Cooling a Birth Distribution of α-particles in a Tokamak with Waves

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Alpha particles, the byproducts of the DT reaction in a tokamak fusion reactor, might be cooled through interactions with waves. Numerical simulations employing two waves, one with frequency about the alpha cyclotron frequency, and one at much lower frequency, show the existence of parameter regimes where more than half of the α-particle power can be diverted to the waves.

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Tokamak reactors might be improved by the so-called "α-channelling" effect, i.e., if α-particle power were diverted into waves which were then employed to drive electron currents [1] or to heat fuel ions [2,3]. The first step to achieving the channelling effect is to demonstrate that waves can cool a birth distribution of α-particles, with α-particles moving to the wall, where they are extracted at lower energy. This paper addresses just this necessary first step, that is, how waves might control the entire birth distribution of α-particles.

The potential benefits of α-channelling are large: If 20% of the α-particle power could be channeled to current drive, the current drive efficiency might be doubled [1]. If 75% of the α-particle power could be diverted to fuel ions, resulting in a hot-ion mode, the fusion power output of the reactor for the same confined pressure might be doubled [3]. In practice, the power into waves is accompanied by α-particle power going to the wall, an inefficiency that is addressed here.

Accomplishing the channelling effect is akin to shaking particles out of a bottle with just a few holes. The 3-D volume here is the energy (ϵ), the magnetic moment (μ), and the canonical angular momentum (P_ϕ) of the α-particles; the boundary of the bottle corresponds to values of these constants of the motion for orbits intersecting the physical boundary of the tokamak. Waves diffuse particles in this constants-of-motion space (ϵ-μ-P_ϕ space). The trick is to place the "holes" in this bottle at low energy, and to devise plasma waves that shake most of the α-particles into these holes.

In devising such a bottle, the simultaneous satisfaction of criteria for different particles can be very frustrating, much as in the case of the analogous child's toy. Here, while waves might be devised to extract energy from a single α-particle [4], getting all of the particles to go into the lowest energy holes in response to the same set of waves is not simple. On the other hand, the parameter space of possible waves and magnetic bottle configurations is immense. What is reported on here is the development of a highly efficient numerical code suited to an exploration of this parameter space, the discovery of promising parameter regimes, and the interesting features exhibited by collections of particles in response to the waves.

Our investigations concentrated on using two waves. It turns out that [4] almost all the energy can be extracted from a single α-particle through the use of two waves, one with $\omega \ll \Omega_\alpha$ (such as the various Alfvén Eigenmodes (AE) [5,6]), and one with $\omega \sim \Omega_\alpha$ (such as the mode converted ion Bernstein wave (IBW) [7]). The high frequency wave extracts perpendicular energy, while the low frequency wave pushes the α-particles to the tokamak periphery and extracts parallel energy. However, unlike the case of one wave only [1], with two waves, there are no constraints on the particle motion, so that some α-particles may be heated while others are cooled.

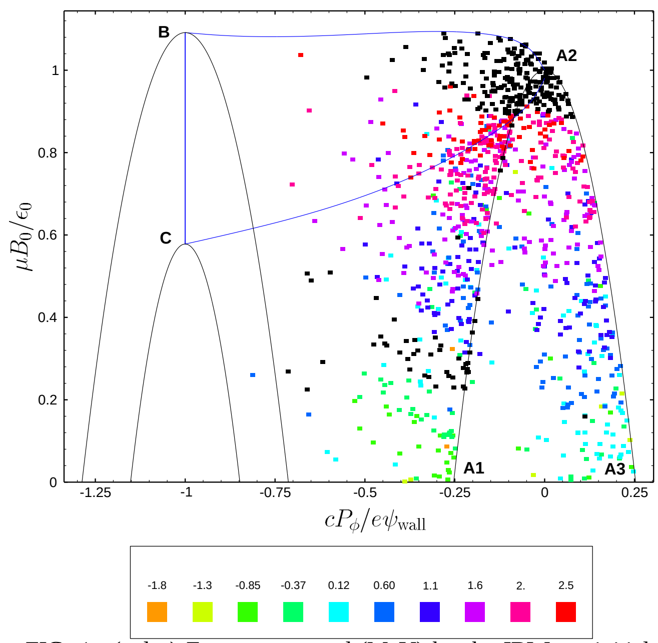

FIG. 1. (color) Energy extracted (MeV) by the IBW vs. initial location of particle in constants-of-motion space.

This behavior is illustrated in the following promising case for a reverse shear tokamak reactor with $A = 3$, $R_0 = 5.4$ m, $B_0 = 6$ T, and $I_p = 16.3$ MA. Here 70% of the power of the ejected α-particles (73% of those born) is diverted to waves, corresponding to 51% of the α-particle power. In Fig. 1 and Fig. 2 [8], the birth locations of 1000 3.5 MeV α-particles are shown in a fixed-energy slice of constants-of-motion space [9]. Those that eventually reach the tokamak periphery are color-coded to show the total energy exchange with each wave. Particles remaining in the tokamak are shown in black. The IBW (Fig. 1) extracts the most energy from those particles which have significant amounts of perpendicular energy. In contrast, the AE (Fig. 2), which must conserve μ, extracts the most energy from those particles which have the most parallel energy. In this case, only 0.2% of the α-particles were heated while being extracted.

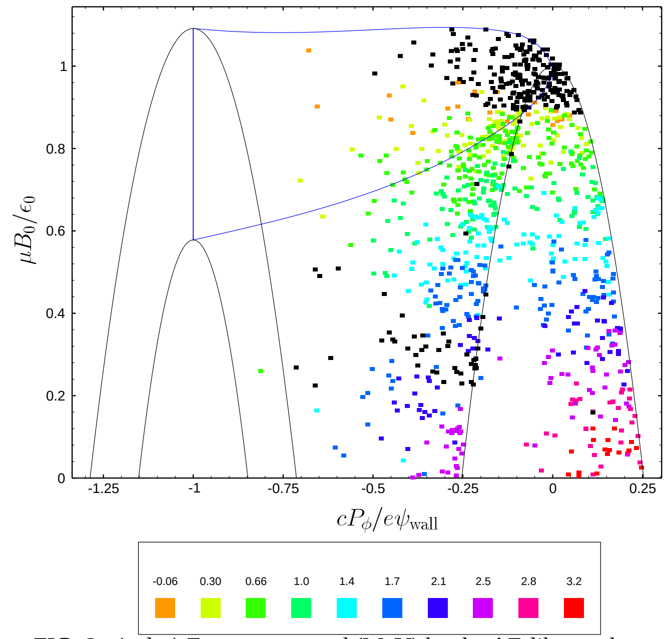

FIG. 2. (color) Energy extracted (MeV) by the AE-like mode vs. initial location of particle in constants-of-motion space.

Note that, in the absence of waves, collisions, and toroidal field ripple, ϵ, μ, and P_ϕ are conserved, so that the poloidal projection of the guiding center orbit is closed. Since the kicks due to the waves in each poloidal transit are small, the particle can be viewed as a tracing a trajectory in $\epsilon - \mu - P_\phi$ space. For the regimes of interest here, many wave modes are present, so it can be assumed that these kicks are uncorrelated.

For the problem at hand, codes that account for the full particle dynamics are needlessly complicated; even guiding center codes, which trace particles in the toroidal and poloidal directions, provide far more information than is required here. Thus, a novel numerical code has been developed to exploit the unique features of this problem. This code simply traces trajectories in $\epsilon - \mu - P_\phi$, thus avoiding integration of the 5 dimensional guiding center equations. The enormous savings in computer time now makes possible extensive scans of parameter space.

In achieving this savings, the wave-particle interactions must be calculated explicitly. Upon interaction with a wave with toroidal mode number n_ϕ and absorbing energy $d\epsilon$, the particle's P_ϕ changes by

$$dP_\phi/d\epsilon = n_\phi/\omega. \tag{1}$$

Thus, cooling α-particles, while moving them to the wall, requires $n_\phi/\omega > 0$. Waves with $\omega \ll \Omega_\alpha$ (e.g. the AE), leave μ invariant. For waves with $\omega \sim \Omega_\alpha$, an α-particle will receive a kick in velocity if $\omega - k_\| v_\| = n\Omega_\alpha$, where n is the harmonic number, such that

$$d(\mu B_0)/d\epsilon = n\Omega_{\alpha 0}/\omega \;\Rightarrow\; d\epsilon_\perp/d\epsilon = n\Omega_\alpha/\omega. \tag{2}$$

While the amplitude of the kick depends on the details of the interaction, the direction in ϵ-μ-P_ϕ-space is completely determined by Eq. (1) and Eq. (2). It is assumed in these simulations that the wave amplitudes are sufficient to eject particles in a time short compared to the collisional slowing down time.

Note that either wave acting alone is insufficient to accomplish significant cooling. The IBW is constrained to $1 < \Omega_\alpha/\omega < 3/2$ in DT plasmas. From Fig. 1, $\Delta P_\phi \sim e\psi_{\rm wall}$/c is needed to move an α-particle to the wall. The maximum change in energy is $\Delta\epsilon \sim \epsilon_0$. From Eq. (1), $n_\phi \Omega_{\alpha 0}/\omega \sim (R_0^2/\rho_0^2)\psi_{\rm wall}/(B_0 R_0^2)$. For large aspect ratio, A, $\psi_{\rm wall}/(B_0 R_0^2) \sim 1/(A^2 q_a)$. This implies $n_\phi \sim 1000$ would be necessary to remove α-particles from the reactor described above. Such a large n_ϕ is probably unachievable experimentally. The AE has $\Omega_\alpha/\omega \sim 1000 - 10000$, producing the opposite concern, i.e., α-particles ejected with little energy extracted.

Since the IBW extracts predominantly perpendicular energy, the most energy that could be extracted from particles with $\mu B_0/\epsilon_0 \ll 1$, is $(\epsilon_0 - \epsilon)/\epsilon_0 \sim \mu B_0/\epsilon_0$. The AE, on the other hand, extracts parallel energy while moving the energetic α-particles out, so that in principle it could nicely complement the IBW. This is just what is seen in Fig. 1 and Fig. 2.

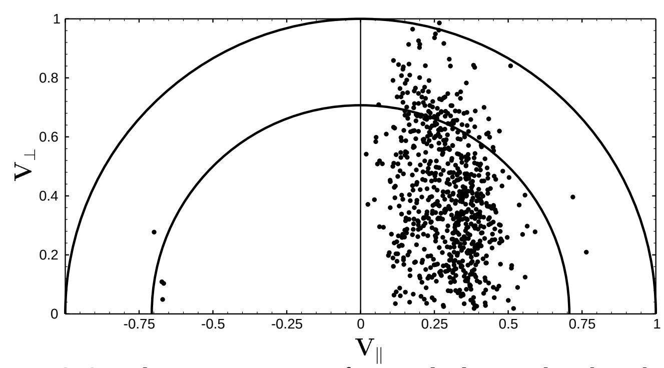

FIG. 3. Velocity space position for particles leaving the tokamak. Semi-circles represent energies of 3.5 MeV and 1.75 MeV.

The distribution function of the exiting α-particles exhibits interesting features. Fig. 3 shows the position in velocity space of the α-particles that hit the wall. Note the bunching in $v_\|$, with a range of perpendicular velocities.

Bunching also occurs in the poloidal exit angle. In Fig. 4, the distribution of the α-particles on the wall vs. poloidal angle is plotted where $0°$ is at the outer midplane, $180°$ at the inner midplane and whether the loss occurs on the upper or lower half of the tokamak depends on the direction of the ∇B drift. The loss is on the outer midplane, because, if the wall of the tokamak is a flux surface, cogoing ions whose orbits are slowly deformed outward will eventually scrape off at $0°$. By changing the size of the last kick the α-particle receives, this loss can be localized or distributed.

Note that in these simulations the amount of power flowing into the wall (20–30%) is much larger than the expected tolerance of future reactors (1–5%), if the loss is localized. While the loss might be tolerable if it were not localized, the interesting challenge is to exploit the bunching in phase space for further energy extraction.

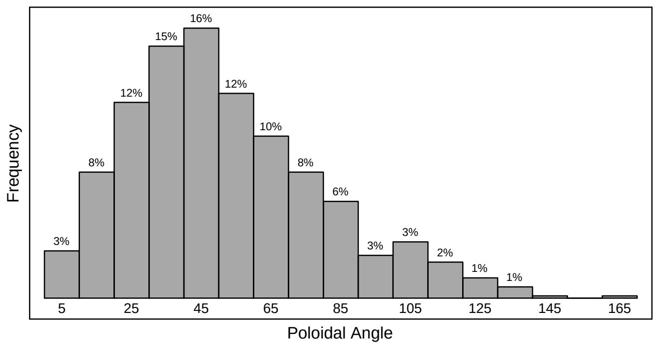

FIG. 4. Histogram of losses vs. poloidal angle in degrees.

In these simulations, the IBW is assumed to exist between two mod B surfaces with a wide range of $k_{\|}$ between $\sim n_\phi/R_0$ to $-n_\phi/R_0$. That $k_{\|}$ can be opposite in sign to n_ϕ [10], the so-called "$k_{\|}$ flip," is important because cogoing α-particles then satisfy a resonance condition with the IBW that is correctly phased for energy extraction.

For simplicity, we neglect the the structure of the AE and the dependence of the resonance condition on the details of the orbit. It is assumed also that so many modes are excited, that all particles whose orbits remain inside a specified flux surface, diffuse in P_ϕ until part of their orbit is outside of that flux surface. In addition to specifying where the mode exists, it is necessary to specify what n_ϕ/ω is acting on particles. We find from our simulations that energy extracted is sensitive to the functional dependence of n_ϕ/ω on $\mu B_0/\epsilon$.

For the case described above, we used $n_\phi \Omega_{\alpha 0}/\omega = 3500$ for particles with $\mu B_0/\epsilon > .85$, and $n_\phi \Omega_{\alpha 0}/\omega = 3500/3$ for the remaining particles. This choice of waves tends to maximize the energy extraction. The lower resonant frequencies experienced by high $\mu B_0/\epsilon$ particles [11] are consistent with this choice. The AE then extracts almost all the energy from the particles with $\mu \sim 0$ (passing particles), while allowing some particles with $\mu B_0/\epsilon \sim .85$ (trapped particles) to be moved out far enough that they can interact with the IBW, which is located near the edge for this simulation.

On the other hand, as can be seen in Fig. 1, many particles, from which the IBW might extract energy, do not leave the plasma. Instead, as these particles move out, their banana tips move closer to the outer midplane. The particle exchanges parallel energy only until it reaches a stagnation point orbit.

These deeply trapped particles might be taken out to the IBW layer, and then extracted, through stochastic ripple diffusion [12], which does not extract energy as it diffuses particles in P_ϕ. To model this effect, the simulation was modified so that particles with $\mu B_0/\epsilon > 1.0$ were treated as diffusing in P_ϕ with almost no energy extracted. Then, in contrast to 73% of particles extracted, 93% of α-particles are extracted, with 61% of the total power going to waves.

In contrast, with the same setup as above, but $n_\phi \Omega_{\alpha 0}/\omega = 3500$ for all values of $\mu B_0/\epsilon$, only 34% of the α-particle-particle power is extracted, while 64% of the particles leave the plasma. Thus, control over n_ϕ/ω for the AE, appears to be of major importance.

Energy extraction is sensitive also to the wave location. The location for the AE and the IBW (surface 1) for the simulation presented above are shown in Fig. 5. If the IBW power is placed along surface 2, many α-particles are heated by the IBW before being ejected. While the AE still extracts 20% of the α-particle power, the net effect of the IBW is to heat the α-particles by almost that amount, so that 95% of the α-particles leave the plasma but no energy is extracted from the particles. If the IBW power is concentrated at point 3, results about the same as those achieved for power concentrated at surface 1 can be obtained.

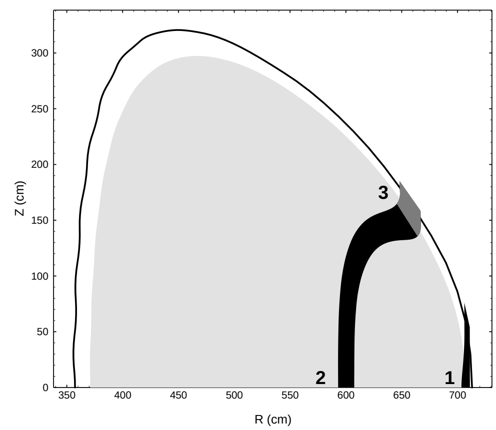

FIG. 5. Locations of IBW (black) and AE-like mode (light gray).

This illustrates the complexities of creating a bottle, from which α-particles can be shaken out of at low energy. For the case where the IBW layer is at surface 2, α-particles are still being cooled, (over 2/3 of the particles ejected lose, on average, 1/2 of their birth energy). But, in contrast to the case with the IBW at surface 1, holes have appeared at energies higher than the particles birth energy. If the location of the IBW moves, the resonant region in constants-of-motion space is changed. The diffusion paths now connect to the boundary, shifting the distribution of holes to higher energies. We have found that the energy extracted is maximized when the IBW layer is close to the outer midplane edge. Here ejecting heated α-particles is unlikely, and the wave can interact with trapped particles.

In these simulations the AE is chosen to exist only part of the way to the wall, but, so long as the relative amplitudes of the IBW and AE can be controlled, the result is not significantly changed if the AE extends to the wall. On the other hand, if the AE does not overlap with the IBW, few particles would be lost and little energy extracted.

The waves utilized in these simulations enjoy substantial experimental documentation. The mode converted ion Bernstein wave has been studied as a means of heating electrons or driving currents [13]. Recent experiments have documented the interaction of these waves with deuterium beam ions in D-He^3 plasmas [14]. Importantly, for the cooling scenarios presented here, experiments have shown that the layer of mode conversion can be controlled quite precisely by varying ω/Ω_α and the species mix of the plasma. The AE, generically used here as a low frequency perturbation, has many different forms (e.g. TAE, EAE, etc.), which have been shown to cause the loss of fast ions. Recent experiments [15,16] indicate that these modes can be launched externally.

What has been shown here is that low frequency waves and ion Bernstein waves can act in concert to extract upwards of 50% of the α-particle power from a tokamak reactor. For α-channelling it is important that the waves that are amplified at the expense of the α-particle power damp on ions. Theoretical calculations show that certain Alfv́en eigenmodes damp on plasma ions, and mode converted ion Bernstein waves in a moderately deuterium rich reactor damp on tritium ions. However, the demonstration of this further requirement for α-channelling goes beyond the scope of this work.

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[8] Each point represents the orbit of a 3.5 MeV α-particle in the reactor with $\mu B_0/\epsilon_0$ plotted vs. $cP_\phi/(e\psi_{\rm wall})$ where μ and P_ϕ are the magnetic moment and the canonical angular momentum, respectively. Particles with orbits which pass through the magnetic axis lie on the curve marked by A1, A2 and A3. A1 is an orbit with $v_\parallel/v = -1$ at the axis. A2 and A3 are orbits with $v_\parallel/v = 0$ and $v_\parallel/v = 1$, respectively. Orbits passing through the outer midplane at the wall, or inner midplane at the wall, would lie on curves B or C respectively. Trapped particles ($v_\parallel$ = 0 somewhere on their orbit) lie in the region bounded by the blue curve. A cogoing or trapped orbit will become lost (hit the wall) if it moves across the right half of curve B from right to left. A countergoing orbit is lost when it crosses the left half of curve C, or if it crosses the curve connecting B and C (i.e. it crosses the passing trapped boundary). The curves for different energies can be obtained by scaling the width of curves A,B,C in proportion to $\sqrt{\epsilon/\epsilon_0}$ and, at the same time, scaling the height of all of the curves by ϵ/ϵ_0.

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