Spatial resolution is an important issue for neutral-beam-based optical diagnostics in tokamak plasmas, such as charge-exchange-recombination spectroscopy (measuring $T_i$ and $v_\phi$) and motional Stark effect (measuring $B_p$). The key geometrical constraint is that the optical sightlines of these diagnostics must be as nearly tangent as possible to magnetic surfaces at the point where they cross the path of the neutral beam. This minimizes the effect of the width of the neutral beam on the spatial resolution of the diagnostic in the direction perpendicular to the flux surfaces. (See figure 1.)
FIG. 1. Optical views of a charge-exchange recombination spectroscopy or motional Stark effect pinhole camera crossing the path of an atomic neutral beam, in the midplane of a tokamak plasma.

It is desirable, in general, to optimize the spatial resolution of these diagnostics near the magnetic axis of the plasma, where peak values of ion temperature and toroidal rotation are observed and where the poloidal field reverses sign. At the same time it is also desirable to optimize resolution near the plasma edge, where strong pressure and flow gradients can exist in the H-mode, and bootstrap currents can be localized. In this note, we derive the optimal geometry for a single "pinhole camera" diagnostic system, based on these considerations.

Figure 2 establishes the geometry for solution of the problem. We assume that the magnetic axis (or other interesting interior point in the plasma) is located at radius $R_X$ measured from the axis of symmetry of the tokamak (point A), and the plasma edge is located at radius $R_E$. The center
of the finite-width neutral beam is assumed tangent to a toroidal circle of radius $R_t$. It is convenient to orient the geometric axes such that the neutral beam travels in the $x$ direction, at $y = R_t$. The $(x, y)$ location of the optimal pinhole camera $(x_c, y_c$, at point D) is given by the intersection of the two sightlines shown. These sightlines are tangent to the circles of radius $R_x$ and $R_e$ at the points where the circles are crossed by the neutral beam.

In order to solve for $(x_c, y_c)$, we construct the circle which passes through the axis of symmetry of the tokamak (point A), the neutral-beam crossing at $R = R_x$ (point B), the neutral-beam crossing at $R = R_e$ (point C) and the optimal camera location (point D). This is possible, of course, because right triangles inscribe semi-circles. The $x$ location of the center of this circle, point E, is clearly the same as the $x$ location of the center of the horizontal line-segment BC.

$$x_E = - \left[ \frac{(R_x^2 - R_t^2)^{1/2} + (R_e^2 - R_t^2)^{1/2}}{2} \right] / 2$$
The equation for the perpendicular bisector of the line segment AB is given by the point-slope formula

\[ x + (R_x^2 - R_t^2)^{1/2}/2 = (y - R_t/2) R_t / (R_x^2 - R_t^2)^{1/2}. \]

Since this line passes through the point E, the y location of E is given by

\[ y_E = \frac{R_t - (R_e^2 - R_t^2)^{1/2}(R_x^2 - R_t^2)^{1/2}}{R_t} / 2 \]

By construction, the optimal camera location is then

\[ x_c = 2 x_E = -(R_x^2 - R_t^2)^{1/2} - (R_e^2 - R_t^2)^{1/2} \]

\[ y_c = 2 y_E = R_t - (R_x^2 - R_t^2)^{1/2}(R_e^2 - R_t^2)^{1/2} / R_t \]

yielding the simple result that
\[ R_c = (x_c^2 + y_c^2)^{1/2} = R_e R_x / R_t \]

(This final result can also be obtained by purely geometrical reasoning.)

Devices with more tangentially aimed neutral beams require optical systems with equivalent pinhole camera geometry closer to the plasma edge. Since the ratio \( R_x / R_t \) tends to decrease for double-pass tangential injection into higher-aspect-ratio plasmas, \( R_c / R_e \) decreases at higher aspect ratio. This simple geometrical result may have a significant impact on the configuration of neutral-beam-based optical diagnostics for the steady-state Tokamak Physics Experiment, TPX, which has both a relatively high aspect ratio, to optimize the generation of bootstrap current, and also highly tangential neutral beams, to maximize beam current drive and provide a wide density range for current-drive experiments.

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