Plasma discreteness effects in the presence of an intense, ultrashort laser pulse

V.I. Savchenko and N.J. Fisch

February 26, 1996

Abstract

Discrete effects of the plasma irradiated by an ultrashort, intense laser pulse are investigated. Although, for most plasmas of interest, the damping of the laser pulse is due to collective plasma effects, in certain regimes the energy absorbed in the plasma microfields can be important. A scattering matrix is derived for an electron scattering off an ion in the presence of an intense laser field.

PACS: 52.40.Nk, 52.50.Jm, 52.25.Rv, 52.25.Kn.

I. INTRODUCTION

The increasing degree of interest in high intensity lasers motivates a theoretical examination of the behavior of plasma in the fields of such lasers. Many recent studies are devoted to analysis of collective behaviour[1]; however, there is a need, addressed here, to examine discrete particle effects inside a very short electromagnetic pulse of linear polarization and of arbitrary amplitude.

The problem of collisional absorption[2] has been studied extensively for low intensity fields, where the electron velocity is not relativistic, and for time scales longer than collisional time. Now, very high intensity fields (10^{18} W/cm^2 and above) can be achieved in very short pulses[3]. For underdense plasma, the duration of such pulses is less than an inverse plasma frequency, and, hence, for ideal plasmas, even less than a collision time. Therefore, to find the amount of energy deposited into the plasma due to its discreteness, a standard approach, such as by using the Fokker-Planck collisional operator, may not be valid. Neither is one allowed to assume that the electron motion is nonrelativistic[4].

In this paper, we address collisional effects in just this ultraintense, ultrashort laser regime, and we find the change in the energy of plasma microfields (usually refered to as a correlational energy) due to the interaction of electrons with the laser pulse. Although, for plasma with an electron temperature $T_e \geq 1$ eV and a very intense, $\eta \sim 1$, short 0.1 ps laser pulse, the damping of the pulse is due to collective effects, and the collective energy sets up a plasma wake. Here the nonlinearity paramter η is $\eta = ea/mc^2$, where a is a wave vector potential, c is the velocity of light, $-e$ the electron charge, and m the electron mass. On the other hand, for a plasma at $T_e \sim 0.01$ eV and for a very short, moderately intense pulse, we find that the correlational energy can be greater than the energy stored in the plasma oscillations. Interestingly, in this regime, in which a plasma is irradiated by the laser waves, discrete (collisional) effects dominate collective effects. In contrast, in an ideal plasma , in the absence of any external fields, collisional effects are always down in magnitude by a factor of $n_e \lambda_d^3$, where n_e is an electron density and λ_d is a Debye length.

The problem of two particle collisions in the presence of an intense laser pulse remains unexplored for flux densities, so intense that the particle motion becomes relativistic. It corresponds to the nonlinearity paramter $\eta = ea/mc$ being of the order unity. For visible light $\eta \sim 1$ for a flux density $\sim 10^{18}$ W/cm^2. We analyze here the case of the scattering of an electron by the ion in the laser field at such intense flux densities. The final answer is presented in terms of a scattering matrix, which describes interaction of two particles both inside and outside the laser pulse.

Suppose a wave packet propagates in the x direction, with vector potential $\mathbf{a}(t - x/c)$. Suppose further that $\mathbf{a}(t - x/c < 0) = \mathbf{a}(t - x/c > T) = \mathbf{0}$. The pulse is then characterized by two time scales: its mean frequency $\bar{\omega}$ and its total phase duration T. The frequency width of the wave packet is $\delta\omega$, such that $T\delta\omega \sim 1$. We assume that the pulse travels at the velocity of light, which is a good approximation for waves in an underdense plasma. This approximation becomes even better for very intense waves[4].

Describing the interaction of plasma with a laser pulse of high intensity ($\eta \sim 1$) is complicated, because one can not use a dipolar expansion[5], which assumes $\eta \ll 1$. But the limit of a very short pulse is tractable[5]. If the pulse spectra is broad enough, i.e. $\omega_p/\delta\omega < 1$, so that the pulse duration is shorter than the time for the electrons to set up a collective response, the plasma collective field can be treated as a perturbation to the laser field. Recent advances in pulse compression[3] now make possible pulses as short as $T \sim 0.1$ ps, for which the above inequality holds for plasmas with densities up to 10^{18} cm^{-3}.

Consider an electron and an ion ($Z = 1$) inside a laser pulse. For distances between them such that $e^2/r^2 < ea\omega/c$, one can treat the ion field as a perturbation to that of the laser. For the fluxes under consideration ($\eta \sim 1$), the minimum distance up to which the above inequality holds is $r_{min} \sim \sqrt{r_e \lambda}$, where $r_e = e^2/mc^2$ is the classical radius of an electron, and λ is the wavelength of the radiation. For visible light, $r_{min} \sim 10^{-9}$ cm. Classical mechanics can be employed if the de Broglie wavelength of an electron $\lambda_e = h/m\gamma v \sim 10^{-9}$ cm ($\gamma = 1/\sqrt{1 - v^2/c^2}$, and v is the typical electron velocity in the laser pulse) is less than the distance between two particles. We see that over the range of distances, from ∞ to $\lambda_e \sim 10^{-9}$ cm, where classical mechanics is valid, the ion field remains less than that of the laser and we can employ perturbation theory.

The paper is organized as follows: In Sec. II we calculate the correlational energy after the pulse. In Sec. III, we study the relativistic interaction of an electron with an ion in the presence of a laser pulse. In Sec. IV, we generalize our results to finite initial velocity and derive the scattering matrix for ultrashort interactions. In Sec. V, our results are summarized.

To simplify the presentation in Sec. III, we use $m = c = -e = 1$, so the nonlinearity parameter η is in fact a; elsewhere, all quantities are expressed in c.g.s. units.

II. CORRELATIONAL ENERGY AFTER THE PULSE

Consider an ultrashort laser pulse, propagating in the x direction, with the width $\delta\omega$ larger than the plasma frequency ω_p. In this limit, the plasma collective field is smaller than that of the pulse. To zeroth order in $\omega_p^2/\delta\omega^2$, the only effect of the laser in the framework of the fluid model is a displacement of each electron in the direction of the pulse by[1]

$$h_0 = \frac{1}{2}\left(\frac{e}{mc^2}\right)^2 \int_0^T a^2(u)du. \tag{1}$$

This displacement sets up a plasma wave behind the pulse with the energy given by[5] $\epsilon_{pl} = 2\pi h_0^2 n^2 e^2$. One can treat this value as a part of the total energy deposited in the plasma by the pulse. Another part comes from the change of the energy stored into the microfields (we neglect the plasma corrections to the exit velocity and displacement for very short pulses[5]), which are always present due to discrete nature of the plasma. This energy is usually refered to as a correlational energy[6]. Its equilibrium quantity is obtained by averaging the potential energy of two particles, using the two-particle equilibrium correlation function

$$g_{ss'}(r) = 1 - \frac{q_s q_{s'}}{T_e}\frac{\exp(-r/\lambda_d)}{r}, \tag{2}$$

where T_e is the plasma temperature, λ_d is the Debye length, and r is the distance between two particles with charges q_s and $q_{s'}$.

While the laser pulse clearly disturbs the plasma two-particle equilibrium distribution, for a short pulse, each Debye cloud is almost intact right after the pulse, since we assume $\delta\omega > \omega_p$. The change in electron temperature due to collisions with the ions inside the pulse is small, $\delta T_e \ll T_e$. Hence, one can use for the quantities λ_d and T_e their initial values before the pulse. The correlational energy density of the plasma consists of three parts,

$$\epsilon_{corr} = \epsilon_{ee} + \epsilon_{ii} + \epsilon_{ei}, \tag{3}$$

representing contributions from electron-electron, ion-ion, and electron-ion correlations respectively. Since, in our model, the only effect of the laser is an instantaneous displacement of each electron by the distance h_0, it is clear that the interaction with the wave will change only the term ϵ_{ei}. Then its value after the pulse, $\tilde{\epsilon}_{ei}$, is determined by the potential energy of the ion in the displaced cloud of electrons, namely

$$\tilde{\epsilon}_{ei} = n\, q_i \phi(\mathbf{r}_i), \tag{4}$$

where $\phi(\mathbf{r}_i)$ is an electric potential of the cloud at the ion's position and $n = n_e = n_i$ is the plasma density.

Let us choose the coordinate system with its origin at the center of a spherically symmetric electron cloud (see Fig.). Then the radius vector of the ion is $-\mathbf{h}_0$. The electrostatic potential is determined from the solution of the Poisson equation

$$\phi(-\mathbf{h}_0) = \int \frac{n_e(r) q_e}{\mid \mathbf{r} - \mathbf{h}_0 \mid} \, dV, \tag{5}$$

where $n_e(r) = n_e g_{ei}(r)$ is the cloud density. Using an expansion in Legendre polynomials,

$$\frac{1}{\mid \mathbf{r} + \mathbf{h}_0 \mid} = \begin{cases} (1/r) \sum_l (h_0/r)^l P_l(\cos\theta), & if \;\; h_0 < r \\ (1/h_0) \sum_l (r/h_0)^l P_l(\cos\theta), & if \;\; h_0 > r \end{cases}$$

the integration in equation (5) can be carried out easily. We find an energy density difference $\Delta\epsilon_{corr} = 2\pi n^2 e^2 h_0 b_0$, where $b_0 = \mid q_e q_i \mid / T_e$ is the classical distance of closest approach. The ratio of the increase in the correlational energy to the energy of collective plasma oscillations is

$$\frac{\Delta\epsilon_{corr}}{\epsilon_{pl}} = \frac{b_0}{h_0} = 0.96 \cdot 10^{-17} \frac{\delta\omega}{2\pi\eta^2 T_e \,[\mathrm{eV}]}, \tag{6}$$

indicating the relative importance of collective effects in comparison to single particle effects. Depending on the plasma temperature and the pulse duration, this fraction can be either greater or less than unity. This is in contrast to an ideal plasma, not subject to any external fields, where discrete (collisional) effects are always down in magnitude by $\sim n\lambda_d^3$. For example, a very short, $T \sim 0.01$ ps, intense, $\eta \sim 0.2$ electro-magnetic pulse, propagating through the Earth ionosphere plasma ($n \sim 10^5$ cm^{-3}, $T_e \sim 0.01$ eV, $n\lambda_d^3 \sim 1.2 \cdot 10^3$) deposits twice as much energy into the plasma microfields than into plasma oscillations. We show the regions, where $\Delta\epsilon_{corr}/\epsilon_{pl} > 1$, in the Fig. 2a.

The quantity $\Delta\epsilon_{corr}$ complements the picture of discrete losses investigated in Ref. [5]. It is interesting to compare it with the incoherent Compton losses $\omega_p^2 S$ (Eqs. (38), (52) in Ref. [5]). Their ratio scales like:

$$\frac{\Delta\epsilon_{corr}}{\epsilon_{compt}} = \frac{1}{4\pi}\left(\frac{\omega_p}{\bar{\omega}}\right)^2 \frac{mc^2}{\eta^2 T_e} = \frac{6.75 \cdot 10^4}{\eta^2 T_e\,[\mathrm{eV}]}\left(\frac{\omega_p}{\bar{\omega}}\right)^2. \tag{7}$$

In Fig. 2b, we distinguish, by regions in $\eta - T_e$ space, where each of these loss mechanisms dominates.

III. SCATTERING OF AN ELECTRON BY AN ION IN THE PRESENCE OF A LASER PULSE

We will briefly review the interaction of a single electron with a pulse of high intensity, and then we will carry out the analysis in the presence of the ion field. Let us start with equations of motion for an electron in a laser pulse of linear polarization along y axis,

$$\frac{d\mathbf{p}}{dt} = \dot{\mathbf{a}} + \mathbf{v} \times (\mathbf{n} \times \dot{\mathbf{a}}), \qquad \frac{d\gamma}{dt} = \dot{\mathbf{a}} \cdot \mathbf{v}, \tag{8}$$

where the wave vector $\mathbf{n}$ is in the x-direction, the electron momentum is denoted by $\mathbf{p}$, and the velocity by $\mathbf{v}$. The dot stands for differentiation with respect to the phase argument, $t - x$, and γ is the relativistic energy. After some algebra, one can find that the quantity $\gamma - \mathbf{n} \cdot \mathbf{p}$ is a constant of motion, which is equal to $\gamma - \mathbf{n} \cdot \mathbf{p} = 1$ for an electron with zero initial velocity. Using this invariant, we solve for the proper time, $\tau = t - x$, and the displacement

$$h_x(\tau) = \frac{1}{2} \int_0^\tau a^2(u) du, \quad h_y(\tau) = \int_0^\tau a(u) du, \quad h_z = 0. \tag{9}$$

In obtaining (9) we have used conservation of canonical transverse momentum, $\mathbf{n} \times \mathbf{p} = \mathbf{n} \times \mathbf{a}$. For an electron initially at the origin, the kinetic energy is then given by

$$\gamma(\tau) = 1 + \frac{1}{2} a^2(\tau). \tag{10}$$

Let us now address the problem of interaction of an electron with an ion in the presence of a laser pulse. We assume the ion with charge state $Z = 1$ to be stationary at the origin and the electron to have zero velocity and position $\vec{r}_i = (x_i, y_i, z_i)$, when it is hit by the pulse at the point A (see Fig. 3). During the body of the pulse the electron will move along the trajectory AB, at the end of which it will gain the exit velocity $\mathbf{V}_B$ due to interaction with the ion. We now proceed to calculate the exit velocity and position of the electron.

The natural length of this problem is λ, the wavelength of a laser radiation. The Coulomb force on the electron in dimensionless form (we express lengths in terms of λ) is then given by

$$\mathbf{f} = -\zeta \frac{\mathbf{r}}{r^3}, \tag{11}$$

where $\zeta = r_e/\lambda$, with r_e being an electron radius. Now one can write down an equation of motion of an electron in the fields both of the ion and of the laser:

$$\frac{d\mathbf{p}}{dt} = \dot{\mathbf{a}} + \mathbf{f} + \mathbf{v} \times (\mathbf{n} \times \dot{\mathbf{a}}), \quad \frac{d\gamma}{dt} = (\dot{\mathbf{a}} + \mathbf{f}) \cdot \mathbf{v}. \tag{12}$$

By assumption, the ion field is smaller than that of the laser for distances up to 10^{-9} cm, so we expand all dynamical quantities h, γ and so on, about the exact result, Eq. (9) and Eq. (10), the expansion parameter being $f/\omega a$. Accordingly, the first order system of equations to be solved is

$$\frac{dp_1^x}{d\tau} = \frac{1}{2}(t_1 - h_1^x)\ddot{a}^2(t_0 - h_0^x) - \zeta\ f_x(\tau_0)\gamma_0(\tau_0) \tag{13}$$

$$\frac{d\gamma_1}{d\tau} = \frac{1}{2}(t_1 - h_1^x)\ddot{a}^2(t_0 - h_0^x) - \zeta\ [f_x(\tau_0)p_0^x + f_y(\tau_0)p_0^y] \tag{14}$$

$$
\begin{aligned}
\frac{dh_1^x}{d\tau} &= p_1^x & (15)\\
\frac{dt_1}{d\tau} &= \gamma_1 & (16)\\
\frac{dp_1^y}{d\tau} &= (t_1 - h_1^x)\ddot{a}(t_0 - h_0^x) - \zeta\ f_y(\tau_0)\gamma_0(\tau_0) & (17)\\
\frac{dp_1^z}{d\tau} &= -\zeta\ f_z(\tau_0)\gamma_0(\tau_0), & (18)
\end{aligned}
$$

where $\tau_0 = t_0 - h_0^x$ and $\mathbf{f}(\tau_0)$ is a force vector, with electron's coordinates lying on its zero order trajectory (part AB in the Fig. 2)

$$
x(\tau_0) = x_i + h_0^x(\tau_0), \quad y(\tau_0) = y_i + h_0^y(\tau_0), \quad z(\tau_0) = z_i. \tag{19}
$$

We can now subtract Eq. (14) from Eq. (13) to obtain an expression for the $\gamma_1 - p_1^x$. Then we solve for the perturbation of the proper time

$$
\gamma_1 - p_1^x = \zeta \int_0^\tau [f_x(u) - f_y(u)a(u)]du \tag{20}
$$

$$
t_1 - h_1^x = \zeta \int_0^\tau ds \int_0^s [f_x(u) - f_y(u)a(u)]du. \tag{21}
$$

With these expressions for the first order quantities Eq. (13) can be solved explicitly. After integration twice by parts we obtain

$$
\begin{aligned}
p_1^x &= \frac{\zeta}{2}\Big(\dot{a^2}(\tau)\int_0^\tau du \int_0^u [f_x(v) - f_y(v)a(v)]dv - \\
&\quad a^2(\tau)\int_0^\tau [f_x(v) - f_y(v)a(v)]dv\Big) - \zeta\ \int_0^\tau [\frac{a^3(u)}{2} f_y(u) + f_x(u)]du,
\end{aligned}
$$

and similar expressions for p_1^y and p_1^z. Now the exit velocity can be calculated using $\mathbf{V} = \mathbf{p_0} + \mathbf{p_1}(T)/(\gamma_0(T) + \gamma_1(T))$ to give

$$V_c^x = -\zeta \int_0^T [f_x(u) + \frac{1}{2}a^3(u) f_y(u)] du, \tag{22}$$

$$V_c^y = \zeta \int_0^T [a(u) f_x(u) - f_y(u)(1 + \frac{3}{2}a^2(u))] du, \tag{23}$$

$$V_c^z = -\zeta \int_0^T f_z(u)(1 + \frac{1}{2}a^2(u)) du. \tag{24}$$

The subscript "c" (cold) indicates that this solution assumed zero initial electron velocity . The exit displacement, given in Appendix, can be obtained by integrating $\mathbf{p_1}$ with respect to the proper time.

The above equations allow simple interpretation. One can treat the zero order trajectory of the electron (Fig. 3) as a finite–mass string in an external force field, its shape given by parameterized Eqs. (19). This string has a mass tensor varying over u (u being a parameter, characterizing a current position on the string). Then the integrations in Eqs. (22), (23), (24) are, in effect, averaging the force of the ion over the inverse mass tensor of the string, the inverse tensor components given by respective coefficients in these equations:

$$m_{xx}^{-1} = 1, \quad m_{xy}^{-1} = \frac{1}{2}a^3(u), \tag{25}$$

$$m_{yx}^{-1} = a(s), \tag{26}$$

$$m_{yy}^{-1} = \left(1 + \frac{3}{2}a^2(u)\right), \tag{27}$$

$$m_{yz}^{-1} = m_{zy}^{-1} = m_{xz}^{-1} = m_{zx}^{-1} = 0, \tag{28}$$

$$m_{zz}^{-1} = \left(1 + \frac{1}{2}a^2(u)\right). \tag{29}$$

This analogy will help us to understand dependence of $\mathbf{V}_c$ on the electron's initial position $\mathbf{r}_i$. We fixed $y_i = 0$ and $z_i = 0.1$ and plotted $\mathbf{V}_c$ versus x_i (see Figs. 4 and 5). For simplicity, we chose the form of the pulse to be $a(u) = \sin(u), 0 < u < T$, where $T = 6\pi$. The periodic behavior of $\mathbf{V}_c$ versus x_i (Fig. 4) is due to periodic structure of the electron's zero order trajectory in this direction. The spikes on the curves correspond to the minimum approach to the ion. Their amplitude varies with respect to x_i, because the ion divides the string in varying ratio. The spikes are singular as $z \to 0$, because the distance of minimum approach also tends to 0. The plots of $\mathbf{V}_c$ versus y_i (Fig. 5), x_i and z_i being fixed, do not exhibit periodicity, because of the lack of periodicity in the zero order trajectory of the electron in this direction.

The exit velocity and displacement fully describe the scattering in the presence of the pulse. In Sec. IV, we use these quantities as initial conditions for the electron motion in the field of the ion after the pulse to describe the whole scattering process.

IV. SCATTERING MATRIX

In this Section we extend the analysis given in Sec. III to the case of non-zero V_i initial velocity of an electron and obtain scattering matrix. We assume that V_i is nonrelativistic and find first order $O(V_i)$ corrections to our previous results. First, let us modify the quantities describing the electron motion in the wave alone. The invariant of motion $\gamma_0 - p_0^x$ will be

$$\gamma_0 - p_0^x = 1 - V_i^x. \tag{30}$$

The relation between the phase argument of $\mathbf{a}$ and the proper time τ_0 is then

$$t_0 - h_0^x = (1 - V_i^x)\tau_0. \tag{31}$$

The y component of momentum p_0^y is modified in a straightforward way

$$p_0^y = V_i^y + a. \tag{32}$$

Eqs. (30), (31), and (32) lead to the following expression for the electron kinetic energy

$$\gamma_0(\tau_0) = 1 + \frac{1}{2}a^2(\tau_0) + a(\tau_0)V_i^y + \frac{1}{2}a^2(\tau_0)V_i^x. \tag{33}$$

To find $O(V_i)$ corrections to the exit velocity $\mathbf{V_c}$, given in (22), (23), (24), we will now perform the same analysis as in Sec. III with new values of $t_0 - h_0^x$,

p_0^y *etc.*, given in (30), (31), (32) and (33), to arrive at

$$\begin{pmatrix} V^x \\ V^y \\ V^z \end{pmatrix} = \begin{pmatrix} V_c^x \\ V_c^y \\ V_c^z \end{pmatrix} + \begin{pmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{zx} & \alpha_{zy} & \alpha_{zz} \end{pmatrix} \begin{pmatrix} V_i^x \\ V_i^y \\ V_i^z \end{pmatrix} \tag{34}$$

The matrix α_{ij} is given in the Appendix. It does not exhibit any symmetry, because the electron-laser and electron-ion interactions possess different symmetries.

So we know the exit velocity and displacement after the laser pulse. Next we consider the Coulomb scattering in the field of the ion, after the electron has interacted with the pulse, to obtain the final velocity of the electron at infinity. The exit velocity and displacement upon leaving the pulse are now taken as initial conditions in the scattering by the ion. The electron energy and its angular momentum are invariants of motion. At the very moment the electron exits the pulse,

$$L = \mid \mathbf{V} \times \mathbf{R} \mid, \quad E = \frac{1}{2}V^2 - \frac{\zeta}{R}, \tag{35}$$

where R is the radius vector of the electron at that moment (see Appendix).

Let us introduce contraction coefficient

$$k = \frac{\sqrt{2E}}{V}, \tag{36}$$

which is the ratio of the velocity at infinity V_∞ to the exit velocity V. The impact parameter is then

$$b = \frac{\mid \mathbf{V} \times \mathbf{R} \mid}{Vk}. \tag{37}$$

We are left to find the angle χ between $\mathbf{V}_\infty$ and $\mathbf{V} = \mathbf{V}_B$ (see Fig. 3). It can be done most easily in the plane of collision. Using the exact solution for the electron motion, we relate angles ϕ and ϕ_∞ to R and b

$$\cos\phi = \frac{b/R - b_v/b}{\sqrt{1 + (b_v^2/b^2)}}, \quad \cos\phi_\infty = \frac{(b_v/b)}{\sqrt{1 + (b_v^2/b^2)}}, \tag{38}$$

where $b_v = 1/V_\infty^2$. Note that we use $m = c = e = 1$ units in this Section. The angle of rotation χ is then given by

$$\chi = \begin{cases} \phi - \phi_\infty, & \text{if } (\mathbf{V} \cdot \mathbf{R}) < 0 \\ \phi + \phi_\infty - \pi, & \text{if } (\mathbf{V} \cdot \mathbf{R}) > 0 \end{cases}, \tag{39}$$

where the sign of $\mathbf{V} \cdot \mathbf{R}$ determines whether an electron will follow part BC or BD of the trajectory (Fig. 3). The scattering matrix, which relates $\mathbf{V}_\infty$ to $\mathbf{V}$, can be written in the form

$$C_{ik} = k(\cos\chi\delta_{ik} + \sin\chi\epsilon_{ijk}n_j + (1 - \cos\chi)n_i n_k). \tag{40}$$

Its structure is simple: it contracts the absolute value of velocity from V_B to V_∞ and rotates $\mathbf{V}$ in the plane of collision by the angle χ, given in (39). The axis of rotation is parallel to the vector $\mathbf{n} = (\mathbf{R} \times \mathbf{V})/(R\ V)$, which is normal to the plane of collision. It turns out that matrix (40) can be most easily obtained through the quaternion formalism. The quaternion, which rotates a vector around axis $\mathbf{n}$ by an angle χ , is

$$\Lambda = \cos\frac{\chi}{2} + \mathbf{n}\sin\frac{\chi}{2}. \tag{41}$$

The rotation of an arbitrary vector $\mathbf{b}$ can be then written in the form

$$\mathbf{b}' = \Lambda \circ \mathbf{b} \circ \tilde{\Lambda}, \tag{42}$$

where $\circ$ stands for quaternion multiplication and $\tilde{\Lambda}$ is conjugated quaternion. After some algebra Eq. (42) gives matrix C_{ik} (40).

Now we can write the final velocity $\mathbf{V}_\infty$ as a product of two matrices, we have found:

$$V_\infty^l = C_{lj}(\alpha_{jk}V_i^k + V_c^j). \tag{43}$$

Eq. (43) describes the whole scattering process from point A to ∞ (Fig. 3), which includes the interaction with the laser and Coulomb scattering in the field of the ion.

The expression in parentheses is V_B, the exit velocity of the electron due to the interaction with the field of a single ion. It was obtained via linearization around the zero order trajectory of the electron. This description of the scattering process can be incorporated into a collisional operator by averaging over the initial position $\mathbf{r_i}$ of the electron[7], in order to describe the plasma response to several short pulses. The derivation of such a collisional operator is, however, beyond the scope of this paper.

V. DISCUSSION AND CONCLUSIONS

In summary, in this paper we investigated the role of discrete particle effects in the energy absorption from an ultra-short laser pulse of high intensity. It was shown that for very short ($\leq$ 0.09 ps) and moderately intense ($\eta \sim 0.01$) pulses the change in correlational energy of the plasma at 1 eV temperature is greater than the energy stored in plasma oscillations. This dominance of discrete (collisional) over collective effects, even when $n\lambda_d^3 \gg 1$, is opposite to the usual collisional effects, which are always down in magnitude by $n\lambda_d^3$. We note, however, that for very intense pulses, $\eta \sim 1$ with duration ~ 0.1 ps, the energy of plasma oscillations is greater than the correlational energy, according to Eq. (6).

Although collisions due to initial nonrelativistic thermal velocity do not take place during the laser pulse, each electron acquires a relativistic velocity in the laser pulse and moves a certain distance in the fields of the ions. As a result of these background fields, the exit velocity at the end of the pulse is affected. This process can be called an "induced collision" to distinguish from an ordinary Coulomb collision, when the only fields present are those of the particles themselves.

The scattering matrix for the induced collisions, Eqs. (22), (23), (24), and (34), is applicable to electron–ion collisions in the presence of the intense laser pulses. The corrections to the exit velocity and displacement of the electron can also be used to obtain a collisional operator that would describe the influence of several short pulses on the plasma. This is, however, beyond the scope of the present paper.

VI. ACKNOWLEDGEMENTS

The authors are grateful to J.M. Rax and G. Schvets for useful discussions. This work was supported by the United States Department of Energy under Contract No. DE-AC02-CHO-3073.

APPENDIX: CALCULATION OF THE SCATTERING MATRIX

Let us write first order corrections to the exit displacement due to the ion field. They are obtained by integrating $p_1^x(\tau), p_1^y(\tau), p_1^z(\tau)$ respectively, which gives:

$$
\begin{aligned}
h_1^x &= -\zeta \int_0^T a^2(u)du \int_0^u [f_x(s) - f_y(s)a(s)] - \zeta \int_0^T du \\
&\quad \int_0^u [\frac{a^3(s)}{2} f_y(s) + f_x(s)]ds, \\
h_1^y &= -\zeta \int_0^T 2a(s)ds \int_0^s [f_x(s) - f_y(s)a(s)] + \zeta \int_0^T a(s)[f_x(s) \\
&\quad - f_y(s)a(s)]ds - \zeta \int_0^T du \int_0^u f_y(s)(1 + \frac{1}{2}a^2(s))ds, \\
h_1^z &= -\zeta \int_0^T ds \int_0^s f_z(u)(1 + \frac{1}{2}a^2(u))du.
\end{aligned}
$$

Now we will determine coefficients of the α_{ij} matrix (34).

It is easy to find, using (30), (31), (32), (33), that equations of trajectory with the first order $O(V_i)$ corrections can be written as

$$
\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} + \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \begin{pmatrix} V_i^x \\ V_i^y \\ V_i^z \end{pmatrix} + \begin{pmatrix} h_0^x \\ h_0^y \\ 0 \end{pmatrix}
$$

where the σ_{ij} matrix coefficients are given by

$$\sigma_{xx}(\tau) = \textstyle\int_0^\tau (1 + a^2(u))du, \;\; \sigma_{xy}(\tau) = \int_0^\tau a(u)du,$$

$$\sigma_{yx}(\tau) = \textstyle\int_0^\tau a(u)du, \;\; \sigma_{yy}(\tau) = \sigma_{zz}(\tau) = \tau.$$

And the zero order trajectory is given by Eq. (9).

We will denote the electron's radius vector, following zero order trajectory, by

$$r(\tau) = [(x_i + h_0^x(\tau))^2 + (y_i + h_0^y(\tau))^2 + z_i^2]^{3/2}.$$

Then after analysis similar to that of in Sec. III the α_{ij} matrix components are:

$$
\begin{aligned}
\alpha_{xx} &= 1-\zeta\int_0^T [f_x(s)(1+a^2(s))+\frac{1}{2}a^3(s)f_y(s)-\frac{\sigma_{xx}(s)+\frac{1}{2}\sigma_{yx}(s)}{r^3(s)}]ds,\\
\alpha_{xy} &= -\zeta\int_0^T [a(s)f_x(s)+\frac{1}{2}a^2(s)f_y(s)+\frac{\sigma_{xy}(s)+\frac{1}{2}a^3(s)\sigma_{yy}(s)}{r^3(s)}]ds,\\
\alpha_{xz} &= -\zeta\int_0^T \frac{1}{2}a^2(s)f_z(s)ds,\\
\alpha_{yx} &= -\zeta\int_0^T a(s)ds\int_0^s [f_x(s)-a(s)f_y(s)+\frac{\sigma_{xx}(s)-a(s)\sigma_{yx}(s)}{r^3(s)}]ds\\
&\quad -\zeta\int_0^T \frac{1+\frac{1}{2}a^2(s)}{r^3(s)}ds\int_0^s a(u)du-\zeta\int_0^T f_y(s)(1+a^2(s))ds,\\
\alpha_{yy} &= 1-\zeta\int_0^T a(s)ds\int_0^s [\frac{\sigma_{xy}(u)-a(u)\sigma_{yy}(u)}{r^3(u)}-f_y(u)]du\\
&\quad -\zeta\int_0^T [a(s)f_y(s)+\sigma_{yy}(s)(1+\frac{1}{2}a^2(s))ds,\\
\alpha_{yz} &= -\zeta\int_0^T a(s)ds\int_0^s f_y(u)du,\\
\alpha_{zx} &= -\zeta\int_0^T f_z(s)(1+a^2(s))ds,\\
\alpha_{zy} &= -\zeta\int_0^T f_z(s)a(s)ds,\\
\alpha_{zz} &= 1-\zeta\int_0^T \frac{\sigma_{zz}(s)(1+\frac{1}{2}a^2(s))}{r^3(s)}ds.
\end{aligned}
$$

REFERENCES

[1] P.Sprangle, E. Esarey, and A. Ting, Phys. Rev. Lett. **64**, 2011 (1990); A. Ting, E.Esarey, and P.Sprangle, Phys. Fluids B **2**, 1390 (1990); J.M. Rax and N.J. Fisch, Phys. Fluids B **4** (5), 1323 (1992).

[2] J.M. Dawson and C. Oberman, Phys. Fluids **5**, 517 (1962); V.P. Silin, Sov. Phis. JETP **20**, 1510 (1965); A.B. Langdon, Phys. Rev. Lett. **44**, 575 (1980); W.L. Kruer, Comments Plasma Phys. Cont. Fusion **6**, 167 (1981); S. Kato, R. Kawakami and K. Mima, Physical Review A **43**, 5560 (1991); C.D. Decker, W.B. Mori, J.M. Dawson, T. Katsouleas, Phys. Plasmas **1** (12), 4043 (1994).

[3] M.Pessot, J. Squier, G. Mourou, and D.J. Harter, Opt. Lett. **14**, 797, (1989);M.Pessot, J. Squier, P. Bado, G. Mourou, and D.J. Harter, IEEE J. Quantum Electron. **QE-25**, 61 (1989).

[4] P. Kaw and J. Dawson, Phys. Fluids, **13**, 472 (1969).

[5] J.M. Rax and N.J. Fisch, Phys. Fluids B **4** (5), 1323 (1992).

[6] L.D. Landau and E.M. Lifshits, Statistical Physics, vol.**5**, p.243, (1980).

[7] Note, that one has to perform averaging over x_i, y_i, z_i, while in the case of a Coulomb collisions it is sufficient to average only over the impact parameter.

FIGURES

Fig. 1 Displacement of the Debye cloud due to interaction with the laser pulse.

Fig. 2 Regions of relative importance of $\Delta\epsilon_{corr}$ in comparison with: a) energy of collective plasma oscillations and b) spontaneous Compton losses. Curves $1, 2$, and 3 correspond to the electron temperature, T_e, 1, 0.1 and 0.01 eV respectively. The pulse duration $\delta\omega^{-1}$ is given with respect to $\delta\omega_0^{-1} = 2\pi \cdot 10^{-13}$. The region above each curve in Fig. (a) corresponds to $\Delta\epsilon_{corr}/\epsilon_{pl} > 1$, while in Fig (b) it corresponds to $\Delta\epsilon_{corr}/\epsilon_{compt} < 1$.

Fig. 3 Electron moves along the trajectory AE in the field of the ion (at the origin), when it is hit by the pulse at the point A. It is forced to move along the new trajectory AB, which is almost the same as its trajectory in the field of the pulse alone (drifting figure eight). At the point B it leaves the pulse with the exit velocity V_B and starts to move along hyperbola CD, BF being its axis of symmetry. Note, that trajectories AE, AB and CD do not necessarily lie in the same plane.

Fig. 4 Components of the exit velocity , V_c^x, V_c^y, V_c^z, respectively versus initial x_i position of the electron, with $y_i = 0, z_i = 0.1$. The magnitude of velocity is presented in terms of $c = 3 \cdot 10^{10}$ cm – velocity of light, and all lengths are measured in $\lambda = 10^{-5}$ cm – wavelength of radiation. The form of the pulse is chosen to be $a(u) = \sin(u), 0 < u < T$, where $T = 6\pi$. As $z \to 0$ the spikes on all graphs tend to $\pm\infty$, forming discontinuities. This corresponds to electron crossing the ion at some point on its zero order trajectory. The periodic behaviour of all plots is due to periodicity of the zero order trajectory of the electron in x direction.

Fig. 5 Components of the exit velocity, V_c^x, V_c^y, V_c^z, respectively, versus initial y_i position of the electron, with $x_i = 0, z_i = 0.1$. Plots do not exhibit any periodicity due to the lack of periodicity in the zero order trajectory of the electron in y direction.

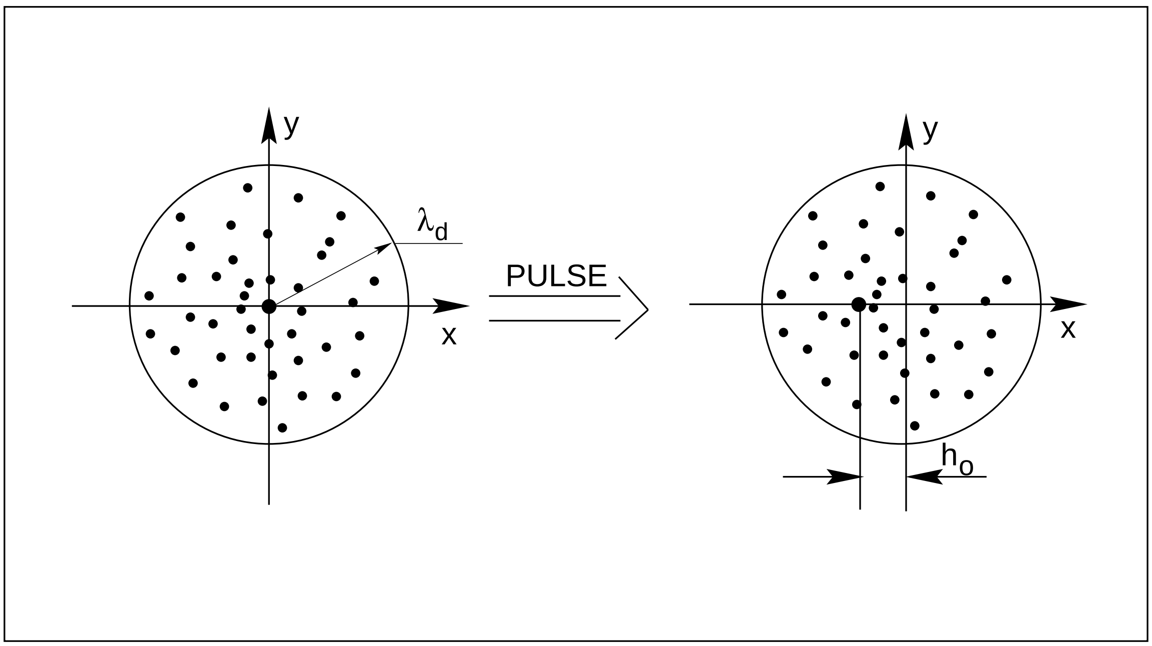


Fig. 1

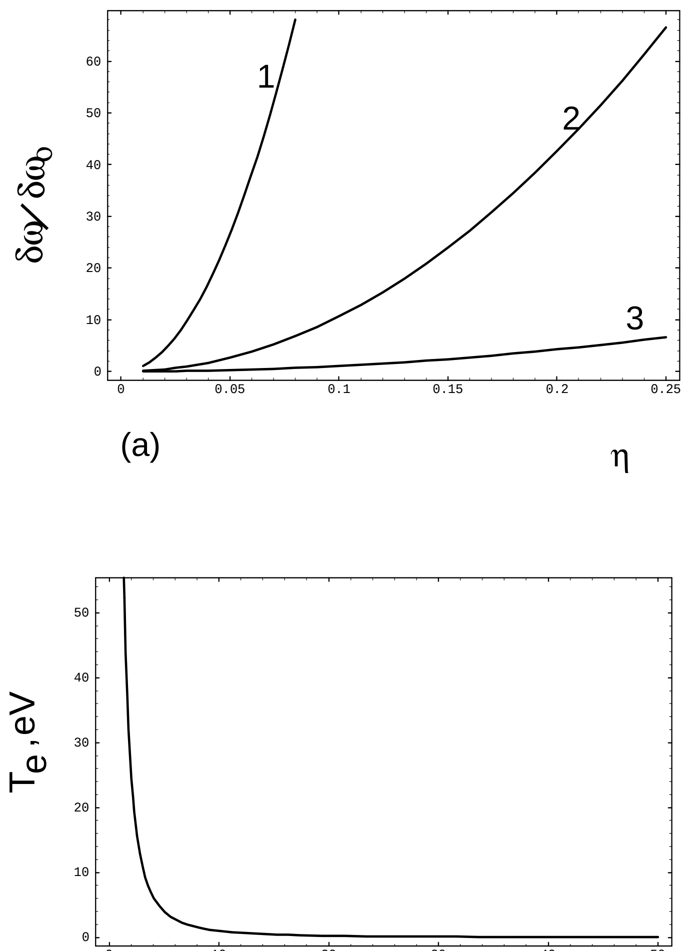


Fig. 2

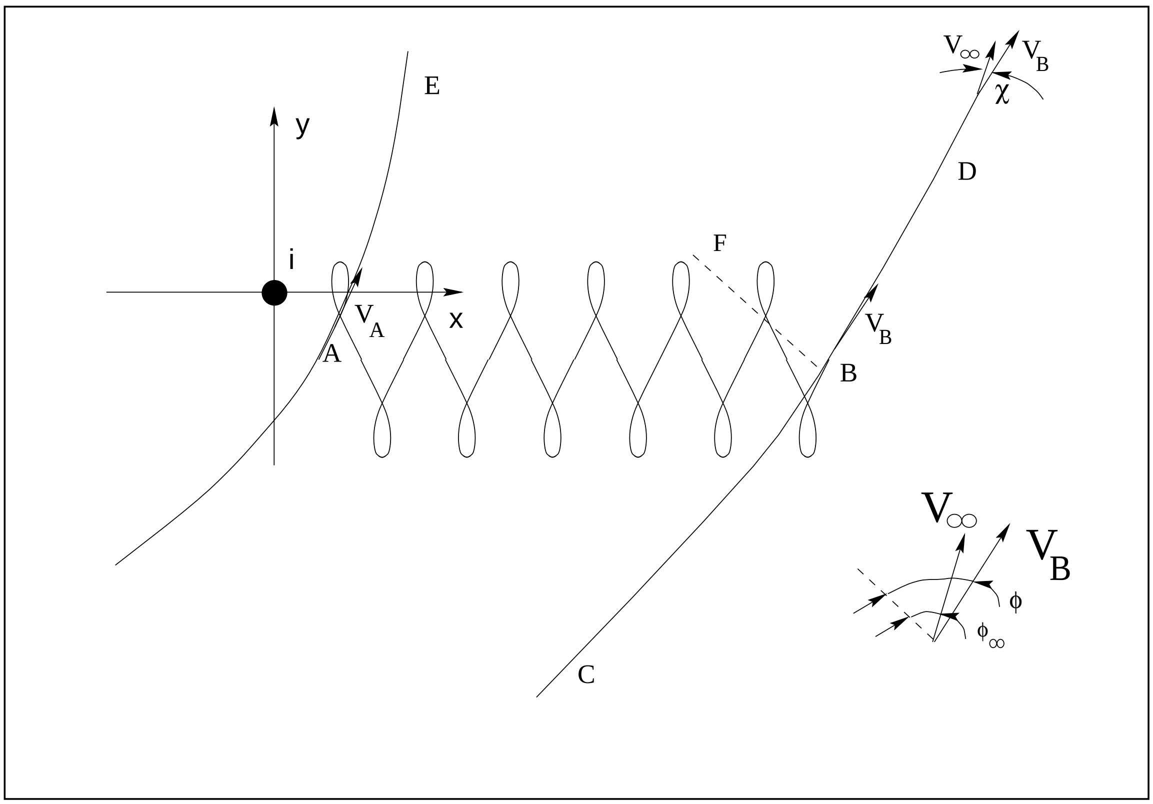
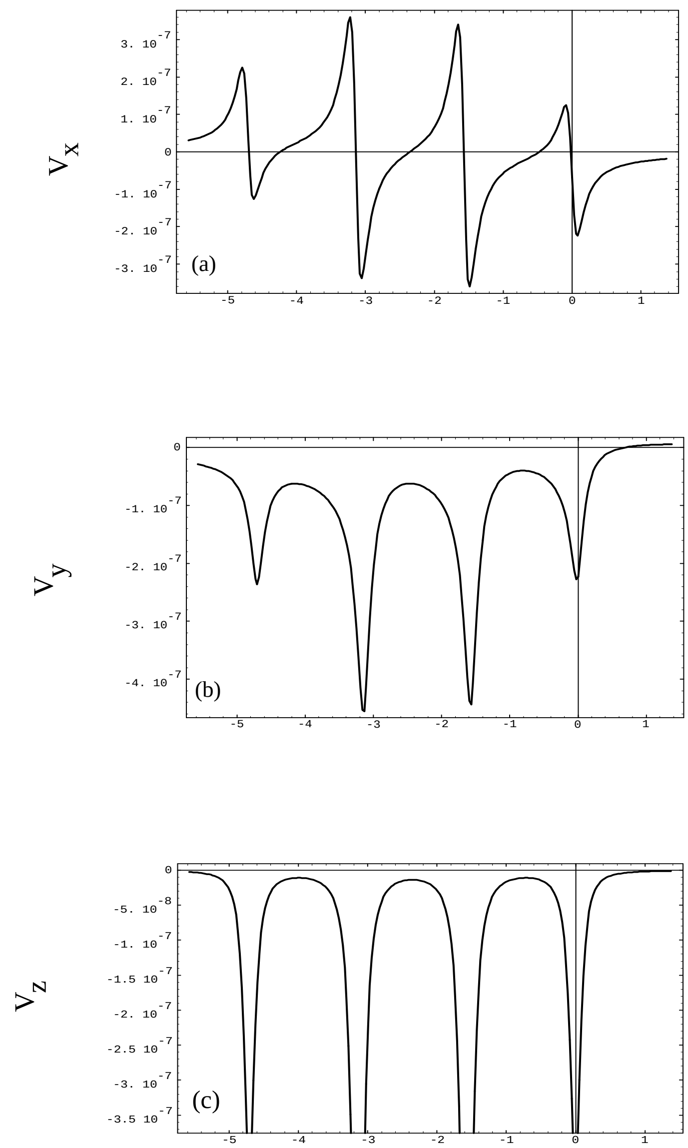
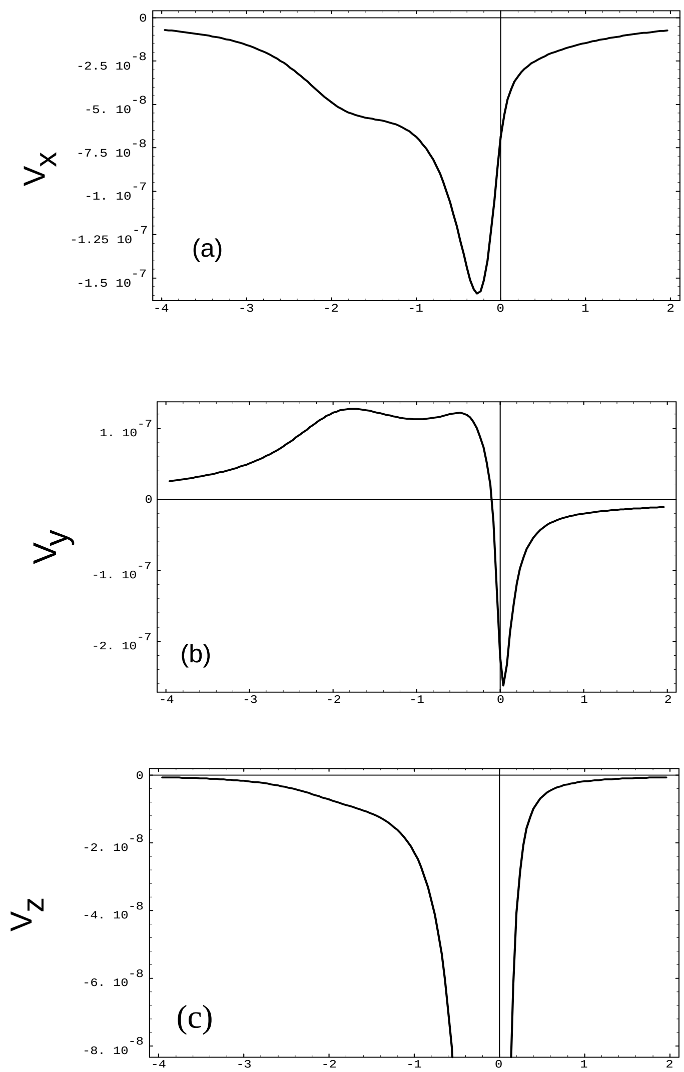


Fig. 3



V_x
3. 10^{-7}
2. 10^{-7}
1. 10^{-7}
0
-1. 10^{-7}
-2. 10^{-7}
-3. 10^{-7}
(a)
-5
-4
-3
-2
-1
0
1
V_y
0
-1. 10^{-7}
-2. 10^{-7}
-3. 10^{-7}
-4. 10^{-7}
(b)
-5
-4
-3
-2
-1
0
1
V_z
0
-5. 10^{-8}
-1. 10^{-7}
-1.5 10^{-7}
-2. 10^{-7}
-2.5 10^{-7}
-3. 10^{-7}
-3.5 10^{-7}
(c)
-5
-4
-3
-2
-1
0
1

Fig. 4



v_x
0
-2.5 10^-8
-5. 10^-8
-7.5 10^-8
-1. 10^-7
-1.25 10^-7
-1.5 10^-7
(a)
-4 -3 -2 -1 0 1 2
v_y
1. 10^-7
0
-1. 10^-7
-2. 10^-7
(b)
-4 -3 -2 -1 0 1 2
v_z
0
-2. 10^-8
-4. 10^-8
-6. 10^-8
-8. 10^-8
(c)
-4 -3 -2 -1 0 1 2

Fig. 5