. On the other hand, the plasma pressure tends to reduce edge iota and enhance the edge magnetic shear.

More recently, we have studied a series of QAS configurations with $N_p = 3$ and a relatively higher aspect ratio of $R/\langle a \rangle = 3.5$. The results of such a study are shown in Fig. 2 which plots the stability diagram of the $n = 1$ external kink mode in phase space of edge shear and edge iota. The solid dots denote the stable equilibria while the circles denote the unstable cases. The variation of transform in these cases are obtained by varying 3D shaping while maintaining good quasi-axisymmetry. It is seen that the stable cases are separated from the unstable cases and the stability boundary is marked approximately by the dashed line. It is also seen that the critical shear for stability increases linearly with the edge iota.

The values of critical shear in Fig. 2 would result in iota profiles which are too low near the magnetic axis for practical experiments. We have thus investigated ways to reduce the shear requirement for the kink stability. It is found that the 3D shaping also plays a significant role in kink stability besides the iota profile. We have found two methods of stabilization via shape control. First, a poloidally localized (on the low-field side of the magnetic axis) low-order helical boundary corrugation is shown to improve the kink stability. The corrugation can be produced by tilted window-frame coils near the outboard midplane of the plasma, a coil topology similar to that proposed by Furth and Hartman[?] for increasing the shear in stellarators. In our case we choose a tilt that produces little change in the iota profile. As a test case for this method, we consider the unstable equilibrium with $\iota(1) = 0.39$ and $\iota(1) - \iota(0.5) = 0.17$ in Fig. 2.. The results show that the $n = 1$ external kink mode is stabilized by a helical corrugation with approximate mode numbers (in coordinate space) of $n = 1$ per period and $m = 4$ and a small amplitude on order of 10% of the plasma minor radius. In the second method, which in principle includes the boundary corrugation generated by the Furth-Hartman coils, we maximize external kink stability by adjusting the general 3D shaping while maintaining quasi-symmetry and the iota profile. To this end, we have included the TERPSICHORE code in a configuration optimizer which maximizes the kink stability together with ballooning stability and quasi-symmetry by varying the 3D boundary shape while maintaining the iota profile. Numerical results obtained using such an optimizer show that the critical shear needed for kink stability can be reduced substantially from that of Fig. 2. As a successful example of this approach, we consider the same test case mentioned above which is stabilized by the boundary corrugation method. Figure 3 shows (a) plasma cross-sections and (b) iota profiles of the test case before stability optimization (solid lines) and the corresponding optimized configuration (dashed lines). The optimized case is stable to both $n = 1$ external kink and ballooning modes at $\beta = 3.9\%$. We note that the edge shear in the optimized case is similar to that of the unstable equilibrium before optimization. It is emphasized here that the quasi-symmetry is preserved in the stability optimization. This demonstrates that the external kink modes can be stabilized by both edge magnetic shear and appropriate 3D shaping without sacrificing quasi-axisymmetry.

The ballooning modes are studied using a local ballooning code[**?**]. For a 4 period QAS with $R/\langle a \rangle = 2.1$, it is found that ballooning beta limit is about 7%, which is similar to that of the equivalent tokamak. At a higher aspect ratio of $A = 3.5$, the ballooning beta limit is found to be somewhat lower, at $\beta \sim 4\%$ for optimized three field period QAS configurations. These beta values are much higher than those of earlier quasi-axisymmetric configurations due to improved strong axisymmetric shaping.

The 3D global MHD stability code CAS3D[**?**] is also being used to calculate the stability of low-n MHD modes. The calculated mode structure and stability beta limits of fixed boundary Mercier-type low-n modes in a two field period QAS agree well with those of the TERPSICHORE code. Work is in progress to benchmark the codes for free boundary external kink modes in stellarator equilibria. The CAS3D code is capable of calculating high-n ballooning modes and vertical stability and is being used to investigates the stability of these modes in QAS.

3 Conclusions

We have investigated systematically the external kink stability and ballooning stability in quasi-axisymmetric stellarators. Results show that magnetic shear as well as plasma boundary shape are two key factors in determining the external kink stability. In tokamaks, the q profiles are intrinsically coupled to the current profile and plasma boundary shape. In contrast, in QAS, the iota profile and plasma boundary shape can be independently varied at fixed current and pressure profile and thus can be used to control external kink modes as well as ballooning modes. The results found here demonstrate that there exists a new class of stable high-beta, high-bootstrap fraction toroidal equilibrium. While the advanced tokamak stabilizes the external kink with a conducting wall and rotation or feedback, it is accomplished here with non-axisymmetric coils.

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References

[1] A. Reiman et al., this conference.

[2] J. L. Johnson et al., Phys. Fluids **1**, 281 (1958).

[3] R. M. Sinclair et al., Phys. Fluids **8**, 118 (1965).

[4] K. Matsuoka et al., Nuclear Fusion **17**, 1123 (1977).

[5] M. I. Mikhajlov and V. D. Shafranov, Nucl. Fusion **30**, 413 (1990).

[6] S. P. Hirshman and J. C. Whitson, Phys. Fluids **26** 3553 (1983).

[7] W. A. Cooper et al., Phys. Plasmas **3**, 275 (1996).

[8] H. P. Furth and C. W. Hartman Phys. Fluids **11** 408 (1968).

[9] W. A. Cooper et al., Nucl. Fusion **29**, 617 (1989).

[10] C. Nuehrenberg, Phys. Plasmas **3**, 2401 (1996).