Relativistic Raman Instability Shifted by Half-Plasma Frequency

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Abstract

A new nonlinear Raman instability in underdense plasma is investigated theoretically. Unlike the usual linear Raman instabilities which grow exponentially in time, this instability takes a finite amount of time to diverge. The explosion time t_∞ depends on the initial level of the perturbation. A general set of equations for spatio-temporal evolution of the forward nonlinear Raman scattering is derived and its temporal evolution is studied in detail. This new instability results in the generation of forward Raman radiation shifted by half the plasma frequency for laser intensities of order or exceeding 10^{18}W/cm^2, something that has been recently observed (Astorre Modena, private communication, 1995).

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I. INTRODUCTION

The advent of high power short pulse lasers makes possible the experimental exploration a new regime in laser-plasma interactions, where the plasma motion is strongly relativistic. A number of nonlinear phenomena occur in this regime, such as plasma wake generation [1–5], relativistic guiding [2,6], parametric instabilities [7,8] and harmonic generation [2,9], to name a few. Parametric instabilities have been studied in the context of Raman backscattering [10], small-angle Raman scattering [11], and Raman forward scattering [8]. A fully relativistic analysis of the Raman forward scattering (RFS) carried out in [8] concludes that, apart from the modification of the growth rate due to the relativistic change in the electron mass, relativity does not bring conceptually new physics into RFS.

This paper introduces a new explosive Raman forward scattering instability (ERFS), which is absent in a non-relativistic plasma. Actually, the motivations for this calculation was the recent experimental observation [13] of a Stokes component in the forward spectrum at a frequency $\omega = \omega_0 - \omega_p/2$, where ω_0 is the frequency of the laser pump and ω_p is the plasma frequency. While the harmonics of the plasma frequency have been observed in experiments and simulations (and are easy to explain by steepening of a strongly driven plasma wave–without even invoking relativity), the Rutherford experiment [13] appears to be the first clear observation of the fractional harmonic of the plasma frequency. The instability that we describe in this paper provides a possible explanation of this phenomenon.

The paper is organized as follows: In Section II, we derive the equations of motion for the plasma electrons in the combined fields of a laser pump and a forward scattered signal. In Section III, we derive the equations describing the forward scattered signal at $\omega = \omega_0 - \omega_p/2$. In Section IV, we show how this signal can be explosively unstable. In Section V, we summarize our results.

II. EQUATIONS OF MOTION

For simplicity we treat the problem as one dimensional and restrict our analysis to forward scattering of circularly polarized light. Extending these results to arbitrary laser polarization, three dimensional particle motion, and finite angle scattering would be straightforward. Electrons are characterized by a Lagrangian displacement $\xi(t, z_0)$, where z_0 is the position of the undisplaced electron. The motion of an individual plasma electron (which is labeled by z_0) can be described by the relativistic Hamiltonian [12]

$$H(t, z_0) = (1 + (\vec{p} + \vec{a}_0 + \vec{a}_1)^2)^{1/2} + \xi^2/2, \tag{1}$$

where we use $e = m = c = 1$, and time is normalized to the *nonrelativistic* plasma frequency. The normalized vector potentials a_0 and a_1 represent the laser pump and the scattered radiation (whose frequency will be later shown to be shifted by half the relativistic plasma frequency), respectively. The last term in the Hamiltonian (1) describes the longitudinal plasma wave. The transverse vector potentials are chosen as

$$\vec{a}_0 = \frac{a_0}{2}(\vec{e}_x + i\vec{e}_y)e^{-i\omega_0(t-z)} + \text{c.c},$$
$$\vec{a}_1 = \frac{a_1}{2}(t, z)(\vec{e}_x + i\vec{e}_y)e^{-i\omega_0(t-z)} + \text{c.c}, \tag{2}$$

where a_1 is assumed to be a slow function of z and t on the $1/\omega_0$ time scale. Since the Hamiltonian (1) is independent of the transverse coordinates x and y, canonical momenta p_x and p_y are conserved and can be chosen, without loss of generality, to vanish. H can be expanded in powers of a_1, which is assumed small:

$$H = H_0 + H_1 + H_2, \tag{3}$$

where

$$H_0 = (M^2 + p_z^2)^{1/2} + \xi^2/2$$
$$H_1 = \frac{a_0 a_1^* + a_0^* a_1}{4(M^2 + p_z^2)^{1/2}}$$
$$H_2 = -\frac{(a_0 a_1^* + a_0^* a_1)^2}{32(M^2 + p_z^2)^{3/2}}, \tag{4}$$

where $M^2 = 1 + \mid a_0 \mid^2$. Note that $\mid a_0 \mid$ *is not* assumed small. On the other hand, $p_z(t, z_0)$ (canonical momentum conjugate to $\xi(t, z_0)$) can be assumed small to study the initial evolution of a_1. From the single-particle Hamiltonian, it is clear that the perturbation term (H_2) is responsible for the instability. Note that a similar term proportional to $(a_0 a_1^* + a_0^* a_1)^2$ enters the right-hand side (RHS) of the Maxwell's equation for a_1.

Indeed, wave-particle energy exchange can be described by an interaction Hamiltonian $H_{int} = \vec{a} \cdot \vec{j}$ which can be expressed as a sum of single-particle contributions:

$$H_{int} = \sum_j \delta(z - z_j(t)) \frac{\vec{P}_j \cdot \vec{a}}{\gamma_j}, \tag{5}$$

where $\vec{P}_j$ is the kinematic momentum of the j'th particle. The term proportional to $(a_0 a_1^* + a_0^* a_1)^2$ is obtained by using the conservation of the transverse canonical momentum and expanding the relativistic γ to first order in a_1.

The unperturbed Hamiltonian H_0 describes a relativistic plasma wave. The first order perturbation H_1 describes the usual Raman forward scattering. Using H_1, in combination with Maxwell's equations, one recovers an exponential growth of a_1, shifted by the relativistic plasma frequency $\Omega_p = 1/\sqrt{M}$. In this paper we concentrate on H_2, which we show describes an explosive instability in a_1, shifted by half the relativistic plasma frequency. We note that H_2 is *absent* in the nonrelativistic treatment, since it describes the relativistic change of mass. Hence, the generation of the radiation shifted by half plasma frequency cannot be described by nonrelativistic equations of motion.

It is interesting to note the experimental data on half harmonic generation. The half harmonics have apparently never been observed in the backscattered spectrum. It would follow from the treatment here that the reason for this may be that the phase velocity of the resulting plasma wave is relatively small, therefore wavebreaking can occur at a relatively low intensity, before relativistic effects become important. Indeed, experimental measurements show a very complicated backscattered spectrum at intensities of order $10^{17}\mathrm{W/cm^2}$ [10], which is indicative of the wavebreaking. In contrast, in RFS the phase velocity of the plasma wave is close to the speed of light and wavebreaking is not an issue for intensities of order

$10^{18}\mathrm{W/cm^2}$, allowing the relativistic effects to become important.

Keeping H_0 and H_2 and assuming that p_z and ξ are small results in

$$\ddot{\xi} + \frac{\xi}{M} = \frac{1}{32M^4}\frac{\partial}{\partial z_0}(a_0 a_1^* + a_0^* a_1)^2, \tag{6}$$

where we used $\partial/(\partial\xi) = \partial/(\partial z_0)$. The nonlinear density $n(z,t)$, for the case of initially uniform plasma of density n_0 is given by

$$n(t,z) = n_0 \int dz_0 \delta(z - z_0 - \xi(t, z_0)), \tag{7}$$

which can be integrated, before wavebreaking, to get $n = n_0/(1+\partial\xi/\partial z_0)$. Linearizing Eq.(7) for small ξ and using Eq.(6) results in equation for $\delta n/n_0 = (n - n_0)/n_0$, we get:

$$\left(\frac{\partial^2}{\partial t^2} + \Omega_p^2\right)\frac{\delta n}{n_0} = \frac{(a_0 a_1^* + a_0^* a_1)^2}{32M^4}. \tag{8}$$

In deriving Eq.(8) we have assumed that, for laser pulses longer than the plasma period, the density wake $\delta n/n_0$ is a function of $\Omega_p(t - z/c)$, so that $\partial^2/\partial z^2 \approx -(\Omega_p)^2$.

III. DERIVATION OF INSTABILITY EQUATIONS

To close the feedback loop of the instability, we apply Maxwell's equation to the perturbed field a_1:

$$\left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2}\right) a_1 e^{-i\omega_0(t-z)} = -\frac{a_0(a_0 a_1^* + a_0^* a_1)}{4M^3}\delta n/n_0 e^{-i\omega_0(t-z)}. \tag{9}$$

Assuming a very underdense plasma, so that $\omega_0 \gg 1$, and introducing moving coordinates $\tau = t$ and $\psi = t - z$, Eq.(9) can be averaged over the fast scale of laser oscillation yielding

$$\left(i\omega_0\frac{\partial}{\partial\tau} - \frac{\partial^2}{\partial\tau\partial\psi}\right) a_1 = -\frac{a_0(a_0 a_1^* + a_0^* a_1)}{8M^3}\delta n/n_0, \tag{10}$$

where we have neglected the $\partial^2/\partial\tau^2$ term, since the instability is assumed to be slow at least in the beginning. Introducing

$$(a_0 a_1^* + a_0^* a_1) = \frac{b}{2}e^{i\Omega_p\psi/2} + \text{c.c.}$$
$$\frac{\delta n}{n_0} = \frac{-i\bar{n}}{2}e^{i\Omega_p\psi} + \text{c.c.}, \tag{11}$$

Eq.(8) can be re-written in the new coordinates as

$$\left(\frac{\partial}{\partial\tau}+\frac{\partial}{\partial\psi}\right)\bar{n}=\frac{b^2}{128M^{7/2}}, \tag{12}$$

and Eq.(10) becomes

$$\frac{\partial b}{\partial\tau}=\frac{a_0^2 b^* \bar{n}}{16\omega_0^2 M^{7/2}}. \tag{13}$$

A nonlinear set of partial differential equations (12-13) describes a spatio-temporal evolution of the explosive Raman instability. Its space-time characteristics are the same as for the usual linear Raman instability [8]. The variable b describes the modulation in laser intensity. As it is clear from Eqs.(12-13), the instability does not occur without an initial modulation in the laser intensity. This is in contrast with the linear Raman instability where *either* the initially present variation in laser light a_1 or the initial density modulation of the plasma $\delta n/n_0$ (which can, for example, originate from the discrete nature of plasma electrons) is unstable. On the other hand, as numerical simulations indicate [14], the main source of the linear RFS instability *is not* associated with initial density noise but, rather, with the profile of the laser which generates a finite density wake, which gets amplified. Hence, the source of both the explosive and linear RFS instabilities is the initial profile of the laser.

The ERFS effect is caused by the relativistic variation of the electron mass and, therefore, by modulation in the laser intensity. The change in the radiation intensity can result from (i) transverse spreading of the laser (diffraction) which leaves the total power in a given longitudinal slice in ψ constant or (ii) longitudinal bunching of the laser power which results from the varying group velocity of the radiation from slice to slice. In the context of the linear Raman scattering, the first process was studied by Antonsen and Mora [11] using paraxial-ray approximation, while the second process was studied by Mori [8]. In this paper we present only a one-dimensional calculation, which relies on mechanism (ii) and neglects small-angle sidescattering. Three-dimensional effects are easily incorporated into the wave equation (9) by adding a $\nabla_\perp^2$ term to the LHS. The wake equation (8) is similarly modified, as demonstrated in [11,15]. The study of the explosive small-angle Raman scattering will be described in the forthcoming publications.

IV. CONVECTIVE AND SELF-SIMILAR SOLUTIONS

The complete analysis of the set of nonlinear partial differential equations (12-13) is rather complicated, but the essential physics for our purposes, can be extracted by considering the purely convective solutions, i.e. by neglecting the ψ derivative in Eq.(12). The resulting set of ordinary differential equations is analogous to the standard three-wave instability (treated, for example, in Ref. [16]), where two out of the three waves are identical.

Two integrals can be easily identified:

$$C = i\mu_2(b^2 n^* - b^{*2} n)/2$$
$$E = \frac{1}{2}\mu_2^2 \mid n \mid^2 - \frac{1}{2}\mu_1\mu_2 \mid b \mid^2, \tag{14}$$

where

$$\mu_1 = \frac{1}{128 M^{7/2}}$$
$$\mu_2 = \frac{1}{16\omega_0^2 M^{7/2}} \tag{15}$$

Equations (12-13) can now be solved by quadratures:

$$\dot{Q} = 2\sqrt{(2E + \mu_1\mu_2 Q)Q^2 - C^2}, \tag{16}$$

where $Q = \mid b \mid^2$. Equation (16) allows to calculate the explosion time given initial conditions. It is straightforward to show that as $t \to t_\infty$, Q diverges as $Q \propto 1/(t - t_\infty)^2$, where t_∞ is the explosion time given by

$$t_\infty = 0.5 \int_{Q_0}^{\infty} \frac{dQ}{\sqrt{(2E + \mu_1\mu_2 Q)Q^2 - C^2}}. \tag{17}$$

Assuming that $\dot{Q} = 0$ at $t = 0$ and $\bar{n} = 0$ we find

$$t_\infty = \frac{32\sqrt{2}\pi M^{7/2}\omega_0}{a_0 Q_0^{1/2}}. \tag{18}$$

For $n_0 = 10^{20}\mathrm{cm}^{-3}$, $\lambda_0 = 1\mu$, and $a_0 = 1$, and an initial noise level of 10%, we find that the instability explodes after about 1mm of propagation through plasma.

An important difference between the explosive and linear RFS instabilities, as we show later, is that the explosion time t_∞ of the nonlinear instability depends on the amplitude of the initial wake $\bar{n}$. Therefore, a linear instability (such as, for example, RFS) can create a strong density wake, reducing the explosion time and resulting in a scaling different from Eq.(18). Taking $C = 0$ (which is always true, in the averaged sense, since C depends on the phasing between density and intensity perturbation) and assuming a large initial density wake,

$$2E \gg \mu_1 \mu_2 Q_0 \tag{19}$$

results in

$$t_\infty \approx \frac{1}{\sqrt{2E}} \ln \frac{4E}{\mu_1 \mu_2 Q_0}. \tag{20}$$

For similar parameters this scaling gives roughly the same result as Eq.(18).

Additional insight into the spatio-temporal evolution of ERFS can be obtained by noting that the system of partial differential equations (12-13) can be reduced to a set of ordinary differential equations by assuming self-similar solutions in the form

$$\begin{aligned} \bar{n} &= \frac{1}{\tau - \psi} \nu(\psi^{1/2}(\tau - \psi)^{1/2}), \\ b &= \beta(\psi^{1/2}(\tau - \psi)^{1/2}), \end{aligned} \tag{21}$$

where ν and β obey an ordinary differential equation

$$\begin{aligned} \frac{d\nu}{du} &= 2\mu_1 u \beta^2 \\ \frac{d\beta}{du} &= 2\mu_2 \beta^* \nu / u, \end{aligned} \tag{22}$$

where $u = \psi^{1/2}(\tau - \psi)^{1/2}$. The solution of the system (22) is similar to that of the system (12-13), with ψ derivatives neglected, and exhibits an explosion in u.

V. CONCLUSIONS

In conclusion, we have theoretically investigated a new nonlinear explosive Raman instability, ERFS. Unlike the standard Raman scattering, ERFS takes a finite amount of time to diverge and has a distinct spectral characteristic, shifted by *half* the plasma frequency from the incident laser. This spectral feature was recently experimentally observed [13], consistent with the theory of ERFS developed here. While there are complicated features to be considered, such as scattering at small angles, this calculation appears to capture the essential physics of the unusual experimental observation of a Raman instability shifted by half a plasma frequency.

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