A threshold for excitation of neoclassical tearing modes

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Abstract

Stability criterion is obtained for neoclassical tearing modes. A finite amplitude of magnetic island is required for their excitation. In both collisional and collisionless regimes the threshold is determined by the ratio of the transversal and the parallel transport near the island, when the flattening of the pressure profile eliminates the bootstrap current. A number of TFTR supershots are compared with the theory. Both the stability condition and the critical island width are consistent with experimental data.

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I. INTRODUCTION

As it is shown earlier theoretically [1,2], the resistive magnetohydrodynamic (MHD) bootstrap current driven (or neoclassical) tearing modes are always unstable in the tokamak plasmas. In this theory, the nonlinear evolution of a magnetic island is determined by the effect of island formation on the bootstrap current. For the small island width, $w < w_{sat}$, (w_{sat} is a characteristic island width proportional to the pressure gradient), the dependence $w \sim t^{1/2}$ (rather than $w \sim t$ as a classical Rutherford regime [3]) was obtained. Depending on the tearing mode stability parameter Δ', the modes either saturate at $w \simeq w_{sat}$ (for $\Delta' < 0$) or enters the Rutherford phase with $w \sim t$ (for $\Delta' > 0$).

Experimentallly, the neoclassical tearing MHD modes were observed in the neutral-beam (NB) heated supershot discharges [4] in the Tokamak Fusion Test Reactor (TFTR) Ref. [5,6]. These modes appear spontaneously, have low frequencies ($f < 50kHz$) and low poloidal and toroidal wave numbers $m/n = 3/2, 4/3, 5/4, 5/3, \ldots$ [6]. The nonlinear evolution of these tearing modes agrees well with predictions of the neoclassical theory [1,2]. In a course of Deuterium-Deuterium (DD) and Deuterium-Tritium (DT) experiments in TFTR, the tearing modes cause deterioration of plasma confinement [6] as is seen on the neutron rate, plasma stored energy, energy confinement time, *etc.*

In this paper we consider conditions of excitation of neoclassical tearing modes which were absent in the original theory. We take into account that the parallel transport in vicinity of the island is finite. Thus, for a small island width, the plasma pressure may be not flattened enough to affect the bootstrap current and to excite the neoclassical tearing modes. This consideration leads to determining both the stability criterion and the threshold of excitation (or a critical initial island width).

Earlier [7], the effect of finite electron parallel thermal conductivity on the flattening the pressure profile was considered in the collisional regime. Our results are valid for arbitrary regimes with the respect to plasma collisionality. We analyze the drift kinetic equation which is more relevant to the high-temperature tokamak plasmas rather than Braginskii equations used in Ref. [7]. In order to test the results and to clarify the remaining problems, we make an extensive comparison of the theory with the observed MHD activity in TFTR. We also presented a comparison with the model of Ref. [7].

The paper is organized as follows. In Section II we present the definitions and the tearing mode evolution equation. In Section III the bootstrap current contribution to the mode evolution equation is calculated for the electron drift kinetic equation. Section IV is devoted to the numerical comparison of the threshold island width in TFTR supershots. In Section V we present the conclusions of this paper.

II. EVOLUTION OF THE TEARING MODE

We start with definition of the coordinate system. The equilibrium magnetic field is described by the equilibrium poloidal flux , Ψ and the toroidal angle ζ:

$$\mathbf{B} = I(\rho)\boldsymbol{\nabla}\zeta + \boldsymbol{\nabla}\zeta \times \boldsymbol{\nabla}\Psi \tag{1}$$

where $I(\rho) = RB_\varphi$ measures the toroidal equilibrium field (R is a major radius, B_φ is the toroidal component of the magnetic field). Without perturbation

$$\mathbf{B}_p = \boldsymbol{\nabla}\zeta \times \boldsymbol{\nabla}\Psi \tag{2}$$

where $\mathbf{B}_p$ is the poloidal component of the magnetic field. The equilibrium helical flux in the vicinity of the rational magnetic surface with $q_s = m/n = q(\rho_s)$ the safety factor has the expansion

$$\Psi^* = \Psi_0 = \frac{q_s'}{q_s}\frac{(\rho-\rho_s)^2}{2}\frac{\partial\Psi}{\partial\rho}\bigg|_s , \tag{3}$$

so that $\boldsymbol{\nabla}\zeta\times\boldsymbol{\nabla}\Psi^* = \mathbf{B}_p^* = \mathbf{B}_p(\rho_s)(\rho-\rho_s)q_s'/q_s$ and $\boldsymbol{\nabla}\zeta\times\boldsymbol{\nabla}(\Psi-\Psi^*) = \mathbf{B}_{ps}$ where $B_{ps} \simeq const$ in the vicinity of magnetic surface, ρ is the minor radius indicator of the magnetic surface and prime means ρ derivative. With the perturbation, the total magnetic field in a low–β plasma can be written as

$$\mathbf{B} = \mathbf{B}_s + \mathbf{B}_{ps} + \mathbf{B}_p^*, \mathbf{B}_s = I(\rho)\boldsymbol{\nabla}\zeta, \mathbf{B}_p^* = \boldsymbol{\nabla}\zeta \times \boldsymbol{\nabla}(\Psi_0 + \Psi_1) \tag{4}$$

where $\Psi_1 = -\tilde{\Psi}_1 \cos\alpha$ is the perturbed helical poloidal flux, $\alpha = m\theta - n\zeta$ is a helical angle, θ is a poloidal angle. The total helical flux is

$$\Psi^* = \Psi_0 + \Psi_1 = \frac{\Psi' q_s'}{2q_s}((\rho - \rho_s)^2 - \frac{w^2}{8}\cos\alpha),$$

where $w = 4\sqrt{\tilde{\Psi}_1 q_s / \Psi' q_s'}$ is the full island width,

$$\tilde{\Psi}_1 = \frac{w^2}{16}\frac{\Psi' q_s'}{q_s} = \frac{w^2}{16}\frac{q_s'}{q_s} B_{ps} J \mid \nabla\theta \mid = \frac{w^2}{16}\frac{q_s'}{q_s^2} B_\varphi R J \mid \nabla\zeta \mid^2,$$

J is Jacobian of the transition to the coordinates ρ, θ, ζ, which satisfies to $J \mid \nabla\theta \mid = R$ for circular Shafranov shifted equilibrium. We define also an useful variable Ω as

$$\Omega = \frac{\Psi^*}{\tilde{\Psi}_1} = \frac{8}{w^2}(\rho - \rho_s)^2 - \cos\alpha. \tag{5}$$

Finally we define the flux surface average operator $\langle f \rangle$, where

$$\langle f \rangle = \frac{\int_{\Omega=const} f dl / \mid \nabla\Omega \mid}{\int_{\Omega=const} dl / \mid \nabla\Omega \mid} = \frac{\oint f d\alpha / (\Omega + \cos\alpha)^{1/2}}{\oint d\alpha / (\Omega + \cos\alpha)^{1/2}} \tag{6}$$

and the poloidal angle average operator $(f)_0$

$$(f)_0 = \frac{1}{2\pi}\oint f d\theta. \tag{7}$$

The poloidal magnetic flux obeys the equilibrium equation, the perturbed part of which reads:

$$\boldsymbol{\nabla} \cdot (R^{-2}\boldsymbol{\nabla}\Psi_1) = -\frac{4\pi}{c}\boldsymbol{\nabla}\zeta \cdot \delta\boldsymbol{j} \tag{8}$$

where $\delta\boldsymbol{j}$ is the perturbed current.T From Eq.(8), the tearing mode stability parameter Δ'

$$\Delta' \overset{\delta \to 0}{=} [\left.\frac{\partial\tilde{\Psi}_1}{\partial\rho}\right|_{\rho_s+\delta} - \left.\frac{\partial\tilde{\Psi}_1}{\partial\rho}\right|_{\rho_s-\delta}]/\tilde{\Psi}_1 \tag{9}$$

can be related to the perturbation of the current density

$$\frac{q_s'}{q_s}\frac{cB_\rho w}{16\sqrt{2}}\Delta' = -\int_{-1}^{\infty} d\Omega \oint \frac{d\alpha \delta j_{||} \cos\alpha}{\sqrt{\Omega + \cos\alpha}}. \tag{10}$$

We extract the inductive contribution in accordance with Ohm's law

$$\delta j_{||} = \frac{\boldsymbol{\nabla}\zeta}{\nabla\zeta}\delta\boldsymbol{j} \simeq \langle\tilde{j}_{||}\rangle + \frac{\langle E_{||}\rangle}{\eta} = \langle\tilde{j}_{||}\rangle - \frac{\langle\nabla_{||}\Phi\rangle}{\eta} - \frac{1}{\eta c}\frac{\partial}{\partial t}\langle\frac{\Psi_1}{J\mid\nabla\theta\mid}\rangle \tag{11}$$

where $\tilde{j}_{||}$ is the noninductive bootstrap current part of the perturbed parallel island current, η is the parallel Spitzer resistivity, $E_{||}$ is the island parallel electric field, Φ is the perturbed electrical potential. Making use of $\langle\nabla_{||}f(\Omega,\alpha)\rangle = 0$ and substituting Eq.(11) in Eq.(10), we find

$$\Delta' + \Delta'_{neo} = \Delta' + \frac{16\sqrt{2}q_s}{cwq'_sB_{ps}}\int_{-1}^{\infty}d\Omega\langle\tilde{j}_{||}\rangle\oint\frac{d\alpha\cos\alpha}{\sqrt{\Omega+\cos\alpha}} = \frac{4\pi I_1}{\eta c^2}\frac{dw}{dt}, \tag{12}$$

where

$$I_1 = \frac{\sqrt{2}}{2\pi}\int_{-1}^{\infty}d\Omega\langle\cos\alpha\rangle\oint\frac{d\alpha\cos\alpha}{\sqrt{\Omega+\cos\alpha}} = .8227$$

and noninductive bootstrap current $\tilde{j}_{||}$ to be determined later.

III. NEOCLASSICAL DRIVE OF TEARING MODES

A. Perturbed electron distribution in the presence of perpendicular transport

For simplicity, we consider only parallel and perpendicular transport terms in the electron drift kinetic equation in the vicinity of the island

$$[\frac{\partial}{\partial t} + v_{||}\boldsymbol{\nabla}_{||} + \boldsymbol{v}_{dr}\boldsymbol{\nabla} + \boldsymbol{v}_{\rho}\boldsymbol{\nabla}]F = C(F) \tag{13}$$

where v_{dr} is the curvature drift, $\boldsymbol{v}_{\rho} = v_{\rho}\boldsymbol{\nabla}\rho/\mid\nabla\rho\mid$, v_{ρ} is due to perpendicular transport and will be specified later, $C(F)$ is the collision operator.

Following Rutherford [8], we solve Eq.(13) by expanding the solution in two small parameters $\Delta = \nu_e/w_b$ and $\delta = \rho_{pe}/(\rho_s\Delta)$, ν_e is Coulomb collision frequency, $w_b = v_{||}/qR$ is the frequency of particle motion on the drift orbit, ρ_{pe} is poloidal electron dyroradius. Because of a slow resistive scale for tearing mode evolution, the first term is of the order of

$(1/w_b)(\partial/\partial t) \sim \delta \cdot \Delta^2$ and does not effect the solution. Other terms in Eq.(13) in vicinity of the resonance surface ρ_s lead to the following equation:

$$[\frac{v_\parallel}{qR}\frac{\partial}{\partial\theta} + \frac{v_\parallel}{qR}[nq_s'(\rho_s - \rho)\frac{\partial}{\partial\alpha} - \frac{1}{\Psi'}(\frac{\partial\Psi_1}{\partial\theta} + m\frac{\partial\Psi_1}{\partial\alpha})\frac{\partial}{\partial\rho}] + \boldsymbol{v}_{dr}\boldsymbol{\nabla} + \boldsymbol{v}_\rho\boldsymbol{\nabla}]F = C(F). \qquad (14)$$

We solve this equation on different scales of $\beta = |\rho - \rho_s|/\rho_s$. We have the following relative orderings for terms in Eq.(14)

$$1 : \beta : \frac{w^2}{\rho_s^2}\beta^{-1} : \delta\Delta : \frac{v_\rho}{\rho_s w_b}\beta^{-1} : \Delta. \qquad (15)$$

The solution can be presented in the form

$$F = f_0 + f_1 + f_2 + \cdots + g_0 + g_1 + g_2 + \cdots = \sum_i (f_i + g_i)$$

where g_i corrections account for δ expansion and i subscript gives Δ expansion. For f we have:

$$[\frac{\partial}{\partial\theta} + nq_s'(\rho_s - \rho)\frac{\partial}{\partial\alpha} - \frac{1}{\Psi'}(\frac{\partial\Psi_1}{\partial\theta} + m\frac{\partial\Psi_1}{\partial\alpha})\frac{\partial}{\partial\rho} + \frac{qRv_\rho}{v_\parallel}\frac{\partial}{\partial\rho}]f = \frac{qR}{v_\parallel}C(f) \qquad (16)$$

We present equilibrium distribution in the form $f = f_M(\rho_s) + f_M'(\rho_s)(\rho - \rho_s) + h$ where f_M is a Maxwellian distribution and $h \ll f_M$ is a correction to provide the heat flux $\Gamma = -n\chi\frac{\partial}{\partial\rho}T$ (χ is a thermal conductivity coefficient) which satisfies

$$v_\rho f_M' = \overline{C}(h), \qquad (17)$$

where $\overline{C}(h) = (RC(h)/v_\parallel)_0/(R/v_\parallel)_0$ is bounce averaged collisional operator, which we take for the sake of simplicity in the form (see Ref. [9])

$$\overline{C}(h) = \bar{\nu}\frac{1}{L(\lambda)}\frac{\partial}{\partial\lambda}\lambda D(\lambda)\frac{\partial}{\partial\lambda}h; \qquad (18)$$

$$\bar{\nu} = 2\nu_e[Z_{eff} + G(\sqrt{\mathcal{E}/T_e})][T_e/\mathcal{E}]^{3/2}; \qquad (19)$$

$\lambda = m_e v_\perp^2 B_0/2EB$ is the pitch angle, $\mathcal{E}$ is the kinetic energy; Z_{eff} is the effective plasma charge, $G(z) = \frac{e^{-z^2}}{\sqrt{\pi}z} + \frac{2}{\sqrt{\pi}}(1 - \frac{1}{2z^2})\int_0^z e^{-t^2}dt$, $L(\lambda) = \oint(\sqrt{1 - \lambda\frac{B}{B_0}})^{-1}d\theta$; $D(\lambda) = \oint\frac{B_0^2}{B^2}\sqrt{1 - \lambda\frac{B}{B_0}}d\theta$.

In order to solve the kinetic equation in a closed form, we assume that v_ρ has the same dependence on the pitch angle as $v_{||}$, i.e. $v_\rho^0\sqrt{1-\lambda}$. Then, the solution can be conveniently written as

$$h = \frac{v_\rho^0}{\bar{\nu}} f_M' I(\lambda). \tag{20}$$

In the passing particle approximation, using the boundary condition $\lim_{\lambda\to 1} h = 0$ and boundness of h, we have a simple solution $I(\lambda) = -2\sqrt{1-\lambda}$. Then, it can be shown that $\mid v_\rho^0 \mid = \sqrt{\chi\bar{\nu}}$ gives the required heat flux

$$\Gamma = \int v_\rho h(\mathcal{E} - \frac{3}{2}T)d^3v = -n\chi T'. \tag{21}$$

Typical experimental value for the perpendicular thermal transport χ is $10^4 cm^2/sec$, which gives $v_\rho/\rho_s w_b \simeq \Delta^{3/2}$.

In order to find the bootstrap current contribution to the evolution equation, we start with the case when $w/\rho_s < w_0$, where $w_0 = 4q_s\sqrt{v_\rho^0 R/mq_s'v}$, i.e. when the perpendicular transport (fourth term in Eq.(16)) exceeds the parallel transport in the island (third term in Eq.(16)). Then outside the island (at $\beta = O(1)$), the perturbed distribution function appears in the form

$$\tilde{f} = f_M' \frac{w^2 \cos\alpha}{16(\rho - \rho_s)} \tag{22}$$

The higher order corrections to the distribution function depend on α as $\cos l\alpha, l = 2, 3, \ldots$, and are resulted from the perturbation Eq.(22) through the collisional integral at $\beta > \sqrt{\Delta}$.

In vicinity of the island $\beta < \sqrt{\Delta}$ the last term on the LHS in Eq.(16) governs the perturbed distribution function which satisfies

$$v_\rho \frac{\partial}{\partial\rho}\tilde{f} = \overline{C}(\tilde{f}).$$

The solution can be constructed as follows

$$\tilde{f}(\rho) \propto (\frac{\bar{\nu}}{v_\rho^0}(\rho - \rho_s) - 2\sqrt{1-\lambda}). \tag{23}$$

The last term does not contribute to the bootstrap current and disappears after velocity integration. The solution to Eq.(23) is determined with an arbitrary factor which can be found by comparing the parallel and perpendicular perturbed heat fluxes and taking the divergence of them. This procedure gives an estimate for $\beta = w_0/\rho_s = o(\sqrt{\Delta})$ (see Ref. [7]). We may conveniently present the solution at $w < w_0$ as follows

$$\tilde{f} = f_M' \frac{w^2}{16} \frac{\rho - \rho_s}{(\rho - \rho_s)^2 + w_0^2} \cos\alpha. \tag{24}$$

In opposite case $w > w_0$ neigther collisions or perpendicular transport can not overcome the parallel heat flux and the lowest order solution is governed by the following equation:

$$(-nq_s'(\rho - \rho_s)\frac{\partial}{\partial\alpha} + m\sin\alpha\frac{w^2}{16}\frac{q_s'}{q_s}\frac{\partial}{\partial\rho})(f + \tilde{f}) = 0, \tag{25}$$

which means that $f + \tilde{f} = g(\Omega)$. This solution gives rise to the non-Maxwelian contribution to the perturbed distribution function through the equation

$$v_\rho \frac{\partial}{\partial\rho} g(\Omega) = \bar{C}(\tilde{h}) \tag{26}$$

Similar to Eq.(20) we present the solution in the form

$$\tilde{h} = I(\lambda)\frac{v_\rho^0}{\bar{\nu}}\frac{4\sqrt{2}}{w}\sqrt{\Omega + \cos\alpha}\frac{\partial}{\partial\Omega}g(\Omega) \tag{27}$$

Comparing two expressions Eq.(20) and Eq.(27) in terms of radial transport through the surface $\rho = const$ we obtain

$$\pm\frac{4\sqrt{2}}{w}\oint\sqrt{\Omega + \cos\alpha}\frac{d\alpha}{2\pi}\frac{\partial}{\partial\Omega}g(\Omega) = f_M' \tag{28}$$

Extracting $\cos\alpha$ component of $g(\Omega)$ one can obtain the lowest order perturbed distribution funcion in the form

$$\tilde{f} = \mp f_M'\frac{w}{\sqrt{2}}\int\frac{\sin\alpha^2 d\alpha}{(\oint\sqrt{\Omega + \cos\alpha}d\alpha)\Big|_{\rho=const}}\cos\alpha + G(\rho) \tag{29}$$

where $G(\rho) = \int g(\Omega)|_{\rho=const}\, d\alpha/2\pi$ and $\oint\sqrt{\Omega + \cos\alpha}d\alpha = 4\sqrt{2}kE(1/k)$, $k = \sqrt{(\Omega+1)/2}$. In the limit $|\ \rho - \rho_s\ | \gg w/2$ Eq.(29) gives the dependence similar to Eq.(22) and in the opposite limit $|\ \rho - \rho_s\ | \ll w/2$ for $\cos\alpha$ harmonic of $\tilde{f}$ Eq.(29) yields

$$\tilde{f} \propto \mp w f_M' (\frac{\rho - \rho_s}{w})^3 \cos\alpha$$

and for

$$G(\rho) \simeq -f_M'(\rho - \rho_s).$$

B. Bootstrap Current Contribution to Δ'

Note that second and third terms in Eq.(13) can be combined into the form

$$[v_{\|}\boldsymbol{\nabla}_{\|} + \boldsymbol{v}_{dr}\boldsymbol{\nabla}]F = \frac{v_{\|}}{w_c}\boldsymbol{\nabla}\varphi \times \boldsymbol{\nabla}P_\varphi \cdot \boldsymbol{\nabla}F \tag{30}$$

where $P_\varphi = e\Psi/mc - v_{\|}R$. Then the equation for neoclassical correction g reads:

$$\frac{v_{\|}}{w_c}\boldsymbol{\nabla}\varphi \times \boldsymbol{\nabla}P_\varphi \cdot \boldsymbol{\nabla}(f+g) = C(f+g)$$

The zero order on Δ gives $f + g = F(P_\varphi) + H$ or

$$g_0 = -\frac{\partial f}{\partial \rho} v_{\|} R \frac{mc}{\Psi' e} = -\frac{\partial f}{\partial \rho}\rho_{pe}\frac{v_{\|}}{v} \tag{31}$$

where H is the correction to g_0 to satisfy the next order equation and having a form:

$$H = \frac{\partial f}{\partial \rho}\rho_{pe}\Theta(\lambda_t - \lambda)\int_\lambda^{\lambda_t} \lambda^{-1}\left(\frac{B_0^2}{B^2}\sqrt{1 - \lambda\frac{B}{B_O}}\right)_0^{-1} d\lambda, \tag{32}$$

Θ is the Heaviside step function. This equation gives the expression for the perturbed bootstrap current [10]

$$\tilde{j}_{\|} = -\frac{1.46\sqrt{\epsilon}c}{B_{ps}}\frac{\partial \tilde{P}}{\partial \rho}, \tag{33}$$

$\epsilon = \rho/R$. In the case of $w < w_0$, calculating the perturbed pressure in terms of the perturbed temperature using the perturbed distribution function from Eq.(24) gives the neoclassical contribution to Eq.(12):

$$\Delta'_{neo} = -4.63\frac{q_s}{q_s'}\beta_{pe}\sqrt{\epsilon}\frac{1}{T_e}\frac{\partial T_e}{\partial \rho}\frac{w}{w_\chi^2}, \tag{34}$$

where

$$w_\chi = 2.93 * 4q_s\sqrt{\sqrt{\chi\nu}R/mq_s' v_{T_e}} \tag{35}$$

accounts for perpendicular transport effect, and numerical coefficient includes the integration of collisional frequency Eq.(19) with the equilibrium Maxwellian distribution function and $v_{T_e} = \sqrt{2T_e/m_e}$. In the case of $w > w_0$, one can obtain the well known result [1,2] after island averaging Eq.(33) and making use of Eq.(28)

$$\Delta'_{neo} = -4.63\frac{q_s}{q_s'}\beta_{pe}\sqrt{\epsilon}\frac{1}{T_e w}\frac{\partial T_e}{\partial\rho} \tag{36}$$

An expression for the bootstrap current contribution in Eq.(12) which approximates both limits can be written in the form

$$\Delta'_{neo} = -4.63\frac{q_s}{q_s'}\beta_{pe}\sqrt{\epsilon}\frac{1}{T_e}\frac{\partial T_e}{\partial\rho}(\frac{w}{w^2 + w_\chi^2}), \tag{37}$$

where β_{pe} is the ratio of the electron pressure to the pressure of the poloidal magnetic field .

Note, that this approach can be also used to calculate the ion contribution to the tearing mode evolution equation. However the stability criteria is determined by electrons. To illustrate this let us take TFTR supershots plasma parameters [14] at the resonance point of the mode $m/n = 3/2$:

$$T_e = 4KeV, T_i = 7KeV, n_e = 0.3 \times 10^{14} cm^{-3}, R_0 = 2.5m, a = 0.9m. \tag{38}$$

Then the ratio of characteristic width w_χ calculated for electrons and ions is

$$\frac{w_{\chi e}}{w_{\chi i}} = \left[\frac{\chi_e}{\chi_i}\left(\frac{T_i}{T_e}\right)^{5/2}\frac{Z_{eff}}{1+(Z_{eff}-1)Z_{imp}}\sqrt{\frac{m_p}{m_i}\frac{1}{30}}\right]^{1/4},$$

where Z_{eff} is effective plasma charge, Z_{imp} is impurity ion charge, and m_p is proton mass. Assuming that $Z_{eff} = 2$ and $Z_{imp} = 6$ we obtain $w_{\chi e} \simeq 1$ cm and $w_{\chi e}/w_{\chi i} \simeq 1/3$. It means that electrons determine the triggering tearing modes.

IV. STABILITY CONDITION AND THRESHOLD FOR NEOCLASSICAL TEARING MODES

Because the parallel plasma transport is finite, the perturbation of the bootstrap current is small $\tilde{j}_{||} \propto w^2$ (in the collisionless case) for small enough islands $w < w_\chi$. In contrast to the original theory [1,2], where the neoclassical contribution $\Delta'_{neo} \propto w^{-1}$, now Δ'_{neo} is bounded and is small for the small islands, $\Delta'_{neo} \propto w$.

As a result of boundness of Δ'_{neo}, a stability condition arises for neoclassical tearing modes

$$\Delta' + \Delta'_{neo}(w) < 0 \tag{39}$$

which should be satisfied for all values of w. It can be violated only in some range of w, $w_{cr} < w < w_{sat}$. This gives a threshold $w = w_{cr}$ for ecxitation the tearing modes from finite amplitude which should be provided by some extra source. The second root $w = w_{sat}$ serves as a saturation level of magnetic island.

In this section we compare these predictions of the theory with the tearing mode MHD activity in TFTR. We calculate Δ' in an approximate way by considering cylindrical approximation for the current distribution and also include explicitly the Glasser-Green-Johnson effect [11]

$$\Delta' = \Delta'_{GGJ} + \Delta'_{cyl}, \tag{40}$$

in the form [12,13]

$$\Delta'_{GGJ} = -5.4 \frac{\beta_p \epsilon^2 P'}{\rho (q')^2 P} \frac{1-q^2}{w}. \tag{41}$$

The second term in Eq.(40) is obtained numerically by solving the tearing mode equation Eq.(8) with the equilibrium current density profiles taken from TRANSP simulation of TFTR plasma [14].

In figure 1 presented are the dependencies $|\Delta'|$ and Δ'_{neo} versus w for the tearing mode 3/2 at 3.3 s in the TFTR discharge #66869, where this mode was unstable. We used the transport coefficients and plasma parameters from TRANSP [14]. From Fig.1 one can see that if the pressure gradient is not too small there is the condition for the instability when $\Delta'_{neo} > |\Delta'|$. If this condition is fulfilled there is a range for the island width, $w_{cr} < w < w_{sat}$, where the mode is unstable. The w_{sat} and w_{cr} are determined from the equation $\Delta'_{neo} = |\Delta'|$. The width w_{sat} is the neoclassical saturation width [1,2,6]. The threshold width w_{cr} is the initial value of the island width which is necessary for the tearing mode to be unstable.

Figure 2 shows the time evolution of the threshold w_{cr} for the mode 3/2 in two similar TFTR shots ##66869, and 66859 those compared in Ref. [5]. Only in discharge #66869 the tearing mode 3/2 was observed. The MHD activity starts at time $3.3s$, which correspond to the minimum of $w_{cr}(t)$ on Fig.2. In shot #66859, also an activity of 4/3 mode was observed.

Figures 3, 4 show the statistical results for 3/2 and 4/3 modes in TFTR. 46 TFTR supershots (in the range $\#\#66840 - 66896$) were scanned with calculation the instability criterion $\Delta'_{neo} > |\Delta'|$ for modes $3/2, 4/3$ and the critical island width. Fig. 3 shows the minimum value $w_{cr}(t)$ versus q_0 for 3/2 mode. Discharges with an observed activity are marked by the solid circles and without 3/2 mode by the open circles. Figure 4 shows data for mode 4/3.

It is found, that in the case of presence of the 3/2 mode for Fig.3 and 4/3 for Fig.4 the start of MHD activity coinsides approximately with the minimum in $w_{cr}(t)$. One can see some scattering in results especially for tearing mode 4/3. The reason is uncertainties in plasma current density profile which leads to local change in second derivative and essential change in Δ'_{cyl}.

We have not found discharges, where the instability criterion $\Delta'_{neo} < |\Delta'|$ is not satisfied and the mode was seen. All unstable shots tend to have $q_0 < 1$ and low threshold width w_{cr}. We argue that to make the modes unstable one need the initial island width which may be provided for m/n mode by the toroidal coupling with mode $(m = n)/n$. Possibly such $(m = n)/n$ modes could give the initial push or initial $w = w_c r$.

V. CONCLUSIONS

In this paper, the contribution to the tearing mode nonlinear evolution from the pressure gradient driven bootstrap current was calculated basing on the model where the flattening of the plasma pressure near the island includes the unperturbed perpendicular diffusion and finite parallel diffusion along the magnetic field lines. The tearing mode can be unstable if two conditions are satisfied. First, the instability criterion $\Delta'_{neo} > |\Delta'|$ should be satisfied. This condition is consistent to TFTR experiments with the observed MHD tearing mode activity. The second, an initial perturbation should be provided for initial island width $w > w_{cr}$ in order to the instability be excited. Thus, the tearing mode activity is more likely in discharges with smaller critical island width w_{cr}.

The initial island width can be provided by the excitation of other modes, e.g., 2/2 or 3/3. These modes always satisfy the instability criterion $\Delta'_{neo} > |\Delta'|$ if $q_0 < 1$ and have very low $w_{cr} \leq 0.1cm$. The nonlinear dynamic of these modes should be studied more carefully both theoretically and experimentally.

Note, that other stabilizing effect e.g., ion polarization drift, was considered in Ref. [15]. It results in critical width of order the ion banana width, $w_{cr} \simeq \sqrt{\epsilon}\rho_{pi}$. It is proportional to the $\propto \sqrt{T_i}$ in contrast to our result $w_{cr} \propto T_e^{-1}$, Eq.(35), which is supported by more often experimental observations of unstable modes with rational surface close to the plasma center, where the temperature is higher.

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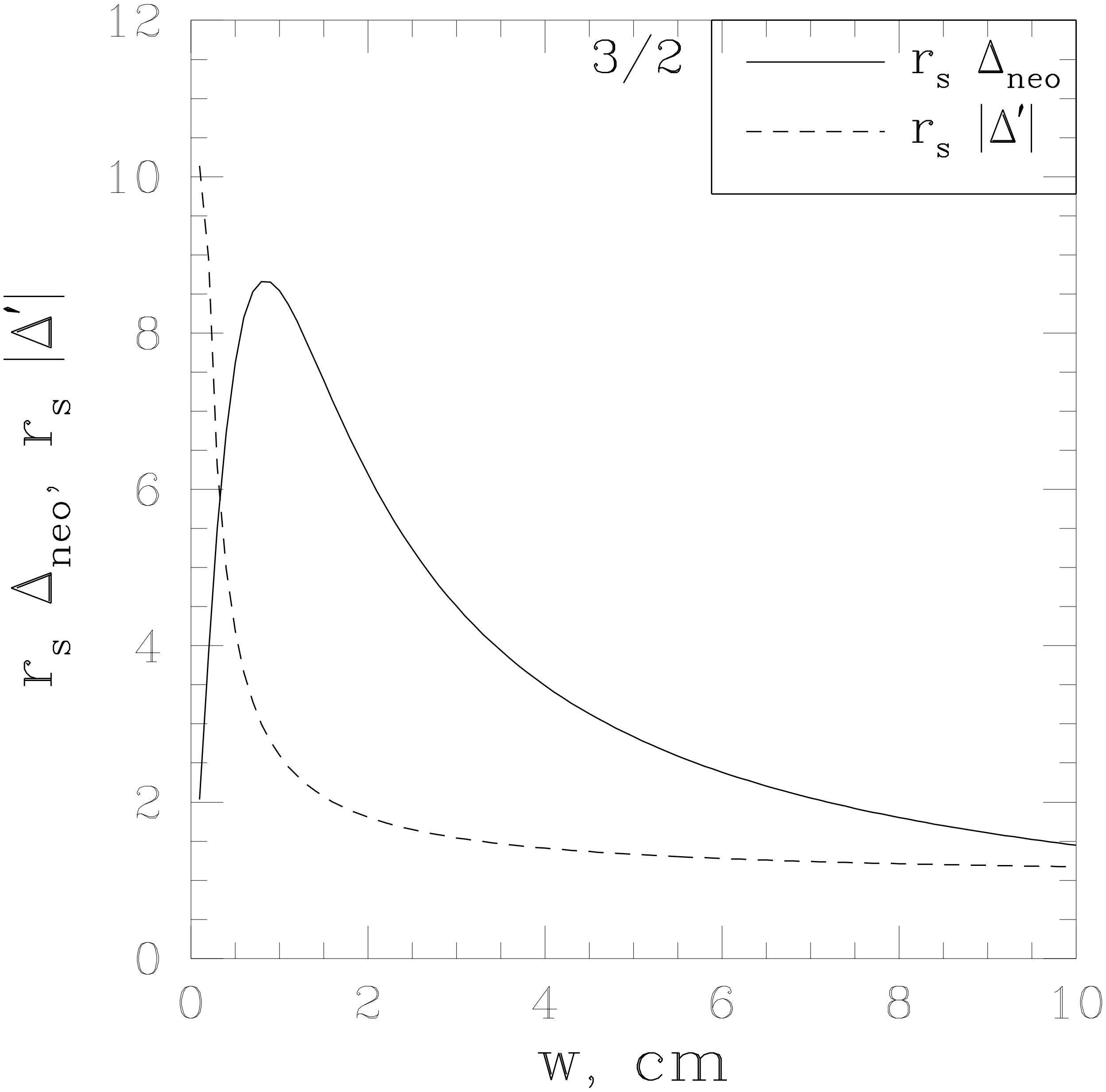

FIG. 1. The dependence of the absolute values of the tearing mode stability parameter $|\Delta'|$ and the neoclassical pressure gradient driven contributions Δ'_{neo} on the island width at $3.3sec$ for the mode 3/2 and the discharge #66869, when this mode was excited.

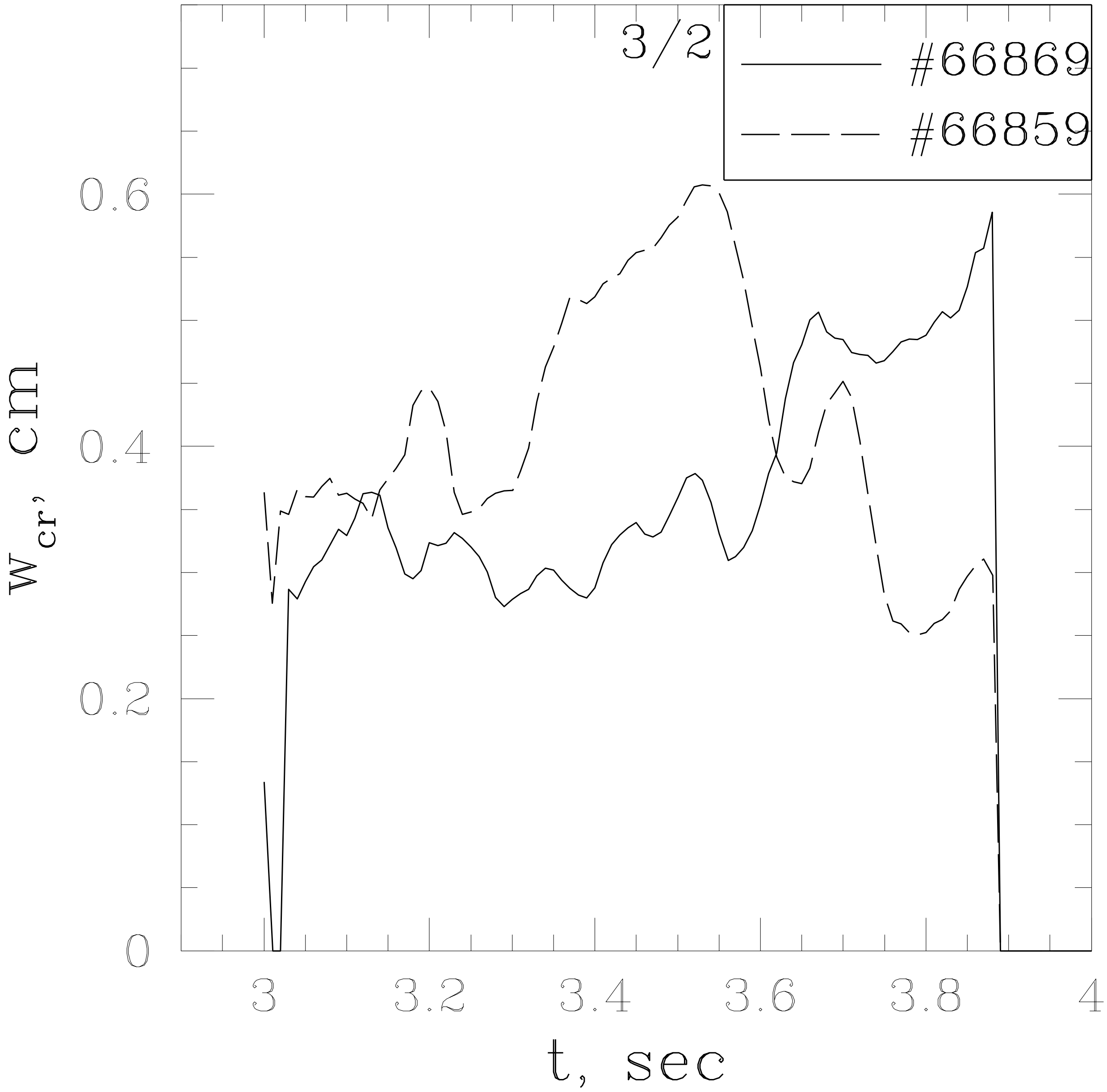

FIG. 2. Comparison of the calculated threshold width evolution $w_{cr}(t)$ for the tearing mode 3/2 in two similar TFTR discharges #66869, and #66859. In one of them #66869 3/2 tearing mode- like MHD activity was observed.

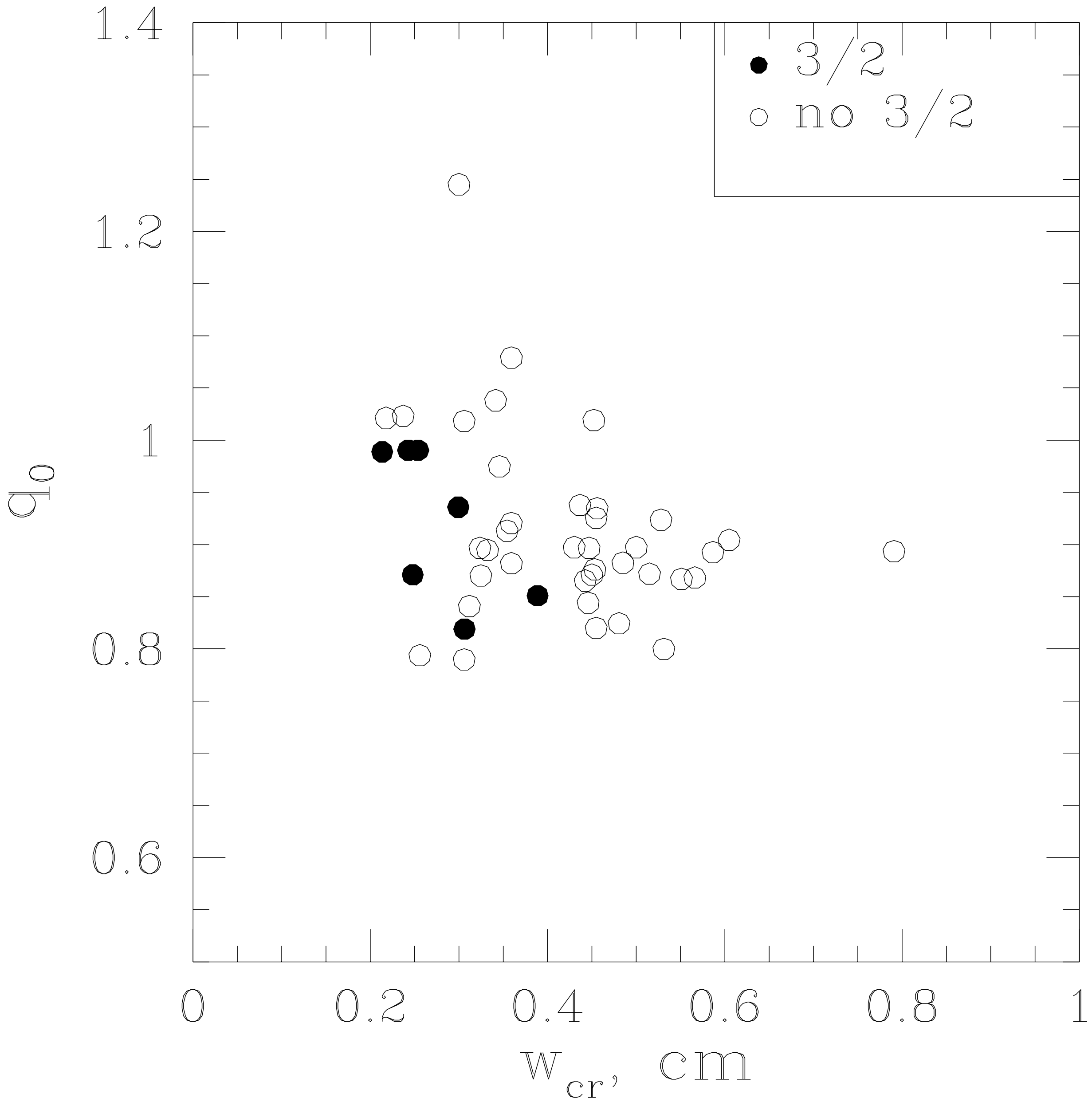

FIG. 3. Calculated points in the plane q_0 vs. w_{cr} for TFTR supershots with (solid circles) and without (open circles) 3/2 mode MHD- like activity. Plotted value w_{cr} is minimum in each discharge.

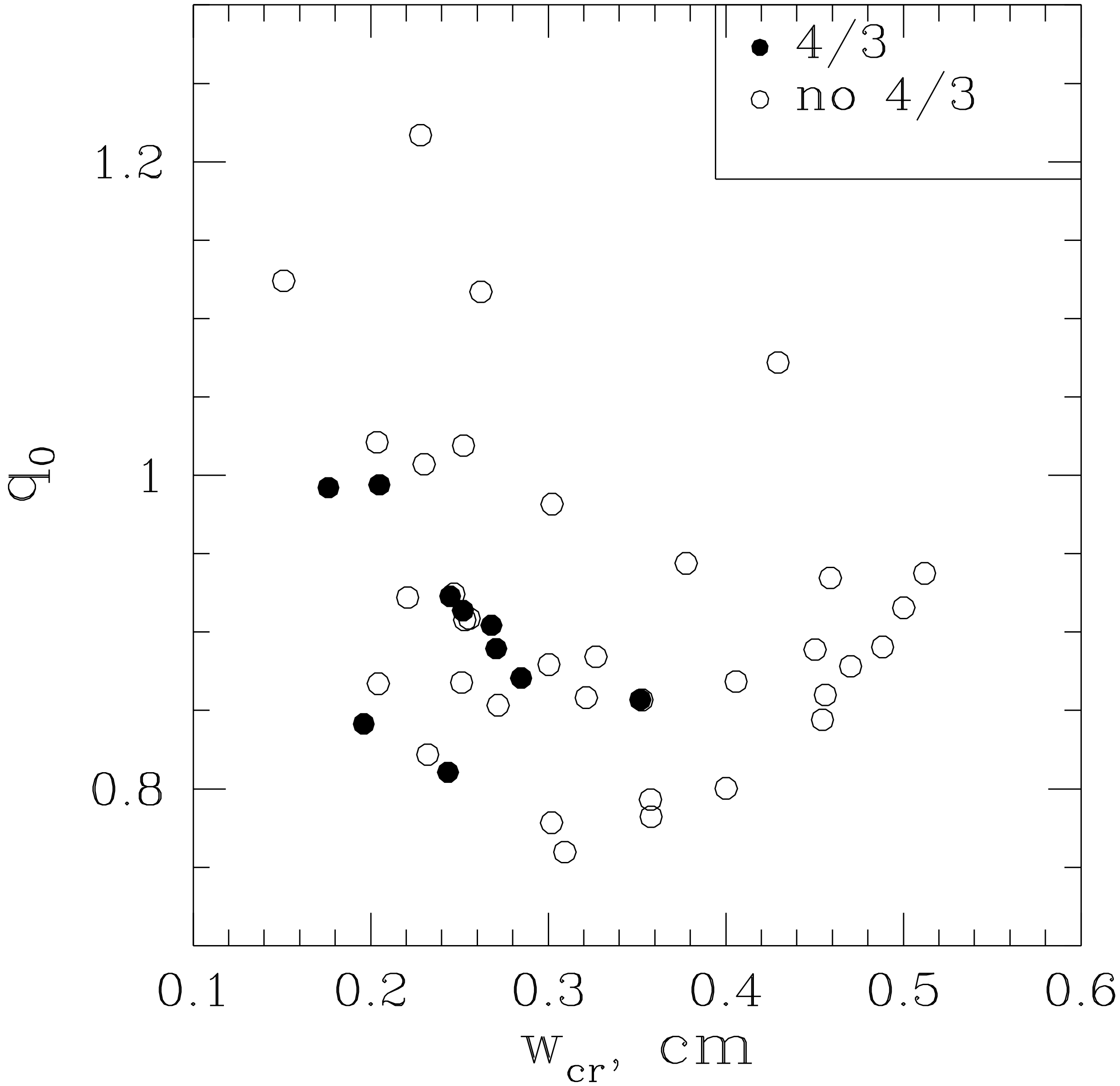

FIG. 4. The same as in Fig.3, but for the tearing mode 4/3.