TAE SATURATION OF ALPHA PARTICLE DRIVEN INSTABILITY IN TFTR*

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A nonlinear theory of kinetic instabilities near threshold [Berk H.L., et al., Plasma Phys. Rep. **23**, 842 (1997)] is applied to calculate the saturation level of Toroidicity-induced Alfvén Eigenmodes (TAE), and to be compared with the predictions of δf method calculations [Y. Chen, PhD thesis, Princeton University, 1998]. Good agreement is observed between the predictions of both methods and the predicted saturation levels are comparable with experimentally measured amplitudes of the TAE oscillations in TFTR [D. J. Grove and D. M. Meade, Nucl. Fusion **25**, 1167 (1985)].

Toroidicity-induced Alfvén Eigenmodes [1,2] (TAE) have attracted much attention in recent years in connection with DT experiments on TFTR and JET tokamaks [3], where α-particles from fusion reactions were generated and their confinement and effects on MHD stability were studied. Both Mirnov coils and reflectometer diagnostics [4] during TFTR DT experiments have shown low level TAE signals that are driven by the fusion produced alpha particles.

In this paper we apply a recently developed nonlinear theory of kinetic instabilities [5] to calculate the saturation level of TAE in TFTR and compare it with the results of numerical simulations, where a δf algorithm was utilized [6,7] taking collisions into account through a Monte-Carlo method. A newly extended version of NOVA-K, which now includes the Finite Orbit Width (FOW) and Larmor Radius (FLR) effects [8], allows the expression for the growth rate and saturation level to be integrated accurately over the phase space for the ensemble of α-particles. Such integration is important for alpha particles which have orbit widths comparable to the mode width and even to the radius and when the ensemble of particles has a broad energy distribution. Thus all the particles have different collisional frequencies and quantitatively have different types of interaction strengths with the TAE mode.

First we give a physical picture for the mechanisms involved in mode saturation and outline the theory of kinetic instabilities near threshold. In TFTR DT experiments, where TAEs were observed, the population of α-particles had relatively low beta $\beta_\alpha \leq 0.1\%$, which means a weak TAE drive. At the same time NBI injected ions contributed to a dominant damping mechanism, Landau damping. Only $100 - 150$ $msec$ after the NBI was turned off and the NBI ions had slowed down, did the TAE become destabilized by fusion α-particles [4]. The mode excitation, at least during the initial excitation, should be close to the near threshold regime, where $\gamma \equiv \gamma_L - \gamma_d \ll \gamma_L$, and $\omega_b/\nu_{\text{eff}} \ll 1$ [9]. During the peak of the activity γ_L might be appreciably bigger than γ_d. Here γ_L is the linear TAE growth rate, γ_d is the TAE damping rate, ω_b is the frequency of α-particle trapping by the perturbed field of the mode, and ν_{eff} is the effective scattering frequency for particle-mode phase decorrelation.

To clarify how the scalings for saturation arise we present the following dimensional arguments. We note that for weakly kinetically driven instabilities the evolution of the wave energy, W_E (for Alfvén-like waves, the wave energy is nearly equally divided between perturbed magnetic energy and wave kinetic energy so that $W_E = \int d^3\mathbf{r}\,\delta\mathbf{B} \cdot \delta\mathbf{B}/4\pi$, where $\delta\mathbf{B}$ is the perturbed magnetic field) can be written as,

$$\frac{\partial W_E}{\partial t} = P_T - 2\gamma_d W_E, \tag{1}$$

with γ_d the rate of background dissipation and P_T the power transfer from resonant alpha particles to the wave, which in general is written as,

$$P_T = \int d^3\mathbf{r}\,\mathbf{j}_r\mathbf{E} = \int d\Gamma\,e_\alpha\mathbf{v}\mathbf{E}f, \tag{2}$$

where $\mathbf{j}_r$ is the perturbed current from the resonant α-particles which have a charge e_α and a velocity $\mathbf{v}$, f is the alpha particle distribution near the resonance region, Γ is six dimensional phase space, and $\mathbf{E} = \mathbf{E}(\mathbf{r}, t)$ is the perturbed electric field. For each particle the resonance condition is determined by

$$\Omega = \omega + n < \omega_d > + l\Omega_b \tag{3}$$

with $< \omega_d >$ is the particle mean toroidal transit frequency, $< \ldots >$ denotes time averaging over unperturbed orbits, Ω_b is the unperturbed bounce frequency, n and l are integers.

In Ref. [5] it was shown that the resonant nonlinear trapping frequency of a particle near the O-point of a finite amplitude is given by,

$$\omega_b^2 = \left\langle e_\alpha\mathbf{E} \cdot \mathbf{v}\frac{n}{\omega}\frac{\partial\Omega}{\partial P_\varphi}\bigg|_{H'} \right\rangle, \tag{4}$$

where $H' = H + \omega P_\varphi/n$, with P_φ and H the particle canonical momentum and energy (note that H' is conserved during the particle motion in a steady wave, whose toroidal angle φ and time dependence varies only as $n\varphi + \omega t$) and the time average is taken for an exactly resonant particle. Further, it was found that at fixed H' and magnetic moment, the steady distribution function satisfies the equation,

$$\Omega\frac{\partial f}{\partial \psi} - \omega_b^2 \sin \psi \frac{\partial f}{\partial \Omega} - \nu_{\text{eff}}^3 \frac{\partial^2 f}{\partial \Omega^2} = -\nu_{\text{eff}}^3 \frac{\partial^2 F}{\partial \Omega^2}, \tag{5}$$

where $\psi = -n\varphi - \omega t - l\Omega_b t$, and $\nu_{\text{eff}}^3 = \nu_c < |\partial P_\varphi/\partial(\mathbf{v}/v)|^2 > (\partial\Omega/\partial P_\varphi|_{H'})^2$ with ν_c the 90° pitch-angle scattering rate, and F is the equilibrium distribution. The two extreme limits of interest where analytic calculations are straightforward are (**a**) $\omega_b^2/\nu_{\text{eff}}^2 \ll 1$ and (**b**) $\omega_b^2/\nu_{\text{eff}}^2 \gg 1$.

For case (**a**) we note that the lowest order estimate of P_T should reproduce linear theory, and we are required to iterate to higher order in powers of $\omega_b^2/\nu_{\text{eff}}^2$ if we wish to determine the saturation amplitude for which $\partial W_E/\partial t = 0$. Several iterations are required as symmetry properties cause some iterants of the P_T integrals to vanish. An expansion in powers of $\omega_b^2/\nu_{\text{eff}}^2$, allows us to write $f = \sum_p f_p$, with

$$\Omega\frac{\partial f_p}{\partial \psi} - \nu_{\text{eff}}^3 \frac{\partial^2 f_p}{\partial \Omega^2} = \omega_b^2 \sin \psi \frac{\partial f_{p-1}}{\partial \Omega}, p > 0, \tag{6}$$

and $f_0 = F$. To estimate f_p we note that it is 2π-periodic in ψ and thus, $\partial f_p/\partial\psi \sim f_p$. Further, the important resonance dependence of f_p on Ω is determined when the two terms on the left side of Eq.(6) are comparable, leading to, $\Omega f_p \sim \nu_{\text{eff}}^3 \partial^2 f_p/\partial^2\Omega \simeq \nu_{\text{eff}}^3 f_p/\Omega^2$. Hence the effective value and range of Ω is $\Omega \sim \nu_{\text{eff}}$, which then implies following ordering,

$$f_1 \sim \frac{\omega_b^2}{\nu_{\text{eff}}} \frac{\partial F}{\partial\Omega}, f_p \sim \frac{\omega_b^2}{\nu_{\text{eff}}^2} f_{p-1}, \int d\Omega f_p \sim \nu_{\text{eff}} f_p. \tag{7}$$

Now we substitute our estimates for f_p in the expression for P_T, using that $\mathbf{vE}$ is proportional to ω_b^2. We find,

$$P_T \simeq \int d\Gamma_\perp d\Omega d\psi \sum_l \alpha_l \frac{\omega_b^4}{\nu_{\text{eff}}} \frac{\partial F}{\partial\Omega} \left[1 - \sum_{p=1}^\infty \lambda_p \left(\frac{\omega_b}{\nu_{\text{eff}}}\right)^{2p} \right], \tag{8}$$

where $d^3\mathbf{r}d^3\mathbf{v} = d\Gamma_\perp d\Omega d\psi$, α_l is appropriate proportionality factor and $\lambda_p = O(1)$. To lowest order we recovered the scaling of linear theory, including that the drive is independent of ν_{eff}. To obtain the next order correction we need to note that the ψ integral in P_T vanishes for all odd values of p and therefore the leading correction term is for $p = 2$. The scaling for the stationary solution to $P_T = 2\gamma_d W_E$ is then found to be the same as in Ref. [9],

$$\omega_b^2 \simeq \nu_{\text{eff}}^2 \sqrt{\frac{\gamma_L}{\gamma_d} - 1}. \tag{9}$$

In the opposite limit (**b**), rigorously treated in Ref. [10], we assume $\nu_{\text{eff}}/\omega_b \ll 1$. In this case the ν_{eff}^3 term is small and we need to solve iteratively the kinetic equation for $f = F + \sum_p f_p$ in the following form,

$$\Omega \frac{\partial f_p}{\partial \psi} - \omega_b^2 \sin \psi \frac{\partial f_p}{\partial \Omega} = \nu_{\text{eff}}^3 \frac{\partial^2 f_{p-1}}{\partial \Omega^2}. \tag{10}$$

The lowest order solution, (p=0, where the right hand side vanishes) shows that f_0 varies on a scale $\Omega \sim \omega_b$, and hence $f_0 \simeq \omega_b \partial F / \partial \Omega$ for $\Omega \sim \omega_b$ and $f_0 \sim 0$ for $\Omega \gg \omega_b$. However, in this order P_T vanishes, so we need to estimate f_1 to find P_T. Thus as $\Omega \sim \omega_b$, we have,

$$f_1 \sim \frac{\nu_{\text{eff}}^3}{\Omega^2 \omega_b} f_0 \sim \frac{\nu_{\text{eff}}^3}{\omega_b^2} \frac{\partial F}{\partial \Omega}. \tag{11}$$

Substituting this estimate for f_1 in Eq.(2) gives for P_T,

$$P_T \sim \int d\Gamma \sum_l \alpha_l \omega_b \nu_{\text{eff}}^3 \frac{\partial F}{\partial \Omega} \simeq \frac{\nu_{\text{eff}}^3}{\omega_b^3} \gamma_L W_E. \tag{12}$$

As, at saturation, $P_T = 2\gamma_d W_E$, we find,

$$\omega_b^2 \simeq \nu_{\text{eff}}^2 \left(\frac{\gamma_L}{\gamma_d} \right)^{2/3}. \tag{13}$$

In experiments, in general, the ensemble of fast particles may have particles which are in different regimes with the respect to the collisionality ((**a**) and (**b**) limits), so that accurate integration over the whole phase space is needed. In Ref. [5] a simple interpolation formula, which combines two regimes mentioned above, was proposed and supported by numerical simulations:

$$\frac{\gamma_d}{\gamma_L} = \overline{1 + 0.57 U(\Gamma)/(1 + 1.45/U(\Gamma))^{1/3}}, \quad \overline{(\ldots)} = \int d\Gamma Q(\ldots)/\int d\Gamma Q, \tag{14}$$

where $U(\Gamma) = (\omega_b(\Gamma)/\nu_{\text{eff}}(\Gamma))^3$, $Q = \sum_l < eE\mathbf{v} >^2 F'_{H'} \delta(\Omega - \omega)$. Eq.(14) is used to calculate the amplitude of TAE using a recently developed version of NOVA-K [8], which includes FOW and FLR effects.

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δf **method**. Direct simulation of the nonlinear evolution of TAE's is carried out based on a collisional δf algorithm [6] and a Hamiltonian formulation of the guiding center motion [11]. The pitch-angle scattering operator for Coulomb collisions is implemented using the Monte-Carlo technique, and thus the collisional effect is correctly accounted for without explicitly calculating ν_{eff}.

TAE amplitude saturation in TFTR. The observed plasma oscillations in the presence of fusion alpha particles in TFTR DT experiments [4] have been attributed to the excitation of core localized TAE modes. The amplitude of TAEs in those measurements was low with $\delta B_\theta / B \simeq 10^{-5}$. For the purpose of the comparison of the two methods outlined above and the experiment we choose equilibrium of TFTR shot # 103101 at 2.92 sec where $n = 4$ was clearly seen for about $\sim 100 msec$. The safety factor profile was flat at the center and had $q(r = 0) = 1.6$, and $q(r = a) = 5$. The central Alfvén velocity was $v_A = 1.2 \times 10^9 cm/sec$. TAE poloidal harmonics radial structure is calculated by NOVA [12] and is shown in Fig. 1 at eigenfrequency $f = 214 kHz$ which matches the measured frequency at $2.92 sec$. By that time alpha particles born at the end of NBI had slowed down to energy $\mathcal{E}_c \simeq 1.5 MeV$, so that we assume their distribution function to be slowing down with the upper cutoff at energy $\mathcal{E} = \mathcal{E}_c$.

As already noted, NOVA-K calculates the growth rate induced by alpha particles including FOW and FLR effects [8]. Additional calculation had been added to allow an accurate numerical integration of Eq.(14) over the phase space. For the case under consideration the contribution of alpha particles with $\beta(0) = 0.083\%$ to the growth rate is $\gamma_\alpha / \omega = 1.1\%$ without FLR and $\gamma_\alpha / \omega = 0.74\%$ with FLR. The results of using the theory Eq.(14) are shown in Fig. 2 along with the results of using δf method implemented in ORBIT code [7] (both without FLR effects). The results were compared with the ORBIT code for the same alpha particle contribution to the growth rate, which required choosing in the ORBIT code lower α-particle beta $\beta(0) = 0.07\%$. The discrepancy is due to some technical simplifications used in the ORBIT calculations, such as using Shafranov shifted equilibrium with circular surfaces and approximate representation of MHD displacement vector etc. [7]. With this

synchronization we see that the two codes agree quite well with each other if $\gamma_L/\gamma_d \leq 10$. Note that agreement is best if the saturation level or equivalently γ_L/γ_d, is not too large. It has been noted in Ref. [13] that such discrepancy is expected at larger saturation levels, since then the analytic approximation, that assumes the nonlinear displacement of an alpha particle is small compared to the spatial scale of a mode, begins to fail. Note, that the experimental value for the amplitude saturation is $\delta B_\theta/B \simeq 10^{-5}$, which is in reasonable agreement with both calculations, provided we assume that $\gamma_L/\gamma_d \simeq 2$, which is compatible with NOVA-K results of linear TAE stability studies [14].

One issue of the analysis is the assumption that the saturation is at a steady level, rather than in the form of pulsations, that can be regular or pulsating. Theory predicts that if ν_{eff} is too small, the wave amplitude will oscillate in time, and perhaps even show explosive behavior. A detailed criteria has been obtained when the linear instability is due to a single resonance in phase space, and the condition for a steady state response has been found to be $\nu_{\mathrm{eff}}/(\gamma_L - \gamma_d) > 2$, when $(\gamma_L - \gamma_d) \ll \gamma_d$. In the opposite regime, $\gamma_L \gg \gamma_d$, physical energy arguments [15], as well as supporting simulations [16], indicate that $\nu_{\mathrm{eff}}/\gamma_d \gtrsim 1$ is required for achieving a steady state saturation level. An interpolation formula for the two regimes gives $\nu_{\mathrm{eff}}/\gamma_d(1 - \gamma_d^2/\gamma_L^2) \gtrsim 1$. However, in the work presented here, there is a continuum of resonance interactions, and a precise transition to a pulsating response has not been determined. Nonetheless, the basic criteria to achieve a steady response is expected to be the same within the numerical factor with substitution $\nu_{\mathrm{eff}} \to \overline{\nu_{\mathrm{eff}}}$. To ensure that we are in the near threshold regime, we perform averaging over the phase space to find $\overline{\nu_{\mathrm{eff}}}$. NOVA-K gives $\overline{\nu_{\mathrm{eff}}} = 6 \times 10^3 sec^{-1}$ to be compared with $\gamma_L - \gamma_d$, where $\gamma_L = 10 \times 10^3 sec^{-1}$, which provides $\overline{\nu_{\mathrm{eff}}}/\gamma_d(1 - \gamma_d^2/\gamma_L^2) \geq 1.56$ for any value of γ_d/γ_L.

The ORBIT code did show considerable pulsations, and the results plotted in Fig.2 are appropriate time average results, which are obtained when the time averaged amplitude is converged against the number of particles, while the amplitude is still pulsating. The question that must be addressed is whether these pulsations are due to noise intrinsic in the Monte-Carlo algorithm, or due to pulsations that are predicted by the nonlinear dynamics.

Further work is needed to definitively clarify this point. However, the noise does not appear to have a regular frequency pattern, that might be expected from pulsations arising from dynamical noise-free behavior. We have thus treated only time averaged data of the ORBIT as we interpret the data to be statistically stationary in time.

Summary

Good agreement has been demonstrated between two different approaches for determining the saturation amplitude of alpha particle driven TAE modes: phase space integral based on an interpolation formula between two nonlinear analytic limits and a particle simulation based on a combined $\delta - f$ and Monte-Carlo algorithm. The agreement makes both models credible and allows the rapid evaluation of the saturation amplitude in the NOVA-K code.

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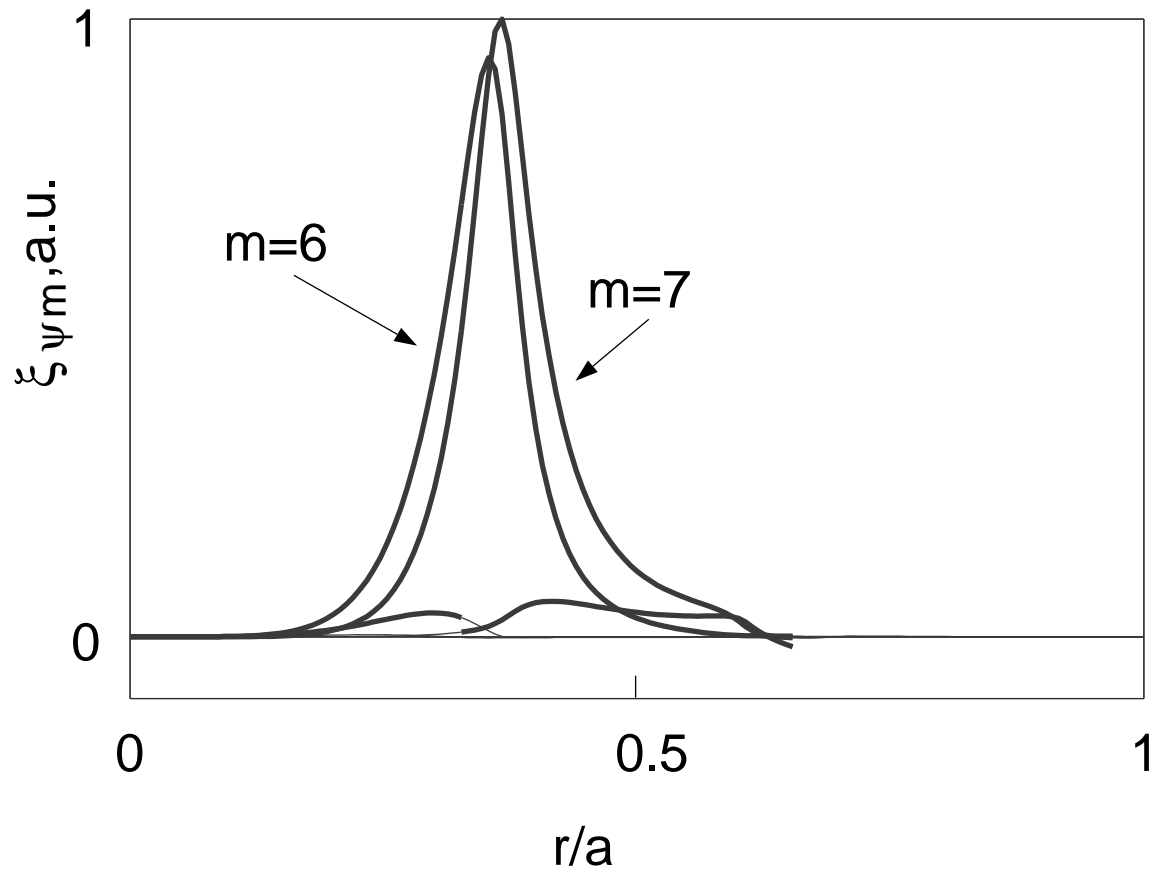[16] H. L. Berk, B. N. Breizman, and M. S. Pekker, Phys. Plasmas **2**, 3007 (1995).

FIG. 1. Radial structure of poloidal harmonics of $n = 4$ core localized TAE mode radial displacement obtained by NOVA for TFTR shot # 103101 at 2.92 sec.
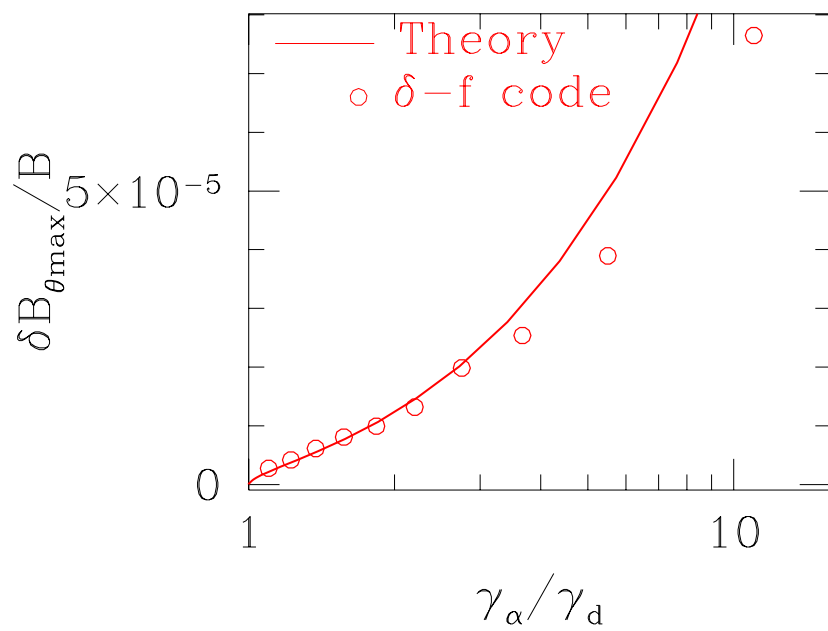
FIG. 2. Theory vs. δf method calculations.